

On the ranking of Test match batsmen

Richard J. Boys

Newcastle University, Newcastle upon Tyne, United Kingdom

Peter M. Philipson

Northumbria University, Newcastle upon Tyne, United Kingdom

Summary. Ranking sportsmen whose careers took place in different eras is often a contentious issue and the topic of much debate. In this paper we focus on cricket and examine what conclusions may be drawn about the ranking of Test batsmen using data on batting scores from the first Test in 1877 onwards. The overlapping nature of playing careers is exploited to form a bridge from past to present so that all players can be compared simultaneously, rather than just relative to their contemporaries. The natural variation in runs scored by a batsman is modelled by an additive log-linear model with year, age and cricket-specific components used to extract the innate ability of an individual cricketer. Incomplete innings are handled via censoring and a zero-inflated component is incorporated into the model to allow for an excess of frailty at the start of an innings. The innings-by-innings variation of runs scored by each batsman leads to uncertainty in their ranking position. A Bayesian approach is used to fit the model and realisations from the posterior distribution are obtained by deploying a Markov Chain Monte Carlo algorithm. Posterior summaries of innate player ability are then used to assess uncertainty in ranking position and this is contrasted with rankings determined via the posterior mean runs scored. Posterior predictive checks show that the model provides a reasonably accurate description of runs scored.

1. Introduction

There is a great deal of discussion in many sports, from the experts through to the fans, about who is the ‘greatest’. Discussions often conclude with the notion that it is impossible to obtain definitive answers. In many cases the game played out in the modern day, in front of the massed media with large teams of supporting staff dedicated to nutrition, fitness and psychology, bears little or no relation to the backdrop at the genesis of the sport. The richness of data now available however suggests that there may be merit in a sophisticated statistical approach to the problem.

The analysis of sports data has undergone something of a boom in recent years with statisticians and data analysts at the forefront. In baseball, for example, sabermetrics has become an accepted term for the use of in-game statistical analysis (Marchi and Albert, 2013), and there is an increasing trend of sports science and data analysis being routinely performed by major sports organisations across the globe.

In this paper we focus on the sport of cricket and look at the performance of Test match batsmen. Cricket is a bat-and-ball game played between two teams of eleven players each on a cricket field, at the centre of which is a rectangular 22-yard-long pitch with a target called the wicket (a set of three wooden stumps topped by two bails) at each end. Each phase of play is called an innings during which one team bats, attempting to score as many runs as possible, whilst their opponents field. In Test matches the teams have two innings apiece and, when the first innings ends, the teams swap roles for the next innings. This

sequence can only be altered by the team batting second being made to ‘follow-on’ after scoring significantly fewer runs than the team batting first. Except in matches which result in a draw, the winning team is the one that scores the most runs, including any extras gained. Individual players start their innings with zero runs and accumulate runs as play progresses, leading to a final score which is a non-negative count. The highest individual score in Test cricket is 400 runs and the average score is around 30 runs. Smaller scores are more likely than larger scores as the aim of the opposition is to bowl out each batsman as quickly as possible and at the cost of as few runs as possible.

The earliest work on the statistical modelling of cricket scores was undertaken by Elderton (1945) and Wood (1945) who considered modelling samples of individual first-class cricket scores from both Test matches and the County Championship (the domestic first-class cricket competition in England and Wales, sitting one level below Test cricket) as a geometric progression and found evidence of a reasonable fit, although Wood commented that the ‘series show discrepancies at each end, and particularly at the commencement’ due to a larger than expected number of scores of zero, or ducks in cricketer parlance. Incomplete (‘not-out’) scores were assumed to continue at the start of the next innings in the former (acknowledged as a ‘pleasant fiction’ by the author) and treated as complete innings by the latter. Later Pollard et al. (1977) investigated the distribution of runs scored by teams in county cricket and found the negative binomial distribution to offer a good fit. Scarf et al. (2011) confirmed this finding for runs scored in both batting partnerships and team innings in Test cricket.

