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Highlights

- We develop a 2-period model where manufacturer and the third-party sell durables.
- The third-party's profitability is affected by degree of manufacturer's upgrading.
- We derive optimal conditions for manufacturer to release an upgraded product.
- We derive optimal conditions for a third-party to enter a secondary market.

The impact of product upgrading on the decision of entrance to a secondary market

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Abstract

In this paper, we examine the impact of manufacturers upgrading strategy of durable products on the decision of third-party entrant in a secondary market. To do so, we develop a two-period model in which a monopolistic manufacturer sells new durable products directly to end consumers in both periods, while a third-party entrant operates a reverse channel selling used products in the secondary market. The manufacturer releases an upgraded product (i.e., one that is technologically superior to the version introduced in the first period). We derive conditions under which it is optimal (1) for the manufacture to release an upgraded product in the second period and (2) for a third party entrant to enter a secondary market. We also find, through numerical analysis, that when upgrades are typically small or moderate, the upgrading of new products can increase a third party entrant's profitability in the secondary market but it does not benefit the third party entrant when upgrades are typically large.

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1. Introduction

Upgrading is the process of replacing a product with a new higher quality version of the same product (e.g., one with a stronger function or higher performance, Fudenberg and Tirole (1998); Martin (2011); Anton and Biglaiser (2013)). Frequent introduction of upgraded products has been recognized as an important means by which firms continuously renew themselves in order to survive and prosper in a rapidly changing business environment (Koufteros and Marcoulides (2006); Anton and Biglaiser (2013))and is particularly noticeable in durable goods industries. For example, a new mobile phone model is introduced into the market with innovative agenda, camera, or Internet functions every month (Martin (2011)), while in the automobile industry, car makers introduce new components with every new model yearly. Similar patterns can be observed in other industries, including PCs, household appliances (e.g., washing machines, dryers, and vacuum cleaners), CRT devices (e.g., TV sets and monitors), and consumer electronics (Anton and Biglaiser (2013)). At the same time, however, trading used products in secondary markets is also a common practice in many durable goods industries (Hendel and Lizzeri (1999); Shulman and Coughlan (2007); Yin et al. (2010); Schiraldi and Nava (2012); Shen and Willems (2014)), including the used car and second-hand PCs, etc. As Computer Business Review (2005) [1]points out, these secondary markets have grown rapidly in recent years with third-party companies, for example, the PC industry building$100+ million per year businesses in buying, selling, or leasing used computer equipment.

In this paper, we focus on the effect of product upgrading on third party used

[1]Computer Business Review (2005). Big Players Emerge in Fragmented Brokerage Market. Accessed September 5, http://www.cbronline.com/news/big players emerge in ragmented brokerage market

product retailer's entrance decision to the secondary market. Because an upgraded product gives consumers a higher utility, it will prompt those consumers who were planning to buy used products in the secondary markets to turn to the new products market for higher quality or performance. In this case, product upgrading will have a negative effect on the sales of used products, which will reduce the entrance propensity of a third party retailer. On the other hand, consumers earn a higher net benefit from replacing a used with an upgraded new product, so product upgrading will have a positive effect on the sales of used products, by ensuring greater availability of used products, which obviously increase the entrance propensity. These observations raise an important question that warrants theoretical analysis: whether the manufacturer upgrading of new products actually affects the third party retailer's entrance decision to secondary market, and if so, how?

Yet the models used in previous research tend to ignore the effect of manufacturer's upgrading decisions on the secondary market and consider only these markets impacts on manufacturer's new product introduction strategies (e.g., Fudenberg and Tirole (1998); Zhao and Jagpal (2006); Yin et al. (2010)). Hence, in this paper, we focus on the effect of manufacturer's upgrading of new products on the sales of used products in the secondary market. To do so, we develop a dynamic two-period model in which a monopolistic manufacturer sells new durable products directly to end consumers in both periods, and a third-party entrant sells used products (i.e., those marketed in the first period) in the second period through a secondary market that is not directly controlled by the manufacturer. Our primary interest is in answering the following questions: Under what conditions it is optimal for a durable goods manufacturer to upgrade new products in the second period? What condition is needed for the third-party entrant to enter the secondary market in the second period? How does manufacturer upgrading degree affect the profits of channel partners? Hence our model differs from those previous studies in that it simultaneously considers an active secondary market, upgrading of new products, consumer market segmentation, and especially, the upgrade degree of new products as a function of consumer demand.

Our analysis reveals that when the investment cost of upgrading products is low, manufacturers do have an incentive to release an upgraded version in the second period, but when the investment cost is higher, they do not. Moreover, although the degree of upgrade always has a negative effect on the price of new products in the first period, its effect on the price of both used and new products in the second period is unimodal depending on intensity. We also find that the third-party entrant is likely to engage in the secondary market when the purchase cost of used products from former consumers is significantly low. Most importantly, we show that the upgrading of new products can increase the third-party entrants secondary market profitability when upgrades are typically minor or moderate but selling used products in the secondary market does not benefit the third-party entrant when upgrades are typically major.

The rest of the paper is organized as follows. Section 2 reviews the related literature and explains our contributions in more detail. Section 3 outlines the key elements of our model, as well as the derivation of the consumer demand function. Section 4 describes the model framework, presents the optimal equilibrium solutions for channel partners, and reports our main findings. Section 5 summarizes our conclusions and suggests opportunities for future research.

2. Relevant Literature

Our paper is closely related to the broader literature on durable goods and new product development strategies; particularly, those studies that address (1) the dynamics between new and used products and (2) the interaction between the secondary market and the introduction of upgrades in the durable goods industry. The first stream of research, which is especially well established, includes Levinthal and Purohit (1989); Fudenberg and Tirole (1998); Shulman and Coughlan (2007). Levinthal and Purohit (1989) examine the optimal sales strategy for a monopolist marketing a durable product in an existing secondary market. They show that not only limiting initial sales lowers new product cannibalization but buying back the earlier version generates greater demand for the new product. Their model, however, assuming that prices are linear functions of the cumulative quantities produced to date, does not allow nonbuyers from the first period to purchase in the second period. Fudenberg and Tirole (1998), in their analysis of firms dynamic pricing strategies in an existing secondary market, assumes that consumers in the market are homogeneous and the used market generates no profits for channel members. Based on heterogeneous consumers, Shulman and Coughlan (2007) show that the manufacturer earns higher profits from allowing used-good sales alongside new-good sales than from shutting down a retailer-profitable secondary market that expands the manufacturers unit sales beyond what is possible when only the primary market exists. These studies, however, ignore the effect of the new products upgrade on channel partners strategies. For further discussion of the relevant issues, see Desai and Purohit (1998, 1999); Desai et al. (2004); Huang et al. (2001); Bhaskaran and Gilbert (2005); Chen et al. (2010); Xiong et al. (2012, 2013); Pangburn and Stavrulaki (2014).

Our study is also related to the literature on the interaction between the secondary market and the introduction of upgrades in durable goods market. Zhao and Jagpal (2006), for instance, examine the effect of secondary markets for durable goods on a firm's dynamic pricing and new product introduction strategies. They find that secondary markets have differentiating effects on pricing across industries depending on the magnitudes of the innovation (major, moderate, or minor), and whether demand externalities are present. Martin (2011) then examines strategic behavior in a durable goods oligopoly where there is a positive probability of upgrade introduction. He argues that the presence of a secondary market not only increases the range of upgrades that are profitable but also raises profitability for a given upgrade quantity because former customers can be charged a higher price for the upgrade. Both studies, however, assume that used products are sold in an isolated channel while in reality, retailers sell used goods for profit in a co-opetition environment, for example, in textbook markets used book sellers not only cooperate with the manufacturers but are in competition with them. The presence of secondary market, therefore, especially one that is not directly controlled by the new product manufacturer, forces new product retailers or manufacturers to take used goods consumers into account when making business decisions. On the other hand, Yin et al. (2010), in their analysis of how the sequential emergence of retailer and P2P used goods markets shape both a manufacturers product upgrade strategy and a primary market retailers pricing strategy, assume that the retailer sells both used and new products for profit simultaneously. They find that frequent product upgrades and rising retail prices in durable product sectors results from the emergence of a P2P used goods market whose interaction with the retail used goods source alters the relative powers of the channel partners. In reaching this conclusion, however, they assume an exogenous segmentation of consumers who return used goods to the retail store or exchange them in P2P markets. For additional insights on this topic, see Fishman and Rob (2000); Kornish (2001); Lim and Tang (2006); Esteban and Shum (2007); Kogan (2011); Oraiopoulos et al. (2012).

Our paper differs from the extant literature in two ways: First, instead of ignoring the new product upgrade degree and paying little attention to its impact on consumer segmentation (particularly, consumer utilities), we endogenize this degree as a function of consumer demand (i.e., an endogenous segmentation of consumers). Second, rather than assuming that used goods are not sold through, or are sold outside, the standard channel, which ignores the effect of upgraded new product introduction in the secondary market, our dynamic model assumes that used products are sold by a third-party entrant in a secondary market not directly controlled by the manufacturer. In particular, we focus on the effect of manufacturer's product upgrading on the third-party entrant's decisions within a two-period context. To do so, we make two assumptions: First, we assume that consumers' quality valuations are heterogeneous, so that used product markets can be explicitly modeled. Second, to provide useful insights on third-party entrance into a secondary market, we assume the existence of an active secondary market that is not controlled by the manufacturer.

