Manufacturer-remanufacturing vs supplier-remanufacturing in a closed-loop supply chain

Yu Xiong a, Quanwu Zhao b, Yu Zhou b,*

a Newcastle Business School, Northumbria University, Newcastle NE1 8ST, UK
b School of Economics and Business Administration, Chongqing University, 174 Shazhengjie, Chongqing 400044, PR China

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A B S T R A C T
Remanufacturing at the component level could be performed by either a manufacturer or a supplier. In this paper, we analyze the performance of manufacturer-remanufacturing and supplier-remanufacturing in a decentralized closed-loop supply chain, and examine their desirability from different stakeholder perspectives. We find that the manufacturer may engage in remanufacturing of used components even if remanufacturing is costlier than traditional manufacturing; given remanufacturing is costlier, the manufacturer may forgo remanufacturing due to a marginal increase in consumer willingness-to-pay for the remanufactured product. If the unit remanufacturing cost is high enough, the manufacturer and consumers prefer manufacturer-remanufacturing, while the supplier and the environment prefer supplier-remanufacturing; otherwise, the manufacturer, the supplier, and consumers prefer supplier-remanufacturing, while the environment’s preference is contingent on the environmental impact discount for the remanufactured product. Finally, the key findings are distilled into a roadmap to guide the development of remanufacturing.

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1. Introduction

In recent years, executives around the world are rallying behind sustainability, and have experienced a dramatic increase of interest in remanufacturing. Successful examples from many industries show that remanufacturing can be both faster-growing and more profitable than traditional manufacturing (Ayres et al., 1997, Guide and Wassenhove, 2003, Geyer et al., 2007). However, the remanufacturability of used products as a whole is restricted by increasing technical complexity, shorted product life cycle, rising costs and uncertainties.

Re-manufacturing at the component level is an alternative that may help maximize the revenue generated from the return stream (Fleischmann et al., 2003), which has been a consensus between researchers and managers. In theory, remanufacturing is defined as “a production strategy whose goal is to recover the residual value of used products by reusing components that are still functioning well” (Debo et al., 2005). In practice, the remanufacturing process of Caterpillar (2010) can be briefly described as:

- First, used products collected from customers are disassembled into their constituent components.
- Next, the individual components are remanufactured to exact specifications to ensure they provide the same quality, reliability and durability as they did when they were new.
- Last, remanufactured components are assembled, tested and made ready for sale as the remanufactured product.

In addition, nowadays, few manufacturers rely on only themselves to design and produce the whole product, which implies that most components are provided by their suppliers. Thus, remanufacturing at the component level can be performed by manufacturers such as Caterpillar, or by their key component suppliers. In 2008, Chinese National Development and Reform Commission launched a pilot program of auto part remanufacturing, and 14 firms were selected and supported to start up remanufacturing, seven of which are auto manufacturers (or their subsidiaries) and the other seven are part suppliers (Sina, 2008).

Thus, a research question is naturally emerging: what is the difference between manufacturer-remanufacturing and supplier-remanufacturing? Our primary objective in this paper is to develop a general understanding of the desirability of manufacturer-remanufacturing and supplier-remanufacturing from different stakeholder perspectives.

The literature on managing the closed-loop supply chain with remanufacturing is abundant, we refer the reader to Atasu et al. (2008a), Guide and Van Wassenhove (2009), and Souza (2013) for a thorough discussion. It has been demonstrated that remanufacturing
can be an effective marketing strategy for manufacturers to defend their market share and render a higher profit (Heese et al., 2005, Atasu et al., 2008b, Chen and Chang, 2013, Wu, 2015). However, to the best of our knowledge, few papers consider the possibility of supplier-remanufacturing and identify the “right” remanufacturer, especially from the perspectives of the consumers and the environment. Xiong et al. (2013) make the first attempt to analyze how the interaction between the manufacturer and the supplier on new product production influences to the economic and environmental performance of remanufacturing. We extend Xiong et al. (2013)’s model to analyze and compare the implications of manufacturer-remanufacturing and supplier-remanufacturing.

More importantly, our model deviates from the literature by allowing remanufacturing a used component does not cost less than manufacturing a new one. On one hand, this deviation is greatly motivated by the industrial practice: although some pioneers have made a profit, most manufacturers have no infrastructure and expertise to remanufacture in a profitable manner (Ferguson, 2010). Specifically, in the globalized world today, remanufacturing is still largely a local business because many countries prohibit the international trade of used products. Huawei (2015), the world’s third largest cell phone producer, capitalizes on recycling in Europe, but does not remanufacture. This may be driven by the possibility that producing a remanufactured cell phone in Europe costs more than producing a new one in China. On the other hand, this derivation leads us to some very interesting findings on firms’ remanufacturing strategy. The prior literature on remanufacturing typically defaults that remanufacturing costs less than traditional manufacturing, e.g., in a seminal research, Ferrer and Swaminathan (2006) use the remanufacturing savings as the key parameter to define the strategy space. To the best of our knowledge, only one paper, Caner et al. (2013), considers the situation where remanufacturing is costlier for an integrated manufacturer. However, it finds remanufacturing is seldom profitable in this situation and suggests the manufacturer focus on situations where remanufacturing costs less. In contrast, our work demonstrates the manufacturer could be better off by engaging in remanufacturing even if it costs more than manufacturing in a decentralized supply chain. In addition, given remanufacturing is costlier, the analytical result shows that the manufacturer may decide to forgo remanufacturing as a result of a marginal increase in consumer willingness-to-pay for the remanufactured product. These findings make an excellent complement to the current literature on remanufacturing.

The rest of this paper is organized as follows. Section 2 delineates our modeling assumptions and notation. Section 3 presents the analysis and solutions of two models with manufacturer-remanufacturing and supplier-remanufacturing, respectively. Section 4 discusses firms’ remanufacturing strategy and identifies the “right” remanufacturer from different stakeholder perspectives. Section 5 concludes this paper. Appendices contains the detailed proofs of all propositions. Hereinafter, for convenience, we use pronouns ‘she’ and ‘he’ to refer to the supplier and the manufacturer, respectively.