Kimber and Hansford (1993) considered the merits of the geometric distribution for samples of individual cricket scores from Test and first class matches, including Australia’s domestic Sheffield Shield competition, along with one day internationals, concluding that ‘there was little evidence against the . . . model in the upper tail’ but rejecting its validity for low scores, mainly due to the excess of ducks in the data. Their work focused on an alternative batting average measure using a non-parametric product-limit estimation approach. Some of these points will be revisited later. They also looked at the independence of cricket scores for a batsman and found ‘no major evidence of autocorrelation’ via a point process approach, surmising that ‘it is quite reasonable . . . to treat scores as if they were independent and identically distributed observations’. Durbach and Thiart (2007) later concluded that batting scores can be considered to come from a random sequence based on a study of sixteen Test match batsmen. We note that studies in other sports of lack of independence of points- or run-scoring, sometimes referred to as the ‘hot hand’, have largely concluded that there is little evidence to support the notion of ‘form’ (Gilovich et al., 1985; Tversky and Gilovich, 1989).

Published work in sports statistics covers a wide range of sports. Initially much of this work centred around the analysis of football and baseball, and focussed on predicting future outcomes but now increasingly looks at gains that might be made using an optimal strategy. The most famous model-based method used in cricket today is, of course, the Duckworth–Lewis–Stern formula (Duckworth and Lewis, 1998, 2004; Stern, 2009) for interrupted one-day cricket matches, with subsequent modification by, for example, McHale and Asif (2013). Other work such as Silva et al. (2015) looks at the effect of powerplay in such matches. In this paper the focus is instead on comparing past and current players, an area where relatively little research has been done (Rohde, 2011; Radicchi, 2011; Baker and McHale, 2014), and study Test cricketers in particular. The innate ability of each player is modelled by taking into account the heterogeneous effect of ageing on sporting performance, any year effects which act as a surrogate for changes to the game that may have made it easier or

Table 1. Timeline of Test match milestones

Test Match #	Year	Elapsed years
1	1877	
100	1908	31
500	1960	52
1000	1984	24
2000	2011	27

more difficult in certain eras, home advantage and some cricket-specific components. Berry et al. (1999) considered how to compare players from different eras in three, predominately US-based, sports: baseball, golf and ice hockey. Their argument, which is adopted here, is that comparisons between modern-day players and players from bygone eras are possible by considering the overlap in playing careers: modern players at the start of their careers will have played against older players at the end of their careers, which started much earlier, and these older players would, in their youth, have played against players from earlier eras once more. In such a way a bridge from present to past is formed.

The paper is structured as follows. The data are described in Section 2. The model description in Section 3 begins by outlining an initial model before introducing modifications to handle some nuances of cricket batting data. Sections 4 and 5 detail the prior and posterior distributions respectively along with the MCMC algorithm. Section 6 describes some of the results such as the posterior mean of player ability, and a ranking by this measure, and summaries of the posterior distribution of player rankings. The paper concludes with some discussion and avenues for future work in Section 7.

2. The data

Cricket is a highly data driven sport, perhaps more than any other with the exception of baseball. Players' entire careers are typically judged by a one-number summary: their average. There is a large amount of data, typically in the form of scorecards, available for all formats of cricket at both international, domestic and even regional level. For some players there is even ball-by-ball data recorded (the Association of Cricket Statisticians and Historians have these data for Sir Jack Hobbs) although such a level of granularity is not generally available and so is not considered further here.

The data used in this paper consists of individual innings by all Test match cricketers ($n = 2855$) from the first Test played in 1877 up to Test 2269, in August 2017. There are currently ten Test playing countries and many more Test matches are played today than at the time of the first Test. Indeed for the first twelve years the combatants were exclusively England and Australia. A demonstration of the growth of Test match cricket is given in Table 1. We note that, in contrast to the standard presentation of historical batting averages such as at <http://stats.espncricinfo.com/ci/content/records/282910.html>, we include all batsmen irrespective of the number of career innings played. However, in keeping with other lists, we do not include World Series Cricket matches as these matches are not considered official Test matches by the International Cricket Council (ICC).