Although a few models have been developed to investigate the existence of new and used products of the same version in dynamic settings (e.g., Ferrer and Swaminathan (2006); Huang et al. (2001); Erzurumlu (2010), they do not capture the manufacturers upgrading strategies. Their focuses are either the competition between new and used products of the same version or how a frictionless used goods market affects the price of new products. In contrast, we consider the competition between new and used products of different versions and examine the effect of the manufacturers product upgrading on the decisions of third-party entrants in the secondary market. Our analytic results show that the upgrading degree of new products is critical to the profits of the channel members.

3. Model Framework

In developing our framework, we consider a two-period model [2] in which a monopolistic manufacturer (M) sells new durable products directly to end consumers in both periods, while a third-party entrant (TPE) sells reverse channeled used products (i.e., cleaned and tested buybacks from former customers) in a secondary market not directly controlled by the manufacturer (see Figure 1). We assume that all products provide only two periods of service (see, Desai et al. (2004); Yin et al. (2010)): "new" in period 1 and "used" in period 2. As a result, only new products are available in period 1, but both new and used products are available in period 2, which means the manufacturer's new product sales face competition from the used products offered by the TPE[3].

Following Desai et al. (2004) and Shulman and Coughlan (2007), we assume that all players in the model are rational and follow a Stackelberg game (see

[2] This assumption is consistent with previous literature (e.g., Desai and Purohit (1998); Desai et al. (2004); Xiong et al. (2012)), and a two-period model not only allows us to study dynamic issues while retaining tractability but simplifies the presentation of our analysis.

[3] In reporting our analytic results, for convenience, we use the pronouns "he" and "she" to refer to the manufacturer and third-party entrant, respectively.

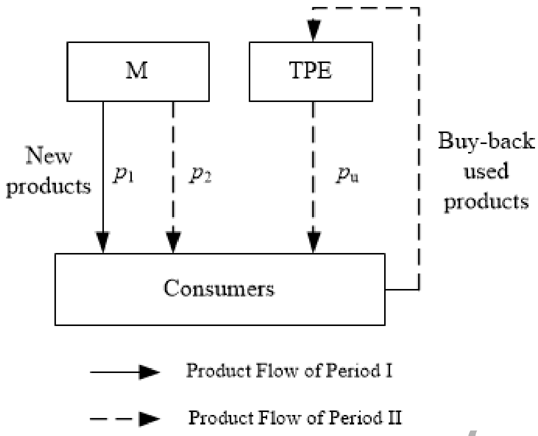

Figure 1: Two-Period Model Framework

Figure 2). In stage 1, period 1, the manufacturer decides whether or not to introduce an upgraded version [4] of the product in period 2, since upgrade decision is normally strategic and require significant lead time [5]. If an upgrade is introduced, it increases customer valuations of the new product in period 2 by a factor of $1 + \alpha$, where α (the product upgrade degree) is greater than zero [6]. Such a decision, however, involves an investment by the manufacturer whose amount depends on the upgrade degree.

Then, the manufacturer determines the unit price of the first-period new products p_1 in stage 2, period 1, and the unit price of the second-period new products [7] p_2 in stage 1, period 2. The TPE then decides the price of used

[4] We label the new product in the first period the "original version."

[5] Edition upgrades of college textbooks, for example, often take over a year, and the decision to update is made years in advance (e.g., Friscia (2009); Yin et al. (2010)).

[6] This factor refers to the degree of quality differentiation between the original and upgraded product versions. For instance, firms in the computer hardware industry continuously introduce technological innovations that lead to improvements in, for example, memory and speed.

[7] The version sold depends on the manufacturers decision in the first stage. Like Yin et al. (2010), we assume that original and upgraded versions of the new produce are not marketed simultaneously.

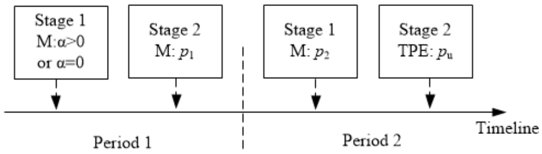

Figure 2: Sequence of Events. (Note: The notations $\alpha > 0$ and $\alpha = 0$, respectively, denote whether or not the manufacturer decides to release an upgraded version in period 2.)

products p_u in stage 2, period 2.

3.1. Product

To model the difference between new and used products, we designate the durability of the products produced in period 1 by factor δ ($0 \leq \delta \leq 1$), which represents how well a unit sold in period 1 holds up in period 2 (when it is classified as used). If $\delta = 1$, the product is perfectly durable and shows no deterioration over time, meaning that in period 2, used units are identical to new units. If $\delta = 0$, the product is nondurable and deteriorates fully after one period of use. In this paper, we consider only $0 < \delta < 1$.

3.2. Manufacturer

The manufacturer's problem is to set upgrade degree and price p_i so as to maximize his profits. Here, $i = 1, 2$ denotes period 1 or 2. If the manufacturer introduces an upgraded product in period 2, we denote it as $\alpha > 0$; otherwise $\alpha = 0$ (i.e., the manufacturer sells the original product in period 2). We further assume that if an upgraded version is introduced, it requires an investment cost for the manufacturer, the amount of which depends on the upgrade degree. Without loss of generality, we normalize the manufacturer's marginal cost of production and selling to zero.

3.3. Third-Party Entrant

The TPE's problem is to choose the price of used products p_u in a way that maximizes her profits. Suppose that used products have a residual value for all customers, the TPE occurs a reverse cost c from buying back used products for profitable resale. We also normalize the TPE's marginal cost of reselling to zero without loss of generality.

3.4. Consumer Strategies

We first assume that the size of the consumer population does not change over time and can be normalized to 1 and that no consumer can use more than one unit of the product in any period. We can then derive the inverse demand functions from the consumer utility functions. We do so by modeling heterogeneous consumers using parameter θ to represent a consumers valuation of the services provided by a durable, which is distributed uniformly in the interval [0, 1]. Consumer with type θ thus has a valuation of θ for a new product. Recall that the durability of the product is δ, which represents how well a unit sold in period 1 holds up as a used product in period 2, then consumer with type θ has a valuation of $\delta\theta$ for one used product. We assume that no consumers sell their used products directly to each other.

In period 1, the consumer can either buy a new product or choose not to. In period 2, consumers who bought a new product in period 1 can either replace it with a new version from the manufacturer or continue using the same product and abstain from the market at the end of period 2. On the other hand, consumers who did not buy a new product in period 1 can either buy a new or used unit or remain inactive in period 2. Following the same procedure as Desai and Purohit (1998); Desai et al. (2004); Oraiopoulos et al. (2012), we can then use a consumer choice model to derive the consumer demand.

The consumer type space can be divided into five segments: (1) consumers who buy a new product in period 1 and then sell their used product and buy a new product in period 2 (NN), (2) consumers who buy a new product in period 1 and continue using it in period 2 (NU), (3) consumers who do not buy in period 1 but buy a new product in period 2 (ON), (4) consumers who do not buy in period 1 but buy a used product in period 2 (OU), and (5) consumers who do not buy in either period (OO). Like Oraiopoulos et al. (2012), we assume that $\alpha < \delta$ (i.e., $1 + \alpha < 1 + \delta$); otherwise, the one-period utility from the improved product would be larger than the combined first- and second-period utility derived by the consumer from a first-period purchase (in which case, the ON segment would grow rapidly at the expense of all other segments).

Using the above analysis, we derive the total utility for every consumer segment: (1) (NN): $U_{NN} = \theta - p_1 + c + (1+\alpha)\theta - p_2$; (2) (NU): $U_{NU} = \theta - p_1 + \delta\theta$; (3) (ON): $U_{ON} = (1+\alpha)\theta - p_2$; (4) (OU): $U_{OU} = \delta\theta - p_u$; and (5) (OO): $U_{OO} = 0$. In terms of consumer utility, if all five strategies are in equilibrium, then each consumer segment values the product more (i.e., has a higher θ) than the next segment, so that NN valuation>NU valuation > ON valuation > OU valuation > OO valuation (Desai et al. (2004). Solving for marginal consumers, $U_{NN} = U_{NU}$, yields the location point $\theta_1 = \frac{p_2 - c}{1+\alpha-\delta}$ of the consumer who is indifferent between an NN or NU strategy. we can similarly obtain point $\theta_2 = \frac{p_1 - p_2}{\delta - \alpha}$ for the consumer who has the same utility whether adopting an NU or ON strategy, point $\theta_3 = \frac{p_2 - p_u}{1+\alpha-\delta}$ for the consumer who has the same utility whether adopting an ON or OU strategy, and pint $\theta_4 = \frac{p_u}{\delta}$ for the consumer who has the same utility whether adopting an OU or OO strategy. we therefore have $q_{NN} = 1 - \theta_1 = \frac{1+\alpha-\delta+c-p_2}{1+\alpha-\delta}$, $q_{NU} = \theta_1 - \theta_2 = \frac{p_2-c}{1+\alpha-\delta} - \frac{p_1-p_2}{\delta-\alpha}$, $q_{ON} = \theta_2 - \theta_3 = \frac{p_1-p_2}{\delta-\alpha} - \frac{p_2-p_u}{1+\alpha-\delta}$, $q_{OU} = \theta_3 - \theta_4 = \frac{p_2-p_u}{1+\alpha-\delta} - \frac{p_u}{\delta}$, where q_{NN}, q_{NU} and q_{OU} denote the size of each consumer segment, respectively.