2. Assumptions and notation

We consider an industry with only one product but two versions: the new product and the remanufactured product. To focus our attention on the desirability of manufacturer-remanufacturing and supplier-remanufacturing from different stakeholder perspectives, we consider a simple bilateral monopoly, as depicted in Figs. 1 and 2. In this paper, we do not consider the reverse channel choice, which has been widely studied in the existing literature, e.g., Xiong et al. (2014), Hong et al. (2015), and Wei et al. (2015).

Therefore, it is reasonable to assume that used products are collected (by the retailer, or the manufacturer, or the third-party operator) at a constant cost, which is normalized to 0.

To isolate the strategic issue of remanufacturing, our model rules out the distortion due to efficiency variance by assuming that either the manufacturer or the supplier costs $c_r$ to remanufacture a used component. Similar assumptions have been widely used in the literature, e.g., Savaskan et al. (2004) assume a manufacturer and a retailer incur a same cost to collect used products, and demonstrate the retailer-managed collection is always preferred by the manufacturer; Zhou et al. (2013) assume centralization and decentralization within a manufacturer are equivalent in terms of the cost structure, and find decentralization outperforms centralization under certain conditions. For the sake of clarity, we assume that, except for the cost to obtain the new/remanufactured component, the manufacturer’s other operating costs are constant and normalized to 0.

Other key assumptions concerning consumer preference, environmental performance, and decision-making rule are borrowed from the literature on closed-loop supply chain management, e.g., Galbreth et al. (2013), Xiong et al. (2013), Chang et al. (2015), and Gu et al. (2015). Here, we present the following set of assumptions, but skip the detailed discussion on their justification. For convenience, Table 1 summarizes the notation used in the model.

**Assumption 1.** The inverse demand functions for new and remanufactured products are

\[ p_n = 1 - q_n - \delta q_r, \]  \hfill (1)

\[ p_r = \delta(1 - q_n - q_r). \]  \hfill (2)

Assumption 1 implies that the consumer willingness-to-pay for the new product is heterogeneous and distributed over the interval $[0, 1]$ with the density of 1; each consumer’s willingness-to-pay for the remanufactured product is a fraction $\delta \in (0, 1)$ of that for the new one; and each customer buys at most one product that offers the most utility, as long as the net utility is positive. Thus,
the linear inverse demand functions, \((\text{Eqs. (1)) and (2))\), can be derived from consumers’ utility functions.

**Assumption 2.** The weighted production quantity of new products and remanufactured products \(q_n + \phi q_r\), is used as a proxy of the closed-loop supply chain’s environmental performance.

It is broadly agreed that the process of remanufacturing has less negative impact on the environmental. **Assumption 2** implies that the life-cycle environmental impact of one unit remanufactured product is a fraction \(\phi \in (0, 1)\) of that of one unit new product. Therefore, the closed-loop supply chain’s environmental performance is equal to one unit new product’s life-cycle environmental impact multiplied by the weighted production quantity of new products and remanufactured products. Regardless of the value of environmental impact, \(q_n + \phi q_r\), can be a proxy.

**Assumption 3.** All decisions are considered in a steady-state period: the supplier moves first to price the new component (and the remanufactured component), and then the manufacturer responds by determining the production quantity of new and remanufactured products.

Closed-loop supply chain management is a typical multiple-period problem because new products are used for a certain period and then become cores for remanufacturing. The steady-state period model implies that players use the same policy in every period after the ramp-up in the first period in an infinite horizon setting. It enables us to analytically address our research question without the distraction of initial and terminal time-period effect. Thus, in our model, by assuming that each product can be used for one period and remanufactured at most once, the production quantity of remanufactured products in the current period is bounded by the production quantity of new products in the previous period, which is equal to the production quantity of new products in the previous period, i.e., \(q_r \leq q_n\). Admittedly, not all used products could be collected; in practice, we have \(q_r \leq q_n\). However, assuming \(\tau = 1\) in this paper does not change any of qualitative insights.

In addition, we assume that the manufacturer and the supplier are risk-neutral and profit seeking, and have perfect knowledge of the demand and cost information – a reasonable assumption in the steady-state period model. In order to guarantee the market demand of new and remanufactured products is non-negative, our model requires \(c_n \leq 1\) and \(c_r \leq 1\). In the following analysis, we call the firm who performs remanufacturing as the manufacturer; subscript \(i \in \{M, S\}\) refers to the manufacturer and the supplier, respectively; superscript \(j \in \{M, S\}\) manufacturer-remanufacturing and supplier-remanufacturing, respectively. The firms’ strategic decisions are analyzed under various scenarios, which are distinguished by parameters \(c_n, c_r\), and \(\delta\). Subscript \(k \in \{1, 2, 3\}\) indicates the scenario under which our analysis is proceeding.

### 3. Models and solutions

#### 3.1. The model of manufacturer-remanufacturing

In this subsection, we analyze the model of manufacturer-remanufacturing, in which the suppliers supplies only the new component to the manufacturer. The supplier’s and the manufacturer’s profit functions can be written as

\[
\Pi^M_n = (W_n - c_n)q_n, \quad \Pi^M_r = (p_{n} - q_{n} - c_r)q_r, \quad \text{s.t. } 0 \leq q_r \leq q_n.
\]

The interaction between the supplier and the manufacturer can be analyzed using backward induction. For a given \(w_n\), the manufacturer determines \(q_n\) and \(q_r\) to maximize his profit. The optimal production quantity decisions are characterized by the following proposition.

**Proposition 1.** In the model of manufacturer-remanufacturing, the supplier’s optimal production quantity decision with respect to the supplier’s new component wholesale price is

\[
\begin{align*}
(1) \quad q^M_n &= (1 - w_n)/2, \quad \phi q^M_r = 0, \quad \text{if } w_n < c_r/\delta; \\
(2) \quad q^M_n &= (1 - w_n - \delta + c_r)/2(1 - \delta), \quad \phi q^M_r = (\delta w_n - c_r)/2(1 - \delta), \quad \text{if } c_r/\delta \leq w_n \leq (c_r + \delta c_r + \delta - \delta^2)/2\delta; \\
(3) \quad q^M_n &= q^M_r = (1 - w_n - \delta - c_r)/2(1 + 3\delta), \quad \text{if } w_n > (c_r + \delta c_r + \delta - \delta^2)/2\delta.
\end{align*}
\]

Next, when setting \(w_n\), the supplier does so with anticipation that the manufacturer will respond as above. Since the manufacturer’s optimal response is contingent on the value of \(w_n\), the process to derive the supplier’s optimal new component wholesale price consists of two steps: (1) we analyze the scenario in which the supplier induces the manufacturer to choose a certain decision; and then (2) we identify the optimal solution by comparing the supplier’s profit in all scenarios. For parsimony we restrict our following analysis to the case of \(\delta < 1/2\). The optimal new component wholesale price for the case of \(\delta > 1/2\) is available in the Proof of Proposition 2, which is a simplification of that for the case of \(\delta < 1/2\).