In addition to runs scored, the data contain other useful information such as the venue, the opposing team and whether or not the batsman's innings is incomplete (this can happen for a variety of reasons; see Section 3.1.1). Thus we can determine whether a Test match is played at 'home' and investigate the extent of any home advantage. Note that there have

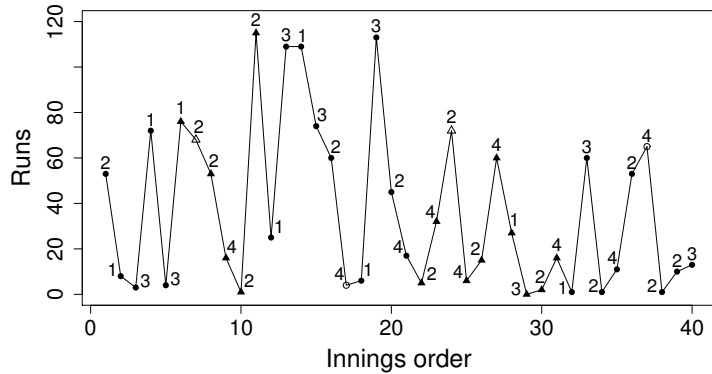


Fig. 1. Innings played by Ian Bell against Australia between 2009 and 2015. Triangles – away matches; open symbols – not outs; numbers – innings number.

been twenty-nine Tests played at neutral venues and we class these as away matches for both teams. The data also include the match innings index which is potentially important as (generally) the conditions in the final innings of a Test match are at their worst for batting and the pressure, due to the game situation, is often at its highest – it is an axiom of cricket that batting last is difficult.

Some aspects of the game that have changed over time are not explicitly recorded in the data: at one stage Tests were ‘timeless’, continuing until a result was achieved; the number of balls in an over has varied between four, five, six and eight; pitches were uncovered and left exposed to the elements up to around 1960; the introduction of limited-overs international cricket in 1971 along with the recent advent of Twenty20 cricket in 2003 and the abolition of ‘amateur’ status in 1962. Together these aspects may well have affected the performance of batsmen, particularly possible changes to pitches and changes to the game dynamic induced by the shorter formats. We will consider these on annual and decade scales respectively.

A typical profile of batting scores is given in Fig. 1. This plot shows the runs scored in the Test match innings of England batsman Ian Bell against Australia between 2009 and 2017. The innings are shown in sequential order and away matches and not outs are indicated by the plotting symbol. Note that, although Bell was in the England side throughout this period, he did not bat in many innings. This feature is typical and can be due to many factors such as big wins where the follow-on was deployed and the winning team did not need to bat a second time, or if the match is drawn due to bad weather or running out of time. The figure highlights the capricious nature of batting and suggests that, while year, ageing and game-specific effects may affect run-scoring on an overall level, the innings-by-innings variation is considerable.

3. The model

Runs scored in an innings are counts and a natural starting point is to consider modelling them via the Poisson distribution, with

$$X_{ijk} | \lambda_{ijk} \stackrel{indep}{\sim} \text{Po}(\lambda_{ijk}), \quad i = 1, \dots, 2855; \quad j = 1, \dots, n_i; \quad k = 1, \dots, n_{ij},$$

where i is the player index, j is the year index and k is the innings index so that X_{ijk} represents the number of runs scored by player i in his j th year during his k th innings of that year. Also n_i and n_{ij} denote respectively the number of years in the career of player i and the number of innings played during year j in the career of player i .

Notation for other available information is as follows. For player i in the k th innings of their j th career year: y_{ijk} is the year the innings was played, a_{ijk} is the age of the player, h_{ijk} indicates whether the innings was played in the batsmen's home country (1 = home, 2 = away), m_{ijk} is a within-match innings index (different from the within-year innings label k), o_{ijk} is the opposition country, and e_{ijk} is an indicator for the era of play, which here is considered on a decade scale. These last two pieces of information together allow us to study possible changes to the performance of a country over time.