Remark 1. (1) The second period quantities of new products ($q_{NN} + q_{ON}$) for the manufacturer increase in upgrade degree α, while the quantities of used products (q_{OU}) for the TPE decrease in upgrade degree α.

(2) The second period quantities of new products ($q_{NN}+q_{ON}$) for the manufacturer decrease in product durability δ, while the quantities of used products (q_{OU}) for the TPE increase in product durability δ.

(3) The quantities for consumers (q_{NN}) who buy new products in both periods increase in the reverse cost of used products c, while the quantities for consumers (q_{NU}) who buy a new product in period 1 and continue using it in period 2 decrease in the reverse cost of used products c.

(4) The first period quantities of new products ($q_{NN} + q_{NU}$) for the manufac-

turer increase with product durability δ.

It is clear that consumer utility increases with upgrade degree: the higher the upgrade degree of the new products in period 2, the greater the number of consumers who purchase a new product in period 1, want to sell their used product and buy a new product in period 2, or who do not purchase a new product in period 1, favor to buy a new one in period 2. Thus, the second-period quantities of new products increase in the upgrade degree, but used product quantities decrease. This reduction occurs for two reasons: First, as the upgrade degree increases, fewer consumers purchase a new product in period 1, preferring instead to buy an upgraded product in period 2. This choice shrinks used product sales in period 2. Second, from the manufacturer's point of view, higher first-period sales generate higher first-period profits, but also result in a greater quantity of used goods to compete with future new good sales, thereby he prefers to limit the production of first period products. That is, fewer used products are available. Hence, overall, used products quantities decrease in the upgrade degree.

The decrease in second-period quantities of new products as product durability increases also has two explanations: First, as product durability increases, more consumers who purchase a new product in period 1 prefer to continue using that product in period 2. This choice shrinks the market of consumers who purchase a new product in the first period and then sell their used product and buy a new product in period 2. Second, as product durability increases, fewer consumers who do not purchase a new product in period 1 choose to buy a new product in period 2. Moreover, the higher the product durability, the higher the utilities and thus the number of consumers who purchase no new product in period 1 and buy a used product in period 2.

The higher the reverse cost of the used products (i.e., the higher the residual value of used products for consumers), the greater the number of consumers who buy a new product in period 1 and then sell their used product and buy a new product in period 2. This observation implies that fewer consumers who buy a new product in period 1 keep using it in period 2 because the higher residual value of the used products increases their utilities.

Finally, the higher the product durability, the higher the utilities of consumers who buy a product in period 1, meaning that more consumers who want to keep the product or sell it in the secondary market at its resale value in period 2 prefer to buy a new product in period 1. Hence, the first-period quantities of new products increase with the increase of product durability.

4. Model Development

As outlined previously, our model includes a monopolistic manufacturer who sells his new durable products directly to end consumers in both periods, while in period 2, a TPE sells reverse channeled used products in a secondary market not directly controlled by the manufacturer. We now characterize the equilibrium between the players involved with a focus on the following dimensions: Under what conditions it is optimal for the durable goods manufacturer to release an upgraded version in the second period? What condition is needed for the TPE to enter the secondary market in the second period? How does the upgrading of new products affect the profits of the channel partners?

4.1. The TPE's problem

To ensure a subgame-perfect Nash equilibrium, we follow a backward induction method in which we first solve the third-party entrant's optimization problem under the assumption of rational consumer expectations. Denoting the TPE's profit by Π_e, we formulate the TPE's problem [8] as

$$Max_{p_u}\Pi_e(p_1,p_2,p_u) = p_u q_{OU} - c q_{NN} = p_u\left(\frac{p_2-p_u}{1+\alpha-\delta} - \frac{p_u}{\delta}\right) - c\left(1-\frac{p_2-c}{1+\alpha-\delta}\right)$$
$$s.t., 0 < q_{OU} \leq q_{NN} \tag{1}$$

[8] Obviously, when $q_{OU} = 0$ in the second period (i.e., no secondary market exists), the TPE is necessarily a nonparticipant. This case, however, is not the focus of our research.

where the constraint $q_{OU} \leq q_{NN}$ ensures that the sales quantity of used products is not greater than the number of units that can be collected from consumers and p_u and c represent the price of used products and their reverse cost to the TPE, respectively.

4.2. The manufacturer's problem

The manufacturer's problem is to maximize the total profit over the two periods with respect to p_1, p_2, taking into account the TPE's best response and the consumers' two-period strategies. In order to obtain the subgame perfect Nash equilibrium, we follow the method of backwards induction. We first solve the manufacturer's second period problem and then solve the first-period problem. Let Π_1 and Π_2 denote the manufacturer's first-period and second-period profits, respectively.

4.2.1. The manufacturer's second-period problem

The manufacturer's second-period optimization problem can be expressed as

$$Max_{p_2}\Pi_2(p_1,p_2,p_u^*) = p_2(q_{NN}+q_{ON}) = p_2(1-\frac{p_2-c}{1+\alpha-\delta}+\frac{p_1-p_2}{\delta-\alpha}-\frac{p_2-p_u^*}{1+\alpha-\delta}), \tag{2}$$

where p_2 denotes the price of the new products in period 2, and the TPE's best response p_u^* is taken into account in the manufacturer's decision.

4.2.2. The manufacturers first-period problem

In period 1, the manufacturer decides whether to introduce an upgraded version of the new product in period 2. If he does so, the upgrade occurs an investment cost. Suppose that the investment cost function is $\frac{1}{2}K[(1+\alpha)^2-1]$[9], where K represents the manufacturer's investment cost parameter,

[9]Such increasing convex cost functions are common in the literature on investing in improvements (e.g., Gurnani and Erkoc (2008); Esteban and Shum (2007)). Obviously, if no upgraded version is released, then no investment is required.

Table 1 Equlibrium Decisions for the Channel Partners in Model

TPE's decison

$$p_u^* = \frac{\delta(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]+c\delta[(\delta-\alpha)(5\alpha-2\delta+2)+3(1+\delta)]}{2[(1+\alpha)(7\delta-7\alpha+1)-2\delta(\delta-\alpha)][2(1+\alpha)(\frac{1}{\delta-\alpha}+1)-\delta]}$$

Manufacturer's decison

period 1: $p_1^* = \frac{(\delta-\alpha)[2(1+\alpha)(2+c)-\delta(\delta-\alpha)]}{7(1+\alpha)(\delta-\alpha)+(1+\alpha)-2\delta(\delta-\alpha)}$

period 2: $p_2^* = \frac{(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]+c[(\delta-\alpha)(5\alpha-2\delta+2)+3(1+\delta)]}{[(\delta-\alpha)(7-\frac{2\delta}{1+\alpha})+1][2(1+\alpha)(\frac{1}{\delta-\alpha}+1)-\delta]}$

Threshold value of K

$K^*(\alpha) = \frac{2[\overline{\Pi}(p_1^*|_{\alpha>0})-\overline{\Pi}(p_1^*|_{\alpha=0})]}{\alpha(\alpha+2)}$

where

$\overline{\Pi}(p_1^* \mid_{\alpha>0}) = p_1(q_{NN}+q_{NU}) + p_2(q_{NN}+q_{ON}) = G(1-\frac{GN(1+\alpha)H}{N(\delta-\alpha)}) + \frac{(1+\alpha)H^2}{2N(1+\alpha-\delta)(\alpha-\delta)}$;

$G = \frac{(\delta-\alpha)(4+4\alpha+\alpha\delta+2c\alpha-\delta^2+2c)}{1+7\delta+9\alpha\delta-7\alpha^2-2\delta^2-6\alpha}$

$N = 2+2\delta+3\alpha\delta-2\alpha^2-\delta^2$;

$H = \alpha^2 - G(1+\alpha+\delta) + c\alpha - 2\alpha\delta - \delta + \delta^2 - c\delta$;

and, $\overline{\Pi}(p_1^* \mid_{\alpha>0})$ equals to $\overline{\Pi}(p_1^* \mid_{\alpha=0})$, when $\alpha = 0$

the manufacturer's first-period optimization problem is

$$\begin{aligned} Max_{p_1}\Pi(p_1, p_2^*, p_u^*) &= \Pi_1(p_1, p_2^*, p_u^*) + \Pi_2^*(p_1, p_2^*, p_u^*) \\ &= p_1(q_{NN}+q_{NU}) + \Pi_2^*(p_1, p_2^*, p_u^*) - \frac{1}{2}K[(1+\alpha)^2-1], \end{aligned} \tag{3}$$

where the first part is the manufacturer's revenue in period 1, and the second is his revenue in period 2, and the third is his investment cost when he releases an upgraded version in the second period. The equilibrium decisions for the players are given in Table 1, in which the threshold value represents the cost below which the manufacturer will release an upgraded version, otherwise he will retain the old version (for a detailed technical analysis, see the Appendix A).