**Proposition 2.** In the model of manufacturer-remanufacturing, the supplier’s optimal new component wholesale price is

\[
\begin{align*}
(1) \quad w^M_n &= (1 + c_n)/2, \quad \text{if } c_r > c^M_r; \\
(2) \quad w^M_n &= c_r/\delta, \quad \text{if } c^M_r < c_r < c^M_r; \\
(3) \quad w^M_n &= (1 + c_n - \delta + c_r)/2, \quad \text{if } c_r < c^M_r; \\
(4) \quad w^M_n &= (1 + c_n - \delta - c_r)/2, \quad \text{if } c_r \leq c^M_r; \quad \text{here, } c^M_r = \delta(1 + c_r)/2, \\
& \quad c^M_n = \delta(1 + c_n - \delta)/(2 - \delta), \\
& \quad c^M_r = (c_n + \delta c_n + 2\delta^2 - 1 - \delta + \sqrt{A})/2\delta, \\
& \quad A = 1 - 2c_n + c_n^2 + 2\delta^2 - 4\delta c_n + 2\delta c_n^2 + 3\delta^2 + 6\delta^2 c_n - 3\delta^2 c_n^2.
\end{align*}
\]

It is worth noting that, the supplier has taken the manufacturer’s optimal response into account when setting \(w_n\). So, if the supplier’s optimal decision is \(w^M_n\), then the manufacturer’s optimal decision must be \(\{q^M_n, q^M_r\}\). Substituting these optimal decisions back into Eqs. (3) and (4) gives the supplier’s and the manufacturer’s profits, as shown in Table 2.

### 3.2. The model of supplier-remanufacturing

In the model of supplier-remanufacturing, the supplier supplies both the new and the remanufactured components to the manufacturer. Their profit functions are

\[
\Pi^M_n = (W_n - c_n)q_n + (W_r - c_r)q_r,
\]

#### Table 1

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_n, c_r)</td>
<td>The unit production cost of the new/remanufactured component</td>
</tr>
<tr>
<td>(q_n, q_r)</td>
<td>The production quantity of new/remanufactured component</td>
</tr>
<tr>
<td>(p_n, p_r)</td>
<td>The market clearing price of the new/remanufactured product</td>
</tr>
<tr>
<td>(w_n, w_r)</td>
<td>The wholesale price of the new/remanufactured component</td>
</tr>
<tr>
<td>(\delta)</td>
<td>The consumer value discount for the remanufactured product</td>
</tr>
<tr>
<td>(\phi)</td>
<td>The environmental impact discount for the remanufactured product</td>
</tr>
<tr>
<td>(I_f^k)</td>
<td>The player’s profit in scenario (k) of the model of (f)</td>
</tr>
<tr>
<td>(v^k)</td>
<td>The consumer surplus in scenario (k) of the model of (f)</td>
</tr>
<tr>
<td>(E^k)</td>
<td>The supply chain’s environmental performance in scenario (k) of the model of (f)</td>
</tr>
</tbody>
</table>
\[\Pi_{SM}^H = (p_i(q_i, q_r) - w_q)q_i + (p_i(q_i, q_r) - w_q)q_r, \text{ s.t. } 0 \leq q_i \leq q_n. \quad (6)\]

Following from Eqs. (4) and (6), we have, replacing \(c_r\) in the manufacturer's profit function in the model of manufacturer-remanufacturing with \(w_q\), gives his profit function in the model of supplier-remanufacturing. Thus, intuitively, replacing \(c_r\) in Proposition 1 with \(w_q\), gives the manufacturer's optimal production quantity decision in the model of supplier-remanufacturing.

Similarly, we identify the supplier's optimal decision in two steps, as follows.

**Proposition 3.** In the model of supplier-remanufacturing, the supplier's optimal wholesale prices for new and remanufactured components are

\[w_{q1}^o = \frac{1 + c_n}{2}, w_{q2}^o = \delta (1 + c_n)/2, \text{ if } c_r > c_l^3;\]

\[w_{q2}^o = \frac{1 + c_n}{2}, w_{q2}^o = \frac{\delta (1 + c_n)}{2}, \text{ if } c_r \leq c_r \leq c_l^3;\]

\[w_{q3}^o = \frac{1 + 4\delta - c_r^2 + (1 + \delta)(c_n + c_r)}{2(1 + 3\delta)} \text{ if } c_r < c_l^3; \text{ here, } c_r = \delta c_n, \quad c_r = \delta (2c_n + \delta - 1)/(1 + \delta).\]

Substituting these optimal decisions back into Eqs. (5) and (6) gives the supplier's and the manufacturer's profits, as shown in Table 3.

### 4. Comparison and discussion

#### 4.1. Whether to remanufacture

In this subsection, we examine the decision on whether to remanufacturing. Substituting the optimal wholesale price(s) in Propositions 2 and 3 back into the manufacturer's optimal response function gives the production quantity of new and remanufactured products. We say the remanufacturer decides to engage in remanufacturing if \(q_i > 0\). It is easy to get the following Corollaries:

**Corollary 1.** In the model of manufacturer-remanufacturing, there exists a threshold value \(c_r^4\) such that the manufacturer should engage in remanufacturing if \(c_r \leq c_r^4\); the threshold value \(c_r^4\) can be obtained if \(c_n < \delta^2/2\).

**Corollary 2.** In the model of supplier-remanufacturing, there exists a threshold value \(c_r^4\) such that the supplier should engage in remanufacturing if \(c_r > c_r^4\); the threshold value \(c_r^4\) is determined if \(c_n < \delta^2/2\).

**Corollaries 1 and 2** show that in both models, the impact of the remanufacturing cost \(c_r\) on the decision of whether to remanufacture is monotone, i.e., the lower the value of \(c_r\), the more likely the remanufacturer is to engage in remanufacturing. When \(c_r < c_r^4\), the threshold value \(c_r^4\) is determined if \(c_n < \delta^2/2\).