Within this Poisson framework we adopt a log-linear model for the run scoring rate which includes the main components thought to influence its outcome, with

$$\log \lambda_{ijk} = \theta_i + \delta_{y_{ijk}} + f_i(a_{ijk}) + \zeta_{h_{ijk}} + \nu_{m_{ijk}} + \xi_{o_{ijk}} + \omega_{o_{ijk}, e_{ijk}} \quad (1)$$

where θ_i represents the ability of player i , the difficulty of the year is captured through δ_ℓ (the data span 141 years), and $f_i(a)$ is a player-specific ageing function, of which more in a moment. The remaining terms in the model are game-specific, representing respectively the effect of playing at or away from home, the match innings effects, the quality of the opposition and an interaction term allowing for the quality of the opposition to change over different eras. Here we take eras to be decades to reduce the number of parameters in the model.

The player ability parameter captures the contribution to runs scored that can be attributed to the fundamental talent of the player. As mentioned earlier, ageing can have a strong impact upon sporting performance so we incorporate an individual quadratic ageing function as suggested by Albert's discussion in Berry et al. (1999), namely

$$f_i(a) = -\alpha_{2i}(a - \alpha_{1i})^2,$$

where α_{1i} is the age at which the peak is attained and α_{2i} is the curvature of the function which measures the rate at which the individual matures and declines.

The year effects are a composite of several factors: clearcut changes such as depth of competition (more Test playing countries), game focus (scoring rates are far higher in modern times and there are fewer draws) and law changes (e.g fewer bouncers per over allowed to make batting easier) whereas others are more subtle, for instance technological advances and game conditions (most pitches are prepared to last five days to ensure maximum profit). We anticipate that the year effects vary smoothly over time and allow for this by using a random walk prior; see Section 4. The year effects also need to be standardised for identifiability reasons and so we compare these effects relative to the final year in the dataset (2017) by taking $\delta_{141} = 0$.

The remaining terms in the model account for home advantage, which is common in many team sports, and two further context-specific effects to represent that as pitches deteriorate, and the match situation becomes more acute, batting may become more difficult and to take into account the quality of the opposition. We set the home effect as the reference level (by taking $\zeta_1 = 0$) and measure the impact of playing away by ζ_2 . The innings effects are represented through ν_g to reflect the difficulty of innings g where $g = 1, 2, 3, 4$ is the innings of the match in which the runs were scored, and with $\nu_1 = 0$ for identifiability. The quality of the opposition is taken into account via ξ_q for $q = 1, \dots, 10$ to represent

the ten Test playing countries, some of which have traditionally been stronger than others. Here we number the countries alphabetically. For reasons of identifiability we will take Australia, the first team in the alphabetical ordering of the Test match playing nations, as the reference opposition country, with $\xi_1 = 0$. Further, the opposition-decade interactions are compared to the final decade (by taking $\omega_{1:10,14} = 0$) and to that of Australia (by also taking $\omega_{1,1:13} = 0$).

Thus in this model $\exp(\theta_i)$ is the average number of runs per innings scored by player i when he is at his peak age, playing at home against Australia, and in the first innings of a Test match taking place in 2017.

3.1. Poisson random effects model

There is substantial variation in individual innings-by-innings cricket scores. As such, the inherent assumption of equidispersion in the Poisson model is unlikely to hold. Under this model and considering players that score on average ten or more runs per innings one would expect their distribution of scores to be broadly Gaussian. However any follower of cricket would intuitively feel that this is not the case and that excess variability to that provided by the Poisson model is present. The data in Fig. 1 on Ian Bell are typical of many other players and show extra-Poisson variation with censored observations and perhaps more ducks than anticipated. We now augment the model to allow for each of these features.

