Dynamic stochastic EPEC model for competition of dominant producers in generation expansion planning

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Abstract—This paper aims to present a dynamic stochastic equilibrium problem with equilibrium constraints (DSEPEC) model to investigate the generation capacity expansion at a certain time horizon and the presence of dominant producers. The DSEPEC model is proposed while there is an uncertainty in the predicted demand, and it is modeled based on discrete Markov model. Each dominant producer is modeled by a bi-level optimization problem so that the first level and the second level provide models for the investment and operation decisions, respectively. The supply function equilibrium (SFE) is used for short-term electricity market. Then, each bi-level model is converted to dynamic stochastic mathematical problem with equilibrium constraints (DSMPEC). To convert DSEPEC to an auxiliary MILP problem, Karush-Kuhn-Tucker (KKT) conditions as well as primal–dual transformation. A sample two-buses power network is employed for simulation and necessary analysis to confirm the efficiency of the proposed framework.

Keywords—Generation expansion, EPEC, MPEC, multi-stage planning, Dynamic stochastic model

I. INTRODUCTION

The security of supply is the most important feature of the operating system that should be addressed in the short and long periods of time [1-2]. It is worth mentioning that generation expansion must be coordinated with the growth of the demand to keep the balance between electricity generation and energy consumption in the long duration [3-4]. The generation expansion has become an intricate issue by entering the power generation industry into the competitive space which is implemented in order to improve the economic efficiency [5-7]. In limited competition space (incomplete), the independent system operator must have profound information as regards models and computational tools as a means to study the expansion behavior of the power generation sector under presence of uncertainty in power systems [8-10]. It is an essential point to apply appropriate model in order to understand the generation expansion and to be able to investigate the influence of various markets on the investment sectors; consequently, scientists have been addressing a variety of generation expansion models to prevail investment issues.

In [11] a bi-level Cournot model has been presented for generation expansion planning. Correspondingly, the effect of transmission congestion is provided by the Cournot model in [12]. In [13], a probabilistic dynamic programming model has been addressed under presence of demand uncertainty in order to overcome the investment problem. In [14], a stochastic bi-level model is presented as a means to investigate Dominant behavior of producers so that uncertainties are associated to demand and rival offers. In [15], a static model of a dominant producer vision (price maker) has been proposed. In [16], the problem considered in [9] is solved by Bender's decomposition in the presence of uncertainty in demand, non-strategic generation companies while the nature of planning is static. Reference [17] presents a bi-level multi-period framework which encompasses generation expansion planning (GEP) from a price maker perspective. A hybrid DP/GAME framework based on Cournot game is suggested in [18] as a solution for the generation expansion planning problem. In [19], expansion planning is undertaken for a set of non-strategic generation companies in the liberalized power market. A Conjecture price method is used in the second level of the aforementioned market. In [20-21], the investment of dominant producers is solved by EPEEC model without considering the uncertainty for one year. In addition, the strategic offers of producers are based on stepwise supply function. Multi stage planning by thri-level level model is done in [22]. In addition, to model operation mo

In this paper, the GEP is developed in such a way that the strategy of all investors is modeled in a mathematical model form consisting of several dynamic stochastic MPEC (DSMPEC) which is called the dynamic stochastic EPEC (DSEPEC) model. The DSEPEC model is tested in dynamic approach under uncertainty in the demand growth which is modeled by discrete Markov model. The DSMPEC obviously considers offers from the stepwise supply function, which is more line with reality of power markets in comparison with other related models for instance Bertrand, Cournot, or conjectural variations.

Following the introduction, the rest of this paper will be presented as the following sections. In Section II mathematical formulation will be characterized. Different scenarios will be considered in Section III and a case study will be provided in Section IV, where the efficiency of the proposed is examined. Finally, the last section provides some relevant and beneficial conclusions.
II. MATHEMATICAL FORMULATION

The mathematical formulation of the proposed model will be introduced in the following Subsections. A variety of indexes are used in this model, where \( w \) is an index for the scenario, \( r/r' \) show the indexes for years, \( t \) shows the index for the demand blocks, \( y \) is an index for the producers, \( i/k \) are indexes for the new and/or the existing generation unit of dominant producers, \( d \) is an index for demand. In addition, \( f \) represents discount rate and \( \chi_{yri} \) is the capacity investment of new unit \( i \) of the dominant producer \( y \) in year \( r \) (MW). In addition, \( \chi^{yr} _{rri} \) is the available capacity of new unit \( i \) of dominant producer \( y \) in year \( r' \), in years after the installation in the year \( r \) (MW).