### Table 2

<table>
<thead>
<tr>
<th>(c_r \leq c_r^4)</th>
<th>(c_r^4 &lt; c_r \leq c_r^5)</th>
<th>(c_r^5 &lt; c_r \leq c_r^6)</th>
<th>(c_r &gt; c_r^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{q1}^o = \frac{1 + c_n}{2} (1 + c_n + \delta) c_n + 4 (c_n - \delta) c_n + 6 (c_n - \delta) c_n + 2 \delta^2 + 3 \delta )</td>
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</table>

### Table 3

<table>
<thead>
<tr>
<th>(c_r &lt; c_r^4)</th>
<th>(c_r^4 &lt; c_r \leq c_r^5)</th>
<th>(c_r^5 &lt; c_r^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{q1}^o = \frac{1 + c_n}{2} (1 + c_n + \delta) c_n + 4 (c_n - \delta) c_n + 6 (c_n - \delta) c_n + 2 \delta^2 + 3 \delta )</td>
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</table>

#### 4.2. Who is the “right” remanufacturer?

In this subsection, we identify the “right” remanufacturer from different stakeholder perspectives. Following from Propositions 2 and 3, we have six scenarios to examine the desirability of manufacturer-remanufacturing and supplier-remanufacturing, as shown in Table 4 and illustrated in Fig. 3. Here, we list only the
supplier’s optimal decision in each scenario. As said before, if the supplier’s optimal decision is \( w_t^M \) or \( \{ w_t^S, w_t^L \} \), then the manufacturer’s optimal response must be \( \{ q_{tm}, q_{tk} \} \). It is worth noting that, the solid line in Fig. 3 refers to \( c_r = c_{r}^{M} \), which separates the manufacturer’s, the consumers’ and the environment’s preferences to manufacturer-remanufacturing and supplier-remanufacturing, as demonstrated by the following Propositions.

In line with intuition, if the remanufacturing cost is sufficiently low \( c_r < c_{r}^{S} \) or sufficiently high \( c_r > c_{r}^{M} \), all used products or no used product will be remanufactured in both models, and then manufacturer-remanufacturing and supplier-remanufacturing are equivalent from the perspective of all stakeholders (as shown in Proofs of Propositions 4–7). Thus, in what follows, we focus on the scenarios under the condition \( c_{r}^{S} \leq c_r \leq c_{r}^{M} \), i.e., Scenarios 2–5.

From the perspective of the manufacturer, we identify the “right” remanufacturer as the firm who makes the manufacturer obtain a higher profit. Thus, by comparing the manufacturer’s profits in models of manufacturer-remanufacturing and supplier-remanufacturing, we have the following proposition.

**Proposition 4.** From the perspective of the manufacturer, manufacturer-remanufacturing is preferred if \( c_r > c_{r}^{M} \), otherwise, supplier-remanufacturing is preferred.

Following from Eqs. (4) and (6), letting the supplier remanufacture, the manufacturer’s cost to obtain a unit remanufactured component will change from \( c_r \) to \( w_t \); in addition, a profit seeking supplier must price \( w_t \) higher than \( c_r \). Well, why does the manufacturer still benefit from supplier-remanufacturing if the remanufacturing cost is low enough? The economic intuition behind Proposition 4 lies in that, if \( c_r \leq c_{r}^{M} \), remanufacturing is so profitable that the manufacturer is willing to remanufacture all used products, and then new and remanufactured products exhibit the characteristics of complements (Debo et al., 2005, Xiong et al., 2013). Therefore, the supplier can appropriate the remanufacturing benefit by pricing the new component higher, which leads to a smaller optimal production quantity of new and remanufactured products and consequently, makes manufacturer-remanufacturing less attractive for the manufacturer. By contrast, with supplier-remanufacturing, although the cost to obtain a unit remanufactured component is higher, the manufacturer can obtain a lower wholesale price for the new component, e.g., \( w_t^M > w_t^S \). So supplier-remanufacturing is more desirable for the manufacturer if the remanufacturing cost is low enough.

Similarly, from the perspective of the supplier, the “right” remanufacturer is identified as follows.

**Proposition 5.** From the perspective of the supplier, supplier-remanufacturing is always preferred over manufacturer-remanufacturing.

Proposition 5 shows that manufacturer-remanufacturing is always detrimental to the supplier. The economic intuition behind this result is straightforward. On one hand, if \( c_r > c_{r}^{M} \), the remanufactured component is a substitute for the new component, then manufacture-remanufacturing will cannibalize the sales of the new component and hurt the supplier. On the other hand, if \( c_r \leq c_{r}^{M} \), with manufacturer-remanufacturing, the profit seeking supplier will strategically price the new component higher and the profit seeking manufacturer will strategically produce fewer new products, which forms a loss-loss situation. As a result, manufacturer-remanufacturing is never preferred by the supplier.

From the perspective of the consumers, we identify the “right” remanufacturer as the firm who makes consumers obtain a greater surplus. Based on our linear inverse demand functions, consumer’s surplus is calculated as

\[
\upsilon_k = \frac{1}{2} \left( \frac{1}{1-p_{i|k}} q_{ik} + \frac{1}{2} (\delta - p_{i|k}) q_{ik} \right)
\]

Comparing the consumers’ surpluses in models of manufacturer-remanufacturing and supplier-remanufacturing gives the following proposition.

**Proposition 6.** From the perspective of the consumers, manufacturer-remanufacturing is preferred if \( c_r > c_{r}^{M} \), otherwise, supplier-remanufacturing is preferred.

Following from Propositions 4 and 6, it is revealed that consumers have the same preference as the manufacturer. This is because, on one hand, if \( c_r > c_{r}^{M} \), the manufacturer will engage in remanufacturing, but the supplier will not, and then remanufacturing drives down the new product price and provides a low-price alternative to the consumers who cannot afford the new product, so manufacturer-remanufacturing is more preferable; on the other hand, if \( c_r \leq c_{r}^{M} \), as said before, manufacturer-remanufacturing results in a higher new product price and consequently a lower production quantity, which reduce the consumers’ surplus, and then supplier-remanufacturing is naturally more preferable.