In the following, we will analyze the impact of parameters on the equilibrium decisions. First, we look at how the price of used product p_u change with respect to the product durability, the reverse cost of used products and the new product upgrade degree. We have the following proposition.

Proposition 1. *The price of used products p_u increases in product durability δ and the reverse cost of used products c but is unimodal (first increasing, then decreasing) in the new product upgrade degree α.*

The role of product durability is two-fold. On the one hand, higher durability expands the market segment that chooses to continue using the product and shrinks the segment of consumers who decide to sell their used products and buy new ones in the second period. The result is a higher procurement cost for the TPE, which reduces product viability and thus product quantity. On the other hand, increased product durability increases the demand for used products in the second period (see Remark 1 (2)). The TPE can therefore charge consumers a higher price for used products in the secondary market. As the reverse cost of used products increases (i.e., a higher residual value of used products for consumers), more consumers prefer to continue using their original products in period 2, making it more difficult for the TPE to reverse channel used products. As a result, the TPE charges a higher price for the used products in the secondary market. The price of used products is unimodal in the new product upgrade degree because this degree has two effects on the used product price: First (effect A), the higher the new product upgrade degree in the second period, the higher the utilities of consumers who do not buy in the first period but buy a new product in the second period. Hence, a higher upgrade degree makes the second-period new product more competitive at the expense of the used product, meaning that its price increases as the update degree increases. Conversely (effect B), as the upgrade degree increases, the utilities that first period consumers obtain from selling their used products to the TPE and buying a new one in the second period increase, so the TPE can procure used products more cheaply, which has a positive effect on the quantity of used products. As a result, the impact of new product upgrade degree on the used product price depends on the balance of the two effects. The used product price decreases with the new product upgrade degree when the effect

B is dominating, otherwise, the used product price increases with the upgrade degree.

Next, we analyze the properties of the manufacturer's new product price, which is presented in the following proposition.

Proposition 2. *The optimal prices of the manufacturer's new product have the following properties:*

(1) *the optimal prices of new products in both the first and second periods increase with the reverse cost of used products c;*

(2) *the optimal price of new products in the first period decreases with the new product upgrade degree α;*

(3) *the optimal price of new products in the second period is unimodal in the new product upgrade degree α.*

The property (1) is straightforward. Because the higher the reverse cost of used products, the higher the residual value of used products (i.e., the higher the quality of new products in period 1), the optimal price of new products in the first period increases as the reverse cost of used products increases. The manufacturer can then charge consumers a higher price for new products in period 1. On the other hand, a higher price for new products in period 1 results in a lower sales volume for new products in period 1, leading to a lower reverse volume of used products for the TPE in period 2. This reduction implies that in period 2, the TPE faces strong competition from new products from which the manufacturer can benefit by charging consumers a higher price for new products.

As the new product upgrade degree increases, both consumers who buy a new product in period 1 sell it at the end of period 1, and buy a new product in period 2 and consumers who do not buy in period 1 but buy a new product in period 2 enjoy an increase in utilities. Therefore, the second-period new products can become more competitive, meaning that the manufacturer always has higher incentive to charge a lower price for new products in period 1 in order to expand the new product market segment in period 2. This effect is driven by two opposing forces. On the one hand (effect C), a higher upgrade degree makes the second-period new products more competitive, so the manufacturer can charge a higher price for them in period 2; on the other (effect D), as the upgrade degree increases, the TPE's procurement cost decreases, which has a positive effect on the quantity of used products. The latter, however, also leads to greater cannibalization of new products, which lowers the new product price in period 2. When the upgrade degree is low, effect C dominates effect D and a higher upgrade degree leads to an increase in second-period new product prices.

Now, we move to the TPE's decision, and we have the following proposition.

Proposition 3. (1) *If the manufacturer releases an upgraded version in the second period, there is a threshold value $\overline{c}$,*
$\overline{c} = \frac{\delta(\delta-\alpha)(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]}{4(1+\alpha)^2(1+\delta-\alpha)(7\delta-7\alpha+1)+\delta(\delta-\alpha)[(\delta-\alpha)(6\delta-27\alpha-27)-13(1+\alpha)]}$
such that when $0 < c < \overline{c}$, the TPE will choose to enter the secondary market; otherwise, when $c \geq \overline{c}$, she will choose to withdraw.

(2) *Because upgrading new products increases the range of the TPE's reverse costs (i.e., expands her survival space in the secondary market), the product upgrade in the second period benefits a TPE engaged in the secondary market. When upgrades are major, however, no such benefit accrues.*

This proposition shows that the TPE chooses to enter the secondary market when $0 < c < \overline{c}$ because the optimal price of used products is greater than her investment cost (i.e., $p_u^* > c$), which makes it profitable to sell used products. If $c \geq \overline{c}$, however, the optimal price of used products is less than or equal to the TPE's investment cost (i.e., $p_u^* \leq c$), so her engagement in the secondary market is not profitable, which implies that the manufacturer can weaken his secondary market by improving the procurement cost of used products. For example, Cisco requires each buyer of its refurbished equipment to pay high relicensing fees for the proprietary software that runs on the equipment. This practice, in effect, creates a higher procurement cost for the TPE and eliminates the secondary market (Oraiopoulos et al. (2012). Likewise, Sun Microsystems,

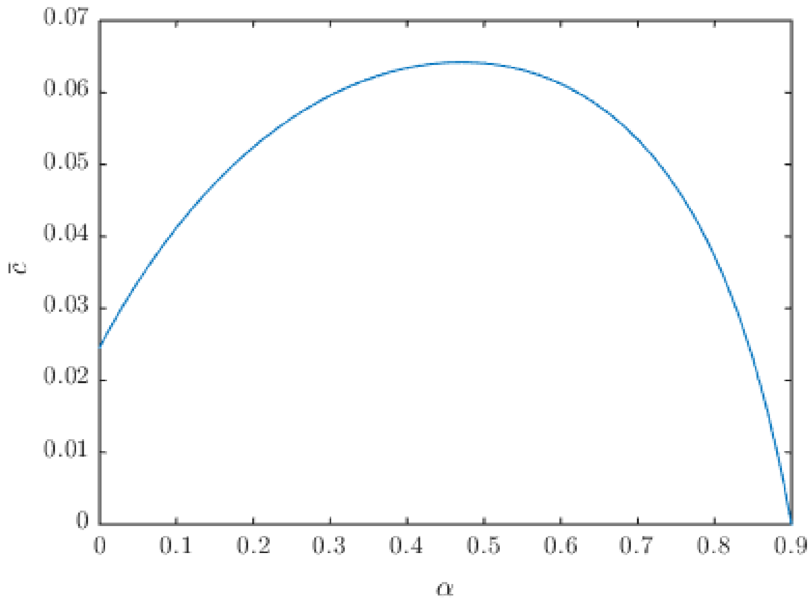

Figure 3: Changing Trends in Reverse Costs

an IT server business, has deliberately attempted to eliminate the secondary market for its machines worldwide through its pricing and licensing schemes (Marion (2004)).

Based on the proof given in the Appendix C, we can see that as the upgrade degree increases, the threshold value of the TPE's reverse cost first increases and then decreases. We depict the intuition underlying proposition 3 (2) in Figure 3 (i.e., $\delta = 0.9$), which displays the range of the TPEs reverse cost.

Here, a lower upgrade degree implies no significant difference between the upgraded and original versions, meaning that more consumers who buy a new product in the first period prefer to continue using it in the second period. This preference in turn leads to an increase in the TPE's reverse cost of used products. However, as the upgrade degree increases, more consumers who buy a new product in the first period prefer to sell their used product to the TPE at the end of period 1 and buy a new product in period 2, thereby reducing the

TPE's procurement cost. Moreover, when the upgrade degree is greater than a certain value, manufacturer's upgrading of new products does not benefit a TPE in the secondary market except in industries where upgrades are typically minor or moderate (in which case, upgrading expands her survival space).