A. The stochastic Bi-level model of each dominant producer

Firstly, the issue of each dominant producer to find optimized decisions (investment and offers) is formulated by a stochastic bi-level model. The first level present investment issue of a dominant producer \( y \) with a view to maximize the total profit present value of the investment which is expressed in (1). \( K_{yri} \) and \( C^E _i / C^S _i \) are annual investment cost of new generating unit \( i \) and Marginal cost of new/existing unit of dominant producer \( y \) (€/MWh). \( \lambda _{yrtnw} \) is market price (locational marginal price).

\[
\min \sum_{r} \left( \frac{1}{1+f} \right)^{r'} \left( \sum_{i} K_{yri} \chi_{yri} + \sum_{i} \chi_{rri} - \sum_{w} \sum_{r} W_{wr} \right) \left[ \sum_{i} P^S_{yri} \left( \lambda _{yrtnw} (i \in n | w) \right) - C^S _i \right] + \sum_{i} P^S_{yriw} \left( \lambda _{yrtnw} (i \in n | w) \right) - C^S _i \right] \right) + \sum_{i} P^E_{yriw} \left( \lambda _{yrtnw} (i \in n | w) \right) - C^E _k \right] \right) \forall y \right)
\]

(1)

The constraints in the first level include the bounds on investment options, limit on investment budget (2), minimum available capacity imposed by the market regulator (3) and the non-negativity of dominant offers. \( K_{yri}^{max} \) is the value of available budget for the investment of dominant producer \( y \) (€/MWh). Furthermore, \( P_{rdw}^{max} \) is the maximum load of demand \( d \) in block \( t \) (MW). \( A \) is supply security factor.

\[
\sum_{r} \left( \frac{1}{1+f} \right)^{r'} \left( \sum_{i} K_{yri} \chi_{yri} + \sum_{i} \chi_{rri} \right) \leq K_{yri}^{max} \forall y \right)
\]

(2)

\[
\left( \sum_{i} Y_{yri} + \sum_{i} \frac{r}{k} \chi^{yr} _{rri} + \sum_{i} p^{E^{max}} \right) \geq A \times \sum_{w} Q_{wr} P_{D^{max}} \forall r, \forall w, \forall y
\]

(3)

In addition, as an obvious result of the dynamic nature of the proposed model, dynamic constraints are envisaged in the first level. Dynamic constraints values on the investment decision variables are represented in constraint (Eqs.4 - 6);

\[
X_{yri}^{T}, = X_{yri} \forall r, \forall i, \forall y
\]

(4)

\[
C_{yri}^{r}, = C_{yri} \forall r, \forall i, \forall y
\]

(5)

\[
O_{yrtnw}^{Sr}, \geq 0 \forall y, \forall r, \forall \forall \forall \forall
\]

(6)

The outputs of the first level problem are consisd of the investment decisions of the producers and the offers that they provide for the spot market. Market-clearing is expressed in the second level problem. An optimization problem is presented as the clearing of the market for various operating conditions, where the priority is to maximize the social welfare. Objective function of second level is given in Eq.7. \( U_{id}^{PD} \) is Price bid of each demand (€/MWh).

\[
\min \sum_{r} \sum_{i} \sum_{w} \sum_{r} O_{yrtnw}^{Sr} - \sum_{r} \sum_{i} \sum_{w} \sum_{r} ^{PD} U_{id}^{PD} \forall r, \forall t, \forall w
\]

(7)

The constraints of second level involve DC power flow equations, transmission network limitations, the limits of units’ capacity and uncertainty in demand. In this model, loads are envisaged to be flexible to the prices. Hence, they are not essentially supplied at their highest levels. The outputs of the second level model include the market prices (i.e., dual variables related to the constraints of power balance), which are fed back to the first level problem. The value of dynamic constraints on the production decision variables are illustrated in (8).

\[
0 \leq p_{yrtnw}^{Sr} \leq X_{yrtnw}^{T}, \forall y, \forall r, r', \forall \forall \forall \forall \forall \forall
\]

(8)

B. The DSMPEC of each producer \( y \) for DSEPEC model

The bi-level problem for producer \( y \) can be transformed into a one level problem (DSMPEC) by imposing Primal-dual transformation to the second level problems. Primal-dual
transformation encompasses some constraints, i.e., primal and dual constraints as well as the strong duality equality.