It is worth noting that, we use the weighted production quantity \( q_{w} + \phi q_{r} \) as a proxy of the closed-loop supply chain’s environmental performance. From the perspective of the environment, the “right” remanufacturer is identified as the firm whose remanufacturing business leads to less impact on the environment, i.e., a fewer weighted production quantity \( q_{w} + \phi q_{r} \).

**Proposition 7.** From the perspective of the environment, if \( c_r > c_{r}^{M} \), supplier-remanufacturing is preferred regardless of the value of \( \phi \); if \( c_r \leq c_{r}^{M} \), supplier-remanufacturing is preferred if \( \phi \) is

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Condition</th>
<th>( w_t^M )</th>
<th>( { w_t^S, w_t^L } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( c_r &gt; c_{r}^{M} )</td>
<td>( w_t^M )</td>
<td>( { w_t^S, w_t^L } )</td>
</tr>
<tr>
<td>2</td>
<td>( c_{r}^{M} &lt; c_r &lt; c_{r}^{S} )</td>
<td>( w_t^S )</td>
<td>( { w_t^S, w_t^L } )</td>
</tr>
<tr>
<td>3</td>
<td>( c_{r}^{S} &lt; c_r &lt; c_{r}^{M} )</td>
<td>( w_t^L )</td>
<td>( { w_t^S, w_t^L } )</td>
</tr>
<tr>
<td>4</td>
<td>( c_{r}^{L} &lt; c_r &lt; c_{r}^{S} )</td>
<td>( w_t^M )</td>
<td>( { w_t^S, w_t^L } )</td>
</tr>
<tr>
<td>5</td>
<td>( c_{r}^{S} &lt; c_r &lt; c_{r}^{L} )</td>
<td>( w_t^S )</td>
<td>( { w_t^S, w_t^L } )</td>
</tr>
<tr>
<td>6</td>
<td>( c_r &lt; c_{r}^{L} )</td>
<td>( w_t^L )</td>
<td>( { w_t^S, w_t^L } )</td>
</tr>
</tbody>
</table>

Table 4. These six scenarios of comparison.
large enough, otherwise, manufacturer-remanufacturing is preferred.

It is worth noting that, if $c_\epsilon > c_\phi^M$, the profit seeking supplier does not remanufacture used products. However, Proposition 7 reveals that supplier-remanufacturing is then preferred from the perspective of the environment. The economic intuition has been discussed by Xiong et al. (2013). Even with $\phi = 0$, although remanufacturing cannibalizes the sales of the new product, the profit seeking supplier will strategically lower the new component price, e.g., $w_{2,\cdot}^M < w_{1,\cdot}^M$, making the manufacturer better off by producing more new products, so remanufacturing is then detrimental to the environment.

On the other hand, if $c_\epsilon \leq c_\phi^M$, in the model of manufacturer-remanufacturing, fewer new products will be produced; however, all used products will be remanufactured. Therefore, manufacturer-remanufacturing leads to fewer new products but more remanufactured products compared with supplier-remanufacturing. Intuitively, the desirability of manufacturer-remanufacturing and supplier-remanufacturing depends on the value of $\phi$.

5. Conclusions

In this paper, motivated by the pilot program of auto part remanufacturing in China, we analyze the performance of manufacturer-remanufacturing and supplier-remanufacturing, and examine their desirability from different stakeholder perspectives. Most of our modeling elements are widely used in the literature, but a main deviation lies in the unit remanufacturing cost is allowed to be higher than the unit manufacturing cost, which is motivated by the fact that not all manufacturers and suppliers have the infrastructure and expertise to remanufacturing cost-efficiently.

Our analytical result confirms that both the manufacturer and the supplier are more likely to engage in remanufacturing as the decreasing of the unit remanufacturing cost. However, a less-intuitive finding is that the manufacturer (and only the manufacturer) may engage in remanufacturing even if remanufacturing a used component is costlier than manufacturing a new one. This finding implies that manufacturers could start up remanufacturing even if its technology is not sophisticated, which is consistent with our observation of the development of remanufacturing in many industries where high-profile manufacturers like Boeing, Caterpillar, General Electric, IBM, Kodak, Volkswagen and Xerox initiate a business model in which remanufacturing is an integral part.

We also find that if remanufacturing costs less, both the manufacturer and the supplier are more likely to engage in remanufacturing as a marginal increase in consumer willingness-to-pay for the remanufactured product; in contrast, if remanufacturing costs more, the manufacturer may forgo remanufacturing due to a marginal increase in consumer willingness-to-pay for the remanufactured product. Furthermore, it is demonstrated that supplier-remanufacturing is a dominant strategy for both the manufacturer and the supplier if the remanufacturing cost is low enough.

These findings delineate a clear trajectory to guide the development of remanufacturing from a business perspective. At the early stage when the remanufacturing technology is unsophisticated, i.e., remanufacturing has a cost disadvantage, manufacturers should pioneer, and then the direction to promote remanufacturing is to invest in process innovation and lower the remanufacturing cost. As the remanufacturing technology becomes more sophisticated, i.e., remanufacturing enjoys a cost advantage, suppliers should be encouraged to engage in remanufacturing, and then tactics such as consumer education could be adopted to increase consumer willingness-to-pay for the remanufactured product and accelerate the development of remanufacturing.

This paper also examines the desirability of manufacturer-remanufacturing and supplier-remanufacturing from the perspective of the consumers and the environment, which may guide consumer groups and environmental organizations to lobby. As Xiong et al. (2013) commented, a simple governmental policy to spur more remanufacturing activities may be detrimental to both the industry and the environment. Given the tensions between different stakeholder perspectives, the government has to make a tradeoff and deliberate on the policy to take the full advantage of remanufacturing for a sustainable future.

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Appendices

Proof of Proposition 1

In the model of manufacturer-remanufacturing, the Lagrangean and the KKT optimality conditions for the manufacturer’s optimization problem are

$$L = \left(p_n (q_n^M - q_n^M) - w_n\right)q_n^M + (p_l (q_l^M - q_l^M) - c_l)q_l^M + \lambda_1 q_l^M - \lambda_2 (q_l^M - q_n^M),$$

(A1)

$$\partial L / \partial q_n^M = 1 - 2q_n^M - 2\delta q_l^M - w_n + \lambda_2 = 0,$$

(A2)

$$\partial L / \partial q_l^M = -\delta (q_n^M + q_l^M) + \delta (1 - q_n^M - q_l^M) - c_l + \lambda_1 - \lambda_2 = 0,$$

(A3)

$$\lambda_1 q_n^M = \lambda_2 (q_n^M - q_l^M) = 0,$$

(A4)

$$q_n^M \geq q_l^M \geq 0.$$  

(A5)

Because the multipliers $\lambda_1$ and $\lambda_2$ can be either zero or positive, we have four scenarios to examine.