4.2.3. The manufacturer's product upgrade problem

Whether a manufacturer decides to release an upgraded new product in period 2 depends on whether doing so is more profitable than continuing to sell the original version product; that is, $\overline{\Pi}(p_1^* \mid_{\alpha>0}) > \overline{\Pi}(p_1^* \mid_{\alpha=0})$. We thus compare total two-period profits in two cases: (1) the manufacturer releases an original version in the first period but an upgraded version in the second period and (2) the manufacturer sells the original version in both periods. We draw the following conclusion:

Proposition 4. *For the monopolistic manufacturer, there exists a threshold value for manufacturer's investment cost parameter K denoted as $K^*(\alpha)$*

$$
\begin{aligned}
&K^*(\alpha) = \frac{2[\overline{\Pi}(p_1^* \mid_{\alpha>0}) - \overline{\Pi}(p_1^* \mid_{\alpha>0})]}{\alpha(\alpha+2)},\\
&\overline{\Pi} = p_1(q_{NN} + q_{NU}) + p_2(q_{NN} + q_{ON}),\\
&\overline{\Pi}(p_1^* \mid_{\alpha>0}) \textit{equals to} \overline{\Pi}(p_1^* \mid_{\alpha=0}), \textit{when} \quad \alpha = 0.
\end{aligned}
$$

such that if $K < K^*(\alpha)$, the manufacturer will release an upgraded version product in period 2, but if $K \geq K^*(\alpha)$, he will keep selling the original version.

Proposition4 suggests that the manufacturer should use two different introduction strategies depending on the investment cost parameter of upgrading new products. When the investment cost parameter is small ($K < K^*(\alpha)$), the manufacturer should release an upgraded version in the second period because a higher upgrade degree increases the competitiveness of second-period new products, a benefit that outweighs the potentially negative cannibalization effect of used products. When the investment cost parameter is large ($K \geq K^*(\alpha)$),

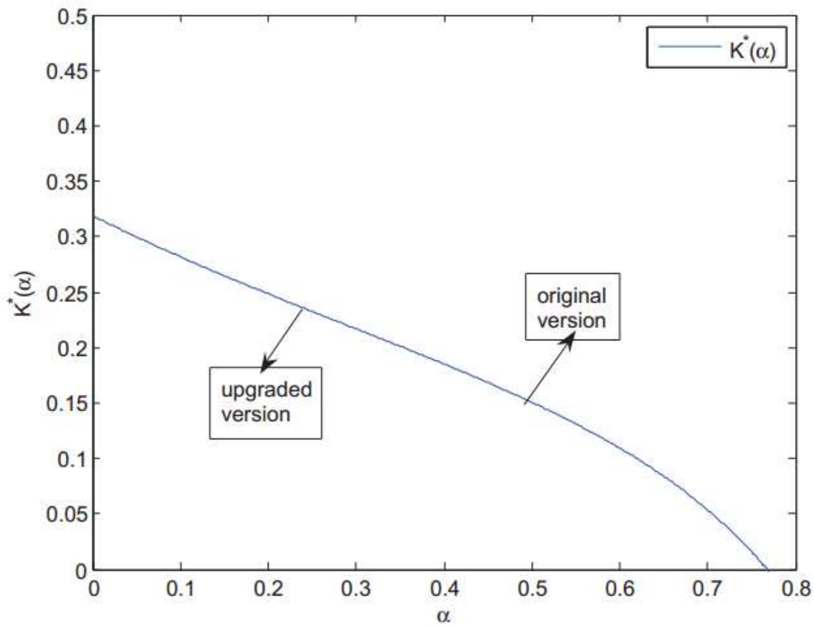

Figure 4: Threshold Values of the Manufacturers Investment Cost

however, the manufacturer should continue selling the original version in the second period.

We then let $\delta = 0.9, c = 0.005$ and numerically illustrate the change of threshold values of the manufacturers investment cost in upgrade degree(see Figure 4). Figure 4 shows a marked decrease of the threshold value in the upgrade degree α. This observation implies that although the manufacturer may prefer to release an upgraded product in period 2, the decrease in the threshold value of investment cost parameters limits his upgrade ability and makes upgrading extremely difficult when the upgrade degree is very large. Hence, the manufacturer must find a balance between the upgrade degree and investment cost.

Having identified the optimal strategies for the manufacturer and TPE with new product upgrade, we now gauge the extent to which upgrading impacts the profitability of both the manufacturer and TPE. The highly nonlinear equations

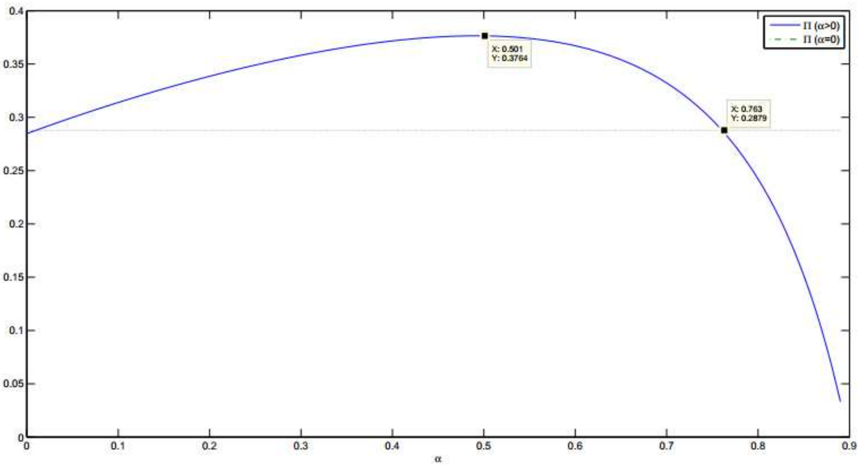

Figure 5: Manufacturers Profits

make analytical comparison difficult. We, therefore, illustrate the impact of upgrading on profits in a series of detailed numerical examples.

Using the same parameters as in the previous examples ($\delta = 0.9, c = 0.005$) and let $K = 0.005$ (we also explored other parameter combinations in their domains, and got similar results), we plot the changing of manufacturer's profit with the increasing of upgrade degree, which is presented in Figure 5. From Figure 5, we have the following observation.

Observation 1. When an upgraded product is released in the second period, the manufacturer's profit first increases and then decreases in the new product upgrade degree α.

According to proposition 4, if the investment cost is feasible for the manufacturer, he prefers to release an upgraded product in the second period. Here, $\Pi(\alpha > 0)$ represents the total two-period profit of the manufacturer who introduces an upgraded product in period 2, while $\Pi(\alpha = 0)$ represents that of the manufacturer who sells the same version during both periods. It is apparent from the figure that the manufacturers profit from releasing an upgraded product in period 2 is greater than his profit from selling the original version when

$\alpha \in (0, 0.763)$. Furthermore, this profit first increases and then decreases in the upgrade degree. This effect of the upgrade degree on the manufacturers total two-period profit is a result of two phenomena: On the one hand (effect E), the higher the new product upgrade degree in period 2, the higher the utilities of those consumers who buy a new product in period 1 then sell their used product and buy a new product in period 2, as well as of those who do not buy in the first period but buy a new product in the second period. These choices result in a higher sales volume for the second-period new product, which increases the manufacturer's profit. On the other hand (effect F), as α increases, the utilities that the first-period consumers obtain from selling their used products to the TPE and buying a new one in the second period also increase, which lowers the TPE's procurement cost for used products. In other words, the sales of used products in the second-period market increase. Nevertheless, because used products cannibalize new products in period 2, the competition from used products becomes stronger, which has a negative effect on the manufacturer's profits. Thus, when the value is low, effect E dominates effect F and the manufacturers profit increases in the upgrade degree, but when value is high, his profit decreases.

Again using the same parameters as in previous examples ($\delta = 0.9, c = 0.005, K = 0.005$), we plot the changing of TPE's profits as the upgrade degree increases, which is depicted in Figure 6. From Figure 6, we have the following observation.

Observation 2. The manufacturer's release of an upgraded product in the second period benefits a TPE in the secondary market, whose profit first increases and then decreases in the new product upgrade degree.

Here, Π_e represents the TPE's profit when the manufacturer releases an upgraded product in period 2. As in the analysis of manufacturer profit, we find that the TPE's profit first increases ($\alpha \in (0, 0.331)$) and then decreases ($\alpha \in [0.331, 0.769)$) as the upgrade degree increases. This effect is also driven by two opposing forces. On the one hand (effect G), as α increases, the sales of

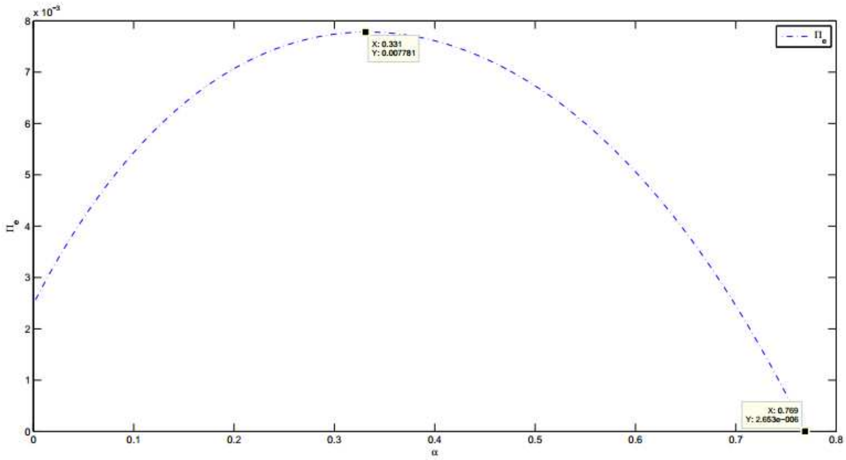

Figure 6: TPE Profits

used products in the second period increase, which leads to an increase in the TPE's profit. On the other hand (effect H), a higher α makes second-period new products more competitive at the expense of used products, so the TPE's profit decreases in the upgrade degree. When effect G dominates effect H, the TPE's profit increases, otherwise, it decreases. Once again, the manufacturer's second-period upgrading of new products benefits the TPE when the upgrade is minor ($\alpha \in (0, 0.331)$) or moderate ($\alpha \in [0.331, 0.769)$) but not when it is major ($\alpha \geq 0.769$)).