In view of the product of continuous variables, the strong duality equalities have nature of the nonlinearities; because of this, the aforementioned equalities cannot be easily linearized. The strong duality equality is obtained through using the primal-dual transformation. This equality is equivalent to the set of complementarity conditions as inequality constraints which can be obtained by the KKT conditions [22].

In addition, complementarity conditions are nonlinear and highly non-convex. It should be noted that for linearizing, each complementarity condition of the following form

\[ a \geq 0, b \geq 0, a \leq \tau M, b \leq (1 - \tau)M, \tau \in [0,1] \]

so that M is envisaged as a large enough constant.

C. Dynamic stochastic EPEC (DSEPEC)

The joint consideration of all DSMPECs constitutes a DSEPEC. In this paper, total profit of all producers was selected as objective function. The summation of Eq.1 for all producers presents the minus total profit of all producers, which is presented in Eq.9.

\[
\min \sum_{y} \left( \frac{1}{1 + f} \left( \sum_{i} K_{yri} x_{yri} + \sum_{r} x_{yrr} \right) - \sum_{w} \sum_{r} \right) \left[ \sum_{i} p^{S}_{yritw} \left( \lambda_{rtn}(i \in n)w - C^{S}_{i} \right) \right] + \sum_{i} p^{S}_{yritw} \left( \lambda_{rtn}(i \in n)w - C^{S}_{i} \right) + \sum_{k} p^{E}_{yrtrk} \left( \lambda_{rtn}(k \in n)w - C^{E}_{k} \right)
\]

In addition to total profit of all producers, annual true social welfare (where production costs of units are considered), Annual social welfare (where dominant offers of units are considered) as well as Minus payment of the demands can be considered as the other objective functions of equilibria [20].

D. Converting DSEPEC to one level model

Owing to the fact that DSMPECs are nonlinear, the primal-dual transformation cannot be directly used as a means to formulate the optimality conditions of all producers. Consequently, by enforcing KKT condition to all DSMPECs, the optimality conditions for the DSEPEC are obtained [20]. The optimality conditions related to the DSEPEC encompass Primal equality constraints of DSMPECs, Equality constraints obtained from differentiating the corresponding Lagrangian related to the DSMPECs with respect to the variables as well as Complementarity conditions associated with the inequality constraints of DSMPECs.

III. CONSIDERING THE UNCERTAINTY IN DEMAND

To reach the more reliable results in planning power network, importance of considering uncertainties is inevitable. In the proposed model, Markov chains is used with a view to model demand uncertainty. The demand growth is envisaged to be 10% and 8% so that related probabilities are 60% and 40%, respectively in any case scenario for each year. Figure 1 presents the uncertainty of Markov chain related to demand growth. In this paper, DSEPEC includes 16 scenarios for 5 years.

IV. CASE STUDY

A network including two buses is considered as a case study depicted in Fig. 2. The information related to candidate unites, existing units as well as demands can be found in [21]. A five years horizon is considered for the planning, where each year is consists of four different demand blocks. The value of annual demand growth is presumed to be 10% and the value of annual discount rate is presumed to be 8.7%, respectively. In addition, 75 million euros is considered for investment budget over the planning period.

This DSEPEC model is solved by means of Solver XPRESS software GAMS. The simulation results of this model in case of static is compared with those of EPEC model.
presented in ref. [20] in order to validate high performance of the proposed model. According to the GEP results of [20], the total profit and total capacity constructed through the planning period for the case 1 of table 5 have been obtained as 11.83 M€ and 200 MW, respectively. Also, the annual true social welfare (ATSW) is obtained as 20.13 M€ in this case. After validating the simulation results in this paper in the case of static approach, that model is extended to DSEPEC so that uncertainties associated with demand growth are considered. In addition, different options are considered for investment.

Table I shows the energy production of dominant producers in the planning period. The first Column in Table I refers to the planning year. The productions for each producer during the planning period are provided in the next columns. The last column provides the total production under planning period. In addition, the value of total production in the planning period is 2634 MWh so that the share of dominant producers in this production is 29.51%, 34.15% and 36.34%, respectively.