Scenario 1 with $\lambda_1 > 0$ and $\lambda_2 = 0$: we have $q_n^M = 0$ according to the optimality condition (A4); substituting $\lambda_2 = 0$ and $q_n^M = 0$ back into Eqs. (A2) and (A3) gives $q_l^M = (1 - w_n)/2$ and $\lambda_1 = c_l - \delta w_n$; here, $\lambda_1 > 0$ requires $w_n < c_l/\delta$.

Scenario 2 with $\lambda_1 = 0$ and $\lambda_2 = 0$: by solving simultaneous Eqs. (A2) and (A3), we have $q_n^M = (1 - w_n - \delta + c_l)/2(1 - \delta)$ and $q_l^M = (\delta w_n - c_l)/(2(1 - \delta))$; the optimality condition (A5) requires $\delta w_n \leq (c_l + \delta c_l - \delta \delta^2)/2\delta$.

Scenario 3 with $\lambda_1 > 0$ and $\lambda_2 > 0$: similar to Scenario 1, we have $q_n^M = q_l^M = (1 - w_n + \delta - c_l)/(2(1 + 3\delta))$ and $\lambda_2 = (2\delta w_n + \delta^2 - \delta - c_l - \delta c_l)/(1 + 3\delta)$, which requires $w_n > (c_l + \delta c_l + \delta - \delta^2)/2\delta$.

Scenario 4 with $\lambda_1 > 0$ and $\lambda_2 > 0$: according to the optimality condition (A4), we have $q_n^M = q_l^M = 0$, which is trivial and discarded.

Proof of Proposition 2

Scenario M1. We assume that in this scenario the supplier behaves to let the manufacturer chooses $\{q_n^M, q_l^M\}$. Thus, her
subject to $\frac{w_{0}^{M}}{c_{0}} < c_{1}/\delta$, which guarantees that the manufacturer will respond by choosing $\{q_{01}^{M}, q_{02}^{M}\}$, see Proposition 1. It is easy to get the unconstrained optimal solution $w_{n_{3}}^{M} = (1 + c_{3}/\delta)/2$, which is the optimal wholesale price if $c_{1} > \delta(1 + c_{3}/\delta)$ according to the constraint $w_{n_{3}}^{M} < c_{1}/\delta$. If $c_{1} \leq \delta(1 + c_{3}/\delta)$, then the optimal wholesale price in this case is infinitely close to $c_{1}/\delta$, which is dominated by $w_{n_{3}}^{M} = c_{1}/\delta$ (we get this solution in the next case).

Scenario M2. In this scenario, we assume that the supplier behaves to let the manufacturer chooses $\{q_{02}^{M}, q_{1M}^{M}\}$. The optimization problem is

$$\max \quad P_{n_{3}}^{M} = (w_{n_{3}}^{M} - c_{0})q_{1M}^{M},$$

subject to $c_{1}/\delta \leq w_{n_{3}}^{M} \leq \left( c_{1} + \delta c_{n} - \delta - \delta^{2} \right)/2\delta$. Similar to the proof of Proposition 1, we have the optimal wholesale price $w_{n_{3}}^{M} = (1 + c_{3} - c_{n} - c_{2}/\delta)/2$ if $c_{1} < \delta(1 + c_{2}/\delta)$.

Next, we want to identify the optimal wholesale price on the whole parameter space. With $\delta < 1/2$, it is easy to prove that $\delta(1 + c_{2}/\delta)/(1 + 2\delta) \geq \delta(1 + c_{3}/\delta)/\delta(1 + c_{2}/\delta) > \delta(1 + c_{3}/\delta)/\delta(1 + c_{2}/\delta)$.

Thus, if $c_{1} > \delta(1 + c_{2}/\delta)/2$, the supplier has two possible solutions: (1) $w_{n_{3}}^{M}$ and letting the manufacturer chooses $\{q_{01}^{M}, q_{02}^{M}\}$, and (2) $w_{n_{3}}^{M-1}$ and the manufacturer chooses $\{q_{01}^{M-1}, q_{02}^{M-1}\}$. Solving for $P_{n_{3}}^{M} = P_{n_{3}}^{M-1}$, we get $w_{n_{3}}^{M} = (1 + \delta c_{n} - c_{2}/\delta)/2$. Thus, in this situation, the wholesale price $w_{n_{3}}^{M}$ is a dominant strategy for the supplier. Similarly, it can be demonstrated that from the perspective of the supplier, $w_{n_{3}}^{M}$ is preferred over $w_{n_{3}}^{M-1}$ if $\delta(1 + c_{2}/\delta)/(1 + 2\delta) < \delta(1 + c_{3}/\delta)/\delta(1 + c_{2}/\delta) > \delta(1 + c_{3}/\delta)/\delta(1 + c_{2}/\delta)$.

If $\delta c_{n} < \delta(1 + c_{2}/\delta)/(1 + 2\delta)$, two possible solutions are: (1) $w_{n_{3}}^{M-2}$ and letting the manufacturer chooses $\{q_{01}^{M-2}, q_{02}^{M-2}\}$, and (2) $w_{n_{3}}^{M-1}$ and the manufacturer chooses $\{q_{01}^{M-1}, q_{02}^{M-1}\}$. Let $\Delta_{M} = P_{n_{3}}^{M} - P_{n_{3}}^{M-1}$, we have $\Delta_{M} < 0$, $\Delta_{M} = \delta(1 + c_{3}/\delta)/(2 - \delta) - 2\delta(1 + c_{3}/\delta)/(2 - \delta) > 0$, and $\Delta_{M} > 0$ if $\delta > 1/2$, $\Delta_{M} = \delta(1 + c_{3}/\delta)/(2 - \delta) - 1/2 > 0$, and then there exists $c_{2}^{*}$ (its expression can be found in the last section) such that $w_{n_{3}}^{M}$ is preferred if $\delta c_{n} < c_{2}^{*}$, otherwise, $w_{n_{3}}^{M-1}$ is preferred.