5. Conclusion

Although the durable product literature has long studied the effect of secondary markets on manufacturers strategies in the primary market, most research assumes either the secondary market is a perfect market or used products are sold outside the standard channel [10]. It thus ignores the effect of introducing upgraded new products on the TPE's entrance decision to secondary market.

[10]See Levinthal and Purohit (1989); Fudenberg and Tirole (1998); Zhao and Jagpal (2006)

We, in contrast, examine how a manufacturers upgrade strategies in the durable goods market affect the TPE's entrance decision to secondary market. We consider several factors simultaneously in our model: an active secondary market, upgrading of new products, consumer market segmentation, and most especially, the upgrade degree of new products as a function of consumer demand.

To focus on the effect of introducing upgraded new products on the secondary market, we developed a dynamic two-period model in which a monopolistic manufacturer sells his new durable products directly to end consumers in both periods, while in the second period, a TPE sells reverse channeled used products in a secondary market not directly controlled by the manufacturer. We derive the condition under which manufacturer's upgrading benefits a TPE. We also identify an investment cost threshold below which the manufacturer's optimal strategy is to release an upgraded version product in the second period. We generate managerial insights into how manufacturer upgrading of new products impact the decision of the TPE in the secondary market by characterizing the optimal strategies of both parties. We find that manufacturer's upgrading of new products can increase a third-party entrant's profitability in a secondary market when upgrades are typically small or moderate. It does not, however, benefit the TPE engaged in selling used products when upgrades are typically large.

The findings reported here suggest two obvious possibilities for further research. First, our model could be extended to a recovery market, a typical assumption in the durable product literature, which would raise such additional issues as how different recovery channels of used products affect the strategies of the manufacturers product upgrade. Likewise, extending the model to an oligopoly market would raise new and interesting questions; especially, whether and how a firm should discriminate between its own former customers and those of its rivals.

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Appendices

Appendix A. Equilibrium Decisions for the Channel Partners in Model

The TPE's optimization problem is

$$\begin{aligned} Max_{p_u}\Pi_e(p_1,p_2,p_u) &= p_u q_{OU} - cq_{NN} \\ &= p_u\left(\frac{p_2-p_u}{1+\alpha-\delta}-\frac{p_u}{\delta}\right)-c\left(1-\frac{p_2-c}{1+\alpha-\delta}\right), \qquad \text{(A.1)} \\ s.t., & 0 < q_{OU} \leq q_{NN} \end{aligned}$$

where all segment sizes have the functional form defined in the paper. Substituting these segment sizes into the TPE's objective function, we find that $\frac{\partial^2 Max_{p_u}\Pi_e}{\partial p_u^2} < 0$.Therefore, the profit function of the TPE is concave in p_u. The Lagrangian for the TPE's problem is

$$L_e(p_u,\lambda_1) = p_u\left(\frac{p_2-p_u}{1+\alpha-\delta}-\frac{p_u}{\delta}\right)-c\left(1-\frac{p_2-c}{1+\alpha-\delta}\right)+\lambda_1\left(1-\frac{p_2-c}{1+\alpha-\delta}-\frac{p_2-p_u}{1+\alpha-\delta}+\frac{p_u}{\delta}\right)$$

and the Kuhn-Tucker conditions for optimality are:

$$\frac{\partial L_e(p_u,\lambda_1)}{\partial p_u} = p_u(\frac{-1}{1+\alpha-\delta} - \frac{1}{\delta}) + \frac{p_2-p_u}{1+\alpha-\delta} - \frac{p_u}{\delta} + \lambda_1(\frac{1}{1+\alpha-\delta} + \frac{1}{\delta}) = 0;$$

$$\lambda_1(1 - \frac{p_2-c}{1+\alpha-\delta} - \frac{p_2-p_u}{1+\alpha-\delta} + \frac{p_u}{\delta}) = 0;$$

$$0 < q_{OU} \leq q_{NN}.$$

We consider two subcases according to whether the Lagrangean multiplier λ_1 is greater than or equats to zero.

Case TPE-a: $\lambda_1 = 0$. Simultaneously solving for the above equations, we have that $p_u^* = \frac{\delta p_2}{2(1+\alpha)}$. Meanwhile, the constraint $0 < q_{OU} \leq q_{NN}$ leads to $0 < p_2 \leq \frac{2(1+\alpha+c-\delta)}{3}$. Therefore, the TPE will choose $p_u^* = \frac{\delta p_2}{2(1+\alpha)}$, when $0 < p_2 \leq \frac{2(1+\alpha+c-\delta)}{3}$.

Case TPE-b: $\lambda_1 > 0$. Simultaneously solving for the above equations, we have that $p_u^* = \frac{\delta(2p_2+\delta-c-1-\alpha)}{1+\alpha}, \lambda_1 = \frac{\delta(3p_2+2\delta-2c-2-2\alpha)}{1+\alpha}$. In addition the Lagrangian multiplier $\lambda_1 > 0$ leads to $p_2 > \frac{2(1+\alpha+c-\delta)}{3}$ Meanwhile, from the utility function, we know that $p_2 < 1+\alpha+c-\delta$. Therefore, we obtain $\frac{2(1+\alpha+c-\delta)}{3} < p_2 < 1+\alpha+c-\delta$.

We now consider the price of new products in the second period under the Case TPE-a. Note that we denote this case in Case M2-a. Replacing the values $p_u^* = \frac{\delta p_2}{2(1+\alpha)}$ and the constraint $0 < p_2 \leq \frac{2(1+\alpha+c-\delta)}{3}$ of the Case TPE-a, we obtain the manufacturer's second period problem is

$$\begin{aligned} Max_{p_2}\Pi_2(p_1,p_2,p_u^*) &= p_2(q_{NN}+q_{ON}) \\ &= p_2(1 - \frac{p_2-c}{1+\alpha-\delta} + \frac{p_1-p_2}{\delta-\alpha} - \frac{p_2-p_u^*}{1+\alpha-\delta}) \\ s.t., 0 < p_2 &< \frac{2(1+\alpha+c-\delta)}{3}, \end{aligned} \tag{A.2}$$

where all segment sizes have the functional form defined in the paper. Substituting these segment sizes into the manufacturer's second-period problem, we find that $\frac{\partial^2 Max_{p_2}\Pi_2}{\partial p_2^2} < 0$. Therefore, the profit function of the manufacturer's second-period is concave in p_2. The Lagrangian for the manufacturer's second-period problem is

$$L_e(p_u,\lambda_1)=p_2(q_{NN}+q_{ON})=p_2(1-\frac{p_2-c}{1+\alpha-\delta}+\frac{p_1-p_2}{\delta-\alpha}-\frac{p_2-p_u^*}{1+\alpha-\delta})-\lambda_2(p_2-\frac{2(1+\alpha+c-\delta)}{3})$$

and the Kuhn-Tucker conditions for optimality are:

$$\begin{aligned}\frac{\partial L_2(p_2,\lambda_2)}{\partial p_2}&=1-\frac{p_2-c}{1+\alpha-\delta}+\frac{p_1-p_2}{\delta-\alpha}-\frac{p_2[2(1+\alpha)-\delta]}{2(1+\alpha)(1+\alpha-\delta)}\\&+p_2[-\frac{1}{1+\alpha-\delta}-\frac{1}{\delta-\alpha}-\frac{2(1+\alpha)-\delta}{2(1+\alpha)(1+\alpha-\delta)}]-\lambda_2=0;\\&\lambda_2(p_2-\frac{2(1+\alpha+c-\delta)}{3})=0;\\&0<p_2<\frac{2(1+\alpha-c-\delta)}{3}\end{aligned}$$

We consider two subcases according to whether the Lagrange multiplier λ_2 is greater than or equals to zero.

Case M2-a-1: $\lambda_2=0$. Simultaneously solving for the above equations, we have that $p_2^*=\frac{(1+\alpha)[(\delta-\alpha)(1+\alpha+c-\delta)+p_1(1+\alpha-\delta)]}{2(1+\alpha)+(\delta-\alpha)[2(1+\alpha)-\delta]}$. In addition, the constraint $0<p_2<\frac{2(1+\alpha-c-\delta)}{3}$ of the manufacturer's second period problem leads to $0<p_1<\frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})]$.