<table>
<thead>
<tr>
<th>Year</th>
<th>Producer 1 (MWh)</th>
<th>Producer 2 (MWh)</th>
<th>Producer 3 (MWh)</th>
<th>Total Production (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>278.141</td>
<td>1481.568</td>
<td>521.395</td>
<td>2281.104</td>
</tr>
<tr>
<td>2</td>
<td>1651.628</td>
<td>1652.997</td>
<td>1856.413</td>
<td>5161.038</td>
</tr>
<tr>
<td>3</td>
<td>1892.554</td>
<td>1860.255</td>
<td>2078.666</td>
<td>5831.475</td>
</tr>
<tr>
<td>4</td>
<td>1477.468</td>
<td>1514.917</td>
<td>2389.141</td>
<td>5381.525</td>
</tr>
<tr>
<td>5</td>
<td>2440.261</td>
<td>2449.193</td>
<td>2689.395</td>
<td>7578.849</td>
</tr>
<tr>
<td>Total</td>
<td>7740.052</td>
<td>8958.930</td>
<td>9535.010</td>
<td>26233.991</td>
</tr>
</tbody>
</table>

Table II shows the generation expansion planning results for Tri-competition. The first column of Table II is referring to the planning year and the second column presents the total constructed unit and the base technology in the parenthesis. The average market prices are provided in the third column. In addition, the profit for each producer for the duration of the planning period and their investment are provided in the next column. The total profit (M€) during the planning period is 70.71 so that the share of dominant producers in this profit is 12.87%, 57.08% and 30.07%, respectively. Also, Social welfare is 127.98 in the planning period.

Figure 3 shows the changes in social welfare compared to demand security coefficient changes. It can be figured out that the level of social welfare increases with the increase in demand security coefficient.

Figure 4 shows the investment by dominant producers compared to demand security coefficient changes in the planning period. As depicted the investment increase and occurs in earlier years with increase in demand security coefficient while all investments are in peak technology.

Figure 5 shows the changes in energy production compared to demand security coefficient changes. The level of energy production increases with increase in demand security coefficient.

V. CONCLUSION

A dynamic stochastic EPEC model has been expressed to study interactions among dominant producers in generation capacity expansion. In this paper, the dynamic nature of the investment decisions has been considered in a network-
constrained electricity pool to find expansion planning equilibria.

Furthermore, various scenarios are considered to model uncertainties related to the demand growth. Discrete Markov model is presented to take into account uncertainties. According to the simulation results, level of social welfare increases with increase in demand security coefficient. Also, the investment increases and occurs in earlier years with an increase in demand security coefficient while all investments are in peak technology. In addition, the level of energy production increases with the demand security coefficient increase. As a future work, there is a need to pay special attention to effect of investment incentives upon function of dominant producers in generation investment problem. In addition to generation expansion, transmission expansion planning can be considered to this model although that is an intricate problem. Also, considering impacts of availability of the gas transmission network in the proposed model is of high importance.

**REFERENCES**


**TABLE II. GENERATION EXPANSION RESULTS FOR TRI-COMPETITION**

<table>
<thead>
<tr>
<th>Year</th>
<th>Added capacity (base) MW</th>
<th>Average price (€/MWh)</th>
<th>Total capacity added (base) MW</th>
<th>Total capacity added (base) MW</th>
<th>Net profit (M€)</th>
<th>Net profit (M€)</th>
<th>Net profit (M€)</th>
<th>Total profit (M€)</th>
<th>Social welfare (M€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300(0)</td>
<td>20.125</td>
<td>0.19</td>
<td>0.19</td>
<td>7.58</td>
<td>3.42</td>
<td>13.03</td>
<td>12.73</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>20.125</td>
<td>0.19</td>
<td>0.19</td>
<td>9.6</td>
<td>3.14</td>
<td>14.78</td>
<td>28.49</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>20.125</td>
<td>0.19</td>
<td>0.19</td>
<td>9.82</td>
<td>2.90</td>
<td>14.73</td>
<td>29.57</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>200(0)</td>
<td>20.125</td>
<td>0.19</td>
<td>0.19</td>
<td>7.31</td>
<td>4.62</td>
<td>13.50</td>
<td>24.66</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>20.125</td>
<td>0.19</td>
<td>0.19</td>
<td>6.05</td>
<td>7.18</td>
<td>14.67</td>
<td>32.53</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>500(0)</td>
<td>20.125</td>
<td>0.19</td>
<td>0.19</td>
<td>40.36</td>
<td>21.26</td>
<td>70.71</td>
<td>127.98</td>
<td></td>
</tr>
</tbody>
</table>