The Scenario with $\delta < 1/2$ is a simplification of that with $\delta > 1/2$. With $1/2 < \delta < 1$, we can skip the comparison of $w_{n_{3}}^{M-1}$ and $w_{n_{3}}^{M}$. The results are all the same in the Scenario with $1/2 < \delta < 1$. Combining the above analysis and comparison gives the supplier's optimal wholesale price of the new component in the model of manufacturer-remanufacturing.

**Proof of Proposition 3**

Scenario S1. In this scenario, we assume that the supplier behaves to let the manufacturer chooses $\{q_{1S}^{S}, q_{2S}^{S}\}$. The optimization problem is

$$\max \quad P_{n_{3}}^{S} = (w_{n_{3}}^{S} - c_{0})q_{1S}^{S} + (w_{n_{3}}^{S} - c_{1})q_{2S}^{S},$$

subject to $w_{n_{3}}^{S} < w_{n_{3}}^{S}/\delta$. It is easy to get that the unconstrained solution is $w_{n_{3}}^{S} = (1 + c_{3}/\delta)$, which is the optimal wholesale price of the new component if $w_{n_{3}}^{S} > \delta w_{n_{3}}^{S}$. Because $P_{n_{3}}^{S}$ is independent in $w_{n_{3}}^{S}$, we do not care about the exact value of $w_{n_{3}}^{S}$. As we will get in the next case, with $(w_{n_{3}}^{S-1}, w_{n_{3}}^{S-1})$, we have $q_{1S}^{S} = q_{2S}^{S} = 0$. Thus, the solutions, $(w_{n_{3}}^{S}, w_{n_{3}}^{S})$ and $(w_{n_{3}}^{S-1}, w_{n_{3}}^{S-1})$, are equivalent.

Scenario S2. In this scenario, we assume that the supplier behaves to let the manufacturer chooses $\{q_{02}^{S}, q_{1S}^{S}\}$. The optimization problem is

$$\max \quad P_{n_{3}}^{S} = (w_{n_{3}}^{S} - c_{0})q_{1S}^{S} + (w_{n_{3}}^{S} - c_{1})q_{2S}^{S},$$

subject to $w_{n_{3}}^{S} < w_{n_{3}}^{S}/\delta$. Similar to the Proof of Proposition 1, we have the optimal wholesale prices of new and remanufactured components

$(1) \quad w_{n_{3}}^{S-1} = (1 + c_{3}/\delta)/2$, $w_{n_{3}}^{S-2} = (1 + c_{3}/\delta)/2 - \delta(1 + c_{n}/\delta)$.

$(2) \quad w_{n_{3}}^{S-1} = (1 + c_{3}/\delta)/2$, $w_{n_{3}}^{S-2} = (1 + c_{3}/\delta)/2 - \delta(1 + c_{n}/\delta)$.

$(3) \quad w_{n_{3}}^{S-1} = (1 + 4\delta - \delta^{2} + (1 + \delta)(c_{n} + c_{1})/2\delta)$.

Clearly, $P_{n_{3}}^{S}$ is concave in the sum of $w_{n}^{S}$ and $w_{n}^{S}$. We have the unconstrained solutions satisfy $w_{n_{3}}^{S} + w_{n_{3}}^{S} = (1 + \delta c_{n} + c_{1}/\delta)/2$. Without loss of generality, we set $w_{n_{3}}^{S} = w_{n_{3}}^{S-2} = (1 + \delta c_{n} + c_{1}/\delta)/2$, and the constraint $w_{n_{3}}^{S} > \delta w_{n_{3}}^{S} + \delta - \delta^{2}/2$. Thus, we say that the solutions, $(w_{n_{3}}^{S}, w_{n_{3}}^{S})$ and $(w_{n_{3}}^{S-2}, w_{n_{3}}^{S-2})$, are equivalent.

Combining the above analysis and comparison gives the supplier's optimal wholesale prices of new and remanufactured components in the model of supplier-remanufacturing.

**Proof of Proposition 4**

Clearly, we have six scenarios to compare the manufacturer's profits. We define $\Delta_{M} = P_{n_{3}}^{M} - P_{n_{3}}^{M}$ in scenario I.

In Scenario 1 with $c_{1} > c_{3}^{*}$, $\Delta_{M} = 0$.

In Scenario 2 with $c_{2}^{*} < c_{1} < c_{3}^{*}$, $\Delta_{M} = 0$.

In Scenario 3 with $c_{1} < c_{1} < c_{2}^{*}$, similar to Scenario 2, we have $\Delta_{M} = 0$.

In Scenario 4 with $c_{2}^{*} < c_{1} < c_{3}^{*}$, $\Delta_{M} = 0$.
In Scenario 5 with \( c_3^2 \leq c_4 \leq c_1^2 \), \( \Delta_{M5} = -\left( \delta + c_1 + \delta c_1 - 2\delta c_n - \delta^2 \right)^2 / 16\delta (1 + 3\delta) (1 - \delta) \leq 0 \).

In Scenario 6 with \( c_4 < c_3, \Delta_{M6} = 0 \).

Combining these results in all six scenarios gives Proposition 4.

**Proof of Proposition 5**

Similarly, we define \( \Delta_4 = P_M^M - P_i \) in Scenario 1.

In Scenario 1 with \( c_1 > c_3^M, \Delta_{S1} = 0 \).

In Scenario 2 with \( c_1 \leq c_3^M, \Delta_{S2} = -\left( \delta + \delta c_1 - 2c_1 \right)^2 / 8\delta < 0 \).

In Scenario 3 with \( c_1^M < \delta c_1 \leq c_3^M, \delta \Delta_4/c_2^M = 1/8(1 - \delta) > 0 \), letting \( \delta \Delta_3/c_2 \delta c_2 = 0 \), we have \( c_4 = c_3^M \), that is to say, \( \Delta_{S3} \) reaches its global minimum at \( c_4 = c_3^M \). We have \( \Delta_{S2} \geq \Delta_{S3} (c_4 = c_3^M) = 0 \).