Case M2-a-2: $\lambda_2>0$. Solving the system, we have that $p_2^*=\frac{2(1+\alpha+c-\delta)}{3}$ and $\lambda_2=\frac{[2(\delta-\alpha)\delta-(1+\alpha)(4+\delta-\alpha)](1+\alpha-\delta+c)+3p_1(1+\alpha)(1+\alpha-\delta)}{3(1+\alpha)(\delta-\alpha)(1+\alpha-\delta)}$. Moreover, Lagrange multiplier $\lambda_2>0$ leads to $p_1>\frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})]$.

Similarly, we consider the price of new products in the second period under the Case TPE-b. Note that we denote this case in Case M2-b. Replacing the values $p_u^*=\frac{\delta(2p_2+\delta-c-1-\alpha)}{1+\alpha}$ and the constraint of the case TPE-b, we obtain that the manufacturer's second period problem is

$$\begin{aligned}Max_{p_2}\Pi_2(p_1,p_2,p_u^*)=p_2(q_{NN}+q_{ON})=p_2(1-\frac{p_2-c}{1+\alpha-\delta}+\frac{p_1-p_2}{\delta-\alpha}-\frac{p_2-p_u^*}{1+\alpha-\delta})\\ s.t.,\frac{2(1+\alpha+c-\delta)}{3}<p_2<1+\alpha+c-\delta.\end{aligned} \tag{A.3}$$

where all segment sizes have the functional form defined in the paper. Substituting these segment sizes into the manufacturer's second-period problem, we find that $\frac{\partial^2 Max_{p_2}\Pi_2}{\partial p_2^2} < 0$. Therefore, the profit function of the manufacturer's second period is concave in p_2. Solving the first derivative of the problem, we have $p_2^* = \frac{(1+\alpha)(\delta-\alpha)(1+\alpha+c-\delta)+p_1(1+\alpha-\delta)}{2(1+\alpha-\delta)(1+2\delta-\alpha)}$. In addition, the constraint $\frac{2(1+\alpha+c-\delta)}{3} < p_2 < 1+\alpha+c-\delta$ leads to $\frac{(1+\alpha+c-\delta)[(1+\alpha)(4+5\delta-\alpha)-4\delta(1+2\delta-\alpha)]}{3(1+\alpha)(1+\alpha-\delta)} <$ $p_1 < (1+\alpha+c-\delta)\frac{[(1+\alpha)(2+3\delta-\alpha)-2\delta(1+2\delta-\alpha)]}{(1+\alpha)(1+\alpha-\delta)}$.

Now we consider the price of new products in the first period under the Case M2-a-1. Note that we denote this case as Case M1-a-1. Replacing the values $p_2^* = \frac{(1+\alpha)[(\delta-\alpha)(1+\alpha+c-\delta)+p_1(1+\alpha-\delta)]}{2(1+\alpha)+(\delta-\alpha)[2(1+\alpha)-\delta]}$ and the constraint in Case M2-a-1, we obtain that the manufacturer's firs period problem is

$$\begin{aligned} Max_{p_1}\Pi(p_1,p_2^*,p_u^*) &= \Pi_1(p_1,p_2^*,p_u^*) + \Pi_2^*(p_1,p_2^*,p_u^*) \\ &= p_1(q_{NN}+q_{NU}) + \Pi_2^*(p_1,p_2^*,p_u^*) - \frac{1}{2}K[(1+\alpha)^2-1], \\ s.t., 0 < p_1 &< \frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})]. \end{aligned} \tag{A.4}$$

where all segment sizes have the functional form defined in the paper. Substituting these segment sizes into the manufacturer's first-period problem, we find that $\frac{\partial^2 Max_{p_1}\Pi_1}{\partial p_1^2} < 0$. Therefore, the profit function is concave in p_1.

The Lagrangian for the manufacturer's first-period problem is

$$\begin{aligned} L_1(p_1,\lambda_3) =& p_1(1-\frac{p_1-p_2^*}{\delta-\alpha} + p_2^*(1-\frac{p_2^*-c}{1+\alpha-\delta}) + \frac{p_1-p_2^*}{\delta-\alpha} - \frac{p_2^*-p_u^*}{1+\alpha-\delta}) \\ &- \frac{1}{2}K[(1+\alpha)^2-1] - \lambda_3(p_1 - \frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})]) \end{aligned} \tag{A.5}$$

The Kuhn-Tucker conditions for optimality are:

$$\begin{aligned}\frac{\partial L_1(p_1,\lambda_3)}{\partial p_1} =& 1-\frac{2p_1}{\delta-\alpha}\\ &+\frac{2(1+\alpha)[(\delta-\alpha)(1+\alpha-\delta)+c(\delta-\alpha)+p_1(1+\alpha-\delta)]+p_1(1+\alpha)(1+\alpha-\delta)}{(\delta-\alpha)[2(1+\alpha)(1+\alpha-\delta)-\delta(\delta-\alpha)]}\\ &-\lambda_3=0;\\ &\lambda_3(p_1-\frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})])=0;\\ &0<p_1<\frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})].\end{aligned}$$

We consider two sub-cases according to whether the Lagrangian multiplier λ_3 is greater than or equals to zero.

Case M1-a-1: $\lambda_3=0$. Simultaneously solving for the above equations, we have that $p_1^*=\frac{(\delta-\alpha)[2(1+\alpha)(2+c)-\delta(\delta-\alpha)]}{7(1+\alpha)(\delta-\alpha)+(1+\alpha)-2\delta(\delta-\alpha)}$. Moreover, p_1^* such that the constraint $0<p_1<\frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})]$.

Case M1-a-2: $\lambda_3>0$. Simultaneously solving for the above equations, we have that $p_1=\frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})]$ and $\lambda_3=1-2(1+\alpha+c-\delta)(4+(\delta-\alpha)(1-2\delta/(1+\alpha)))/(3+3\alpha-3-\delta)/(\delta-\alpha)+((2+2\alpha)((\delta-\alpha)(1+\alpha-\delta)+c(\delta-\alpha)+(1+\alpha+c-\delta)(4+(\delta-\alpha)(1-2\delta/(1+\alpha)))/(3+3\alpha-3-\delta)(1+\alpha-\delta))+(1+\alpha)(1+\alpha-\delta)(4+(\delta-\alpha)(1-2\delta/(1+\alpha)))/(3+3\alpha-3\delta)/(2+2\alpha+(\delta-\alpha)(2+2\alpha-\delta))$.

Similarly, we consider the price of new products in the first period under the Case M2-a-2. Note that we denote this case as Case M1-a-2. Replacing the values $p_2^*=\frac{2(1+\alpha+c-\delta)}{3}$ and the constraint in Case M2-a-2, we obtain that the manufacturer's first period problem is

$$\begin{aligned}Max_{p_1}\Pi(p_1,p_2^*,p_u^*) &= \Pi_1(p_1,p_2^*,p_u^*)+\Pi_2^*(p_1,p_2^*,p_u^*)\\ &= p_1(q_{NN}+q_{NU})+\Pi_2^*(p_1,p_2^*,p_u^*)-\frac{1}{2}K[(1+\alpha)^2-1],\\ s.t.,p_1 &> \frac{(1+\alpha+c-\delta)}{3(1+\alpha-\delta)}[4+(\delta-\alpha)(1-\frac{2\delta}{1+\alpha})].\end{aligned} \tag{A.6}$$

where all segment sizes have the functional form defined in the paper. Substituting these segment sizes into the manufacturer's first-period problem, we find that $\frac{\partial^2 Max_{p_1}\Pi_1}{\partial p_1^2} < 0$. Therefore, the profit function is concave in p_1. Solving the first order derivative of the manufacturer's first-period problem, we have that $p_1^* = \frac{4(1+c)+\alpha-\delta}{6}$. It can be shown that p_1^* does not satisfy the constraint. Therefore, this case will never occur in equilibrium.

Similarly, we consider the price of new products in the first period Case M2-b. We denote this case as Case M1-b. Substituting the value $p_2^* = \frac{(1+\alpha)[(\delta-\alpha)(1+\alpha+c-\delta)+p_1(1+\alpha-\delta)]}{2(1+\alpha-\delta)(1+2\delta-\alpha)}$ and the constraint in Case M2-b, we obtain that the manufacturer's first period problem is

$$\begin{aligned} Max_{p_1}\Pi(p_1, p_2^*, p_u^*) &= \Pi_1(p_1, p_2^*, p_u^*) + \Pi_2^*(p_1, p_2^*, p_u^*) \\ &= p_1(q_{NN} + q_{NU}) + \Pi_2^*(p_1, p_2^*, p_u^*) - \frac{1}{2}K[(1+\alpha)^2 - 1], \\ s.t., p_1 &> \frac{(1+\alpha+c-\delta)[(1+\alpha)(4+5\delta-\alpha) - 4\delta(1+2\delta-\alpha)]}{3(1+\alpha)(1+\alpha-\delta)} \\ p_1 &< (1+\alpha+c-\delta)\frac{(1+\alpha)(2+3\delta-\alpha) - 2\delta(1+2\delta-\alpha)}{(1+\alpha)(1+\alpha-\delta)}. \end{aligned} \tag{A.7}$$

where all segment sizes have the functional form defined in the paper. Substituting these segment sizes into the manufacturer's first-period problem, we find that $\frac{\partial^2 Max_{p_1}\Pi_1}{\partial p_1^2} < 0$. Therefore, the profit function is concave in p_1. Solving the first order derivative of the manufacturer's first-period problem, we have that $p_1^* = \frac{2(\delta-\alpha)(2+\delta+c)}{1+8\delta-7\alpha}$. It can be shown that $p_!^*$ does not satisfy the constraint. Based on the above analysis, we obtain the optimal price of new products in period 1 is $p_1^* = \frac{(\delta-\alpha)[2(1+\alpha)(2+c)-\delta(\delta-\alpha)])}{7(1+\alpha)(\delta-\alpha)+1+\alpha)-2\delta(\delta-\alpha)}$. Substituting p_1^* into p_2^*, p_u^*, we get
$p_2^* = \frac{(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]+c[(\delta-\alpha)(5\alpha-2\delta+2)+3(1+\alpha)]}{[(\delta-\alpha)(7-\frac{2\delta}{1+\alpha})+1][2(1+\alpha)(\frac{1}{\delta-\alpha}+1)-\delta]}$
$p_u^* = \frac{\delta(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]+c\delta[(\delta-\alpha)(5\alpha-2\delta+2)+3(1+\delta)]}{2[(1+\alpha)(7\delta-7\alpha+1)-2\delta(\delta-\alpha)][2\delta(\delta-\alpha)][2(1+\alpha)(\frac{1}{\delta-\alpha}+1)-\delta]}$.
Therefore, the equilibrium decisions for the channel partners are given in Table 1.

Appendix B. Proof of Proposition 1

It is obvious that the price of used products increases in the residual value c. For the sign of $\frac{\partial p_u}{\partial \delta}$, we have $\frac{\partial p_u}{\partial \delta} = \frac{f_2 \frac{\partial f_1}{\partial \delta} - f_1 \frac{\partial f_2}{\partial \delta}}{4[(7\delta-7\alpha+1)(1+\alpha)-2\delta(\delta-\alpha)]^2[2(1+\alpha)(1+\frac{1}{\delta-\alpha})-\delta]^2}$, where $f_1 = \delta(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)] + c\delta(5\delta-2\alpha+7\alpha\delta-5\alpha^2-2\delta^2+3)$, $f_2 = 2[(1+\alpha)(7\delta-7\alpha+1)-2\delta(\delta-\alpha)][2(1+\alpha)(1+\frac{1}{\delta-\alpha})-\delta]$. Therefore, to show that $\frac{\partial p_u}{\partial \delta} > 0$, it is sufficient to show that $f_2 \frac{\partial f_1}{\partial \delta} - f_1 \frac{\partial f_2}{\partial \delta} > 0$. Simple calculation can give the result.

Finally

$$\frac{\partial p_u}{\partial \alpha} = \frac{f_2 \frac{\partial f_1}{\partial \alpha} - f_1 \frac{\partial f_2}{\partial \alpha}}{4[(7\delta-7\alpha+1)(1+\alpha)-2\delta(\delta-\alpha)]^2[2(1+\alpha)(1+\frac{1}{\delta-\alpha})-\delta]^2},$$

thus, the sign of $\frac{\partial p_u}{\partial \alpha}$ is determined by the sign of the numerator. Since $f_2 \frac{\partial f_1}{\partial \alpha} - f_1 \frac{\partial f_2}{\partial \alpha} = (\delta((1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha))+\delta(1+\alpha-\delta)(10\delta-14\alpha-2)+c\delta(-2+7\delta-10\alpha))((2+2\alpha)(7\delta--7\alpha+1)-4\delta(\delta-\alpha))((2+2\alpha)(1+1/(\delta-\alpha))-\delta)-((18\delta-28\alpha-12)((2+2\alpha)(1+1/(\delta-\alpha)((1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha))+c\delta(5\delta-2\alpha+7\alpha\delta-5\alpha^2-2\delta^2+3$ and $\alpha < \delta$. Using extensive numerical analysis by taking values in the whole domain, we can show that $f_2 \frac{\partial f_1}{\partial \alpha} - f_1 \frac{\partial f_2}{\partial \alpha}$ is first positive and then negative as α increases. Namely, the price of used products first increases in the upgraded degree, and then decreases in the upgraded degree.

Appendix C. Proof of Proposition 2

(i) This result is obvious.

(ii)For the sign of $\frac{\partial p_1^*}{\partial \delta}$, we have, $\frac{\partial p_1^*}{\partial \delta} = (-4-8\alpha-2c-2\alpha\delta-4\alpha^2+2\delta^2-2\alpha^2\delta+2\alpha\delta^2-4c\alpha-2c\alpha^2+5\alpha^2\delta^2-10\alpha\delta^3+8c\alpha\delta^2-4c\alpha^2\delta+5\delta^4-4c\delta^3)/(-7\delta+6\alpha-1-9\alpha\delta+7\alpha^2+2\delta^2))^2$. Note that $\alpha < \delta$, so $\frac{\partial p_1^*}{\partial \delta} < 0$.

(iii) Omitted, please refer to the proof of Proposition 1 (iii).

Appendix D. Proof of Proposition 3

To show that $p_u^* - c > 0$, i.e., $p_u^* - c = \frac{\delta(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]+c\delta(5\delta-2\alpha+7\alpha\delta-5\alpha^2-2\delta^2+3)}{2[(7\delta-7\alpha+1)(1+\alpha)-2\delta(\delta-\alpha)][2(1+\alpha)(1+\frac{1}{\delta-\alpha})-\delta]} - c > 0$, it is sufficient to show that $\delta(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]+$

$c\delta(5\delta-2\alpha+7\alpha\delta-5\alpha^2-2\delta^2+3)-2c[(7\delta-7\alpha+1)(1+\alpha)-2\delta(\delta-\alpha)][2(1+\alpha)(1+\frac{1}{\delta-\alpha})-\delta]>0$. That is, $c[[14(\delta-\alpha)(1+\alpha)+2(1+\alpha)-4\delta(\delta-\alpha)][2\frac{1+\alpha}{\delta-\alpha}+2(1+\alpha)-\delta]-\delta(5\delta-2\alpha+7\alpha\delta-5\alpha^2-2\delta^2+3)]<\delta(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]$.

Thus,

$$c<\frac{\delta(\delta-\alpha)(1+\alpha-\delta)[(1+\alpha)(7\delta-7\alpha+5)-3\delta(\delta-\alpha)]}{4(1+\alpha)^2(1+\delta-\alpha)(7\delta-7\alpha+1)+\delta(\delta-\alpha)[(\delta-\alpha)(6\delta-27\alpha-27)-13(1+\alpha)]}.$$

Appendix E. Proof of Propositon 4

If the manufacturer decides to release a new upgraded product in period 2, the profit of releasing a new upgraded product must larger than the profit of selling the original version products in period 2, i.e., $\Pi(p_1^* \mid \alpha>0)>\Pi(p_1^* \mid \alpha=0)$. On the other hand, from the function of manufacturer's optimization problem, it is clear that the profit of the manufacturer $\Pi(p_1)$ decreases linearly in K. Therefore, in order to obtain the threshold value $K^*(\alpha)$, we need to let $\Pi(p_1^* \mid \alpha>0)=\Pi(p_1^* \mid \alpha=0)$. That is, $\overline{\Pi}(p_1^* \mid \alpha>0)-\frac{1}{2}K[(1+\alpha)^2-1]=\overline{\Pi}(p_1^* \mid \alpha=0)$, where $\overline{\Pi}=p_1(q_{NN}+q_{NU})+p_2(q_{NN}+q_{ON})$. Thus, the threshold value $K^*(\alpha)=\frac{2[\overline{\Pi}(p_1^*|\alpha>0)-\overline{\Pi}(p_1^*|\alpha=0)]}{\alpha(\alpha+2)}$, and $\overline{\Pi}(p_1^* \mid \alpha>0)=G(1-\frac{GN+(1+\alpha)H}{N(\delta-\alpha)})+\frac{(1+\alpha)H^2}{2N(1+\alpha-\delta)(\alpha-\delta)}$;

$G=\frac{(\delta-\alpha)(4+4\alpha+\alpha\delta+2c\alpha-\delta^2+2c)}{1+7\delta+9\alpha\delta-7\alpha^2-2\delta^2-6\alpha}$.

$N=2+2\delta+3\alpha\delta-2\alpha^2-\delta^2$;

$H=\alpha^2-G(1+\alpha-\delta)+c\alpha-2\alpha\delta-\delta+\delta^2-c\delta$;

and, $\Pi(p_1^* \mid \alpha>0)$ equals to $\Pi(p_1^* \mid \alpha=0)$, when $\alpha=0$.