In Scenario 3 with \( c_1^M < c_3 < c_3^M, \delta \Delta_4/c_2^M = -\left( 8 - 6\delta \right)/32\delta (1 - \delta) < 0 \), letting \( \delta \Delta_3/c_2 = 0 \), we have \( c_4 = c_3 - 3\delta c_3 - 3\delta c_3 \delta (4 - 3\delta) \), that is to say, \( \Delta_{S3} \) reaches its global maximum at \( c_4 = c_3^M - 3\delta c_3 - 3\delta c_3 \delta (4 - 3\delta) \). In addition, the condition of this scenario implies \( c_1^M < c_3^M < c_3^M < c_3^M \). Thus, \( \Delta_{S3} \) is increasing in \( c_4 \). We have \( \Delta_{S4} > \Delta_{S3} (c_4 = c_3^M) = 3\delta (1 - c_3^2) / 32 > 0 \).

For Scenario 4 with \( c_1^M < c_3 \), similar to Scenario 2, we have \( \Delta_{S4} < 0 \).

In Scenario 5 with \( c_1^M < c_3 \leq c_1^M, \Delta_{S5} = -\left( \delta + c_1 + \delta c_1 - 2\delta c_n - \delta^2 \right)^2 / 32\delta (1 + 3\delta) (1 - \delta) \leq 0 \).

In Scenario 6 with \( c_3 < c_1, \Delta_{S6} = 0 \).

Combining these results in all six scenarios gives Proposition 5.

**Proof of Proposition 6**

Similarly, we define \( \Delta_4 = \vartheta^M - \vartheta^1 \) in Scenario 1.

In Scenario 1 with \( c_1 > c_3^M, \Delta_{S1} = 0 \).

In Scenario 2 with \( c_1^M < c_3 \leq c_1^M, \Delta_{S2} = -\left( \delta + \delta c_1 - 2c_1 \right)^2 / 16\delta < 0 \), letting \( \delta \Delta_2/c_1 = 0 \), we have \( c_4 = c_3^M \), that is to say, \( \Delta_{S3} \) reaches its global minimum at \( c_4 = c_3^M \). We have \( \Delta_{S2} \geq \Delta_{S3} (c_4 = c_3^M) = 0 \).

In Scenario 3 with \( c_1^M < c_3 < c_3^M, \delta \Delta_4/c_2^M = -\left( 8 - 6\delta \right)/32\delta (1 - \delta) < 0 \), letting \( \delta \Delta_3/c_2 = 0 \), we have \( c_4 = c_3 + 3\delta c_3 - 3\delta c_3 \delta (4 - 3\delta) \). In addition, the condition of this scenario implies \( c_1^M < c_3^M < c_3^M < c_3^M \). Thus, \( \Delta_{S3} \) is increasing in \( c_4 \). We have \( \Delta_{S4} > \Delta_{S3} (c_4 = c_3^M) = 3\delta (1 - c_3^2) / 32 > 0 \).

For Scenario 4 with \( c_1^M < c_3 \), similar to Scenario 2, we have \( \Delta_{S4} < 0 \).

In Scenario 5 with \( c_1^M < c_3 \leq c_3^M, \Delta_{S5} = -\left( \delta + c_1 + \delta c_1 - 2\delta c_n - \delta^2 \right)^2 / 32\delta (1 + 3\delta) (1 - \delta) \leq 0 \).

In Scenario 6 with \( c_3 < c_1, \Delta_{S6} = 0 \).

Combining these results in all six scenarios gives Proposition 6.

**Proof of Proposition 7**

Similarly, we define \( \Delta_4 = E^M - E^1 \) in Scenario 1.

In Scenario 1 with \( c_1 > c_3^M, \Delta_{E1} = 0 \).

In Scenario 2 with \( c_1^M < c_3 \leq c_1^M, \Delta_{E2} = (\delta + \delta c_1 - 2c_1) / 4\delta \), clearly, \( \Delta_{E2} \) is decreasing in \( c_4 \). Thus, we have \( \Delta_{E2} \geq \Delta_{E2} (c_4 = c_3^M) = 0 \).

In Scenario 3 with \( c_1^M < c_3 < c_3^M, \delta \Delta_4/c_4 \delta c_4 = 0 \). Thus, \( \Delta_{S3} \geq \Delta_{E3} (\vartheta = 0) = \left( c_4 - \delta c_4 \right) (1 - \delta) \). In addition, the condition of this scenario implies \( c_1^M < c_3^M < c_3^M \). Thus, \( \Delta_{S3} > \Delta_{E3} (c_4 = c_3^M) = 0 \).

In Scenario 4 with \( c_1^M < c_3 < c_3^M, \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = -\left( \delta - c_n \right)/2(1 + 3\delta) \) and \( \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = (1 - \delta) (1 - c_n) / 4 (1 + 3\delta) > 0 \), let \( \vartheta \) in Scenario 4 with \( c_1^M < c_3 < c_3^M, \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = (1 - \delta) (1 - c_n) / 4 (1 + 3\delta) > 0 \), let \( \vartheta \) in Scenario 4 with \( c_1^M < c_3 < c_3^M, \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = (1 - \delta) (1 - c_n) / 4 (1 + 3\delta) > 0 \), let \( \vartheta \) in Scenario 4 with \( c_1^M < c_3 < c_3^M, \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = (1 - \delta) (1 - c_n) / 4 (1 + 3\delta) > 0 \), let \( \vartheta \) in Scenario 4 with \( c_1^M < c_3 < c_3^M, \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = (1 - \delta) (1 - c_n) / 4 (1 + 3\delta) > 0 \), let \( \vartheta \) in Scenario 4 with \( c_1^M < c_3 < c_3^M, \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = (1 - \delta) (1 - c_n) / 4 (1 + 3\delta) > 0 \), let \( \vartheta \) in Scenario 4 with \( c_1^M < c_3 < c_3^M, \Delta_{E4} (\vartheta = 0, c_4 = c_3^M) = (1 - \delta) (1 - c_n) / 4 (1 + 3\delta) > 0 \), let \( \vartheta \) in Scenario 4 with \( c_1^M < c_3, \Delta_{E5} = 0 \).

In Scenario 5 with \( c_3^M < c_4 \), similar to Scenario 4, there exists a threshold value \( \vartheta = \phi^* = 2\delta (1 + 3\delta) \) such that if \( \vartheta > \phi^* \), \( \Delta_{E5} > 0 \); otherwise, \( \Delta_{E5} < 0 \).

In Scenario 6 with \( c_3 < c_3^M, \Delta_{E6} = 0 \).

Combining these results in all six scenarios gives Proposition 7.