We allow for the extra-Poisson variation by introducing random effects, acting multiplicatively on the Poisson mean parameter, so that

$$X_{ijk} | \lambda_{ijk}, v_{ijk} \stackrel{indep}{\sim} \text{Po}(\lambda_{ijk} v_{ijk}).$$

There are many possible choices of distribution for the random effects v_{ijk} , such as gamma, log-normal, inverse Gaussian or general power transforms (Hougaard et al., 1997). We will use the gamma distribution as this gives a negative binomial distribution for the number of runs after integrating over v_{ijk} (Cameron and Trivedi, 1986; Greene, 2008). This choice allows a direct comparison with earlier work, particularly as the geometric distribution is a special case. For further flexibility we allow the random effects distributions to be player-specific, reflecting that player characteristics, such as aggression, can lead to substantial differences in variability between players of comparable ability. Thus we take $v_{ijk} \sim \text{Ga}(\eta_i, \eta_i)$, with $E(v_{ijk}) = 1$ and $\text{Var}(v_{ijk}) = 1/\eta_i$. Therefore (marginally) we use a negative binomial model for runs scored, with

$$X_{ijk} | \lambda_{ijk}, \eta_i \stackrel{indep}{\sim} \text{NB}\{\eta_i, \eta_i / (\eta_i + \lambda_{ijk})\}.$$

Note that introducing the random effects makes no change to the (marginal) mean but has inflated the (marginal) variance to $\text{Var}(X_{ijk}) = \lambda_{ijk}(1 + \lambda_{ijk}/\eta_i)$, with the basic Poisson model being recovered as $\eta_i \rightarrow \infty$. This form of variance function is appropriate for modelling batting scores as the variability is smaller for players of lesser ability (they have a more restricted range of runs scored and rarely achieve high numbers of runs) and larger for players of higher ability (although they score high numbers of runs, they will typically also have innings with very low scores). The form of the negative binomial success probability is cumbersome and so to simplify the exposition we will use $\beta_{ijk} = \lambda_{ijk}/\eta_i$.

We now augment the model to deal with (i) incomplete scores – innings where the batsmen is not dismissed; (ii) potential zero-inflation in the data – more ducks (zero scores) in the data than the model suggests.

3.1.1. Censoring

Approximately 13% of innings are incomplete, referred to as ‘not out’ in cricketing vernacular, and are typically due to the completion of a team innings, which, by necessity, must include one incomplete innings at the fall of the final wicket, or two incomplete innings in a successful run chase (or if the match has not been completed due to adverse weather or running out of time). Incomplete innings can also happen when the team captain ‘declares’ and brings the innings to a premature close (typically to aid the prospect of victory) and this can result in either one or two incomplete innings. Historically, cricket has dealt with incomplete innings in a somewhat *ad hoc* manner whereby the runs are added to the numerator in the batting average without any increment to the denominator. Clearly such innings ought not to be dealt with in the same way as a complete innings and the standard cricketing treatment can exaggerate the contribution of incomplete innings and thereby affect the batting average. From a statistical viewpoint, a not-out is simply a censored observation. Kimber and Hansford (1993) claim that ‘ x not-out is representative of all scores of x or more’ and so we assume non-informative censoring. Thus, denoting a not-out (censored) innings by the binary variable c , the likelihood contribution from player i , for the k th innings of the j th year of his career, is

$$\left\{ \binom{x_{ijk} + \eta_i - 1}{x_{ijk}} \beta_{ijk}^{x_{ijk}} / (1 + \beta_{ijk})^{\eta_i + x_{ijk}} \right\}^{1 - c_{ijk}} \times P(X_{ijk} \geq x_{ijk})^{c_{ijk}}, \quad (2)$$

where X_{ijk} has a negative binomial distribution.

3.1.2. Zero-inflation

After ignoring censored zeroes, ducks account for almost 11% of the observations in the data. Even Sir Donald Bradman (with a Test batting average of 99.94) had a modal score of zero with seven ducks out of eighty innings. This high proportion of zeroes is likely to be due to players being vulnerable early in their innings (Brewer, 2008), taking time to acclimatise to conditions and ‘get their eye in’ rather than owing to some other process that causes scores to be necessarily zero. Thus the proportion of ducks is likely to be higher than expected using the Poisson random effects model and so we modify the model to allow for this inflation of zeros. We also allow for different levels of zero-inflation for each player.

There are two basic ways of dealing with zero inflation. One way is to model the probability of getting a zero by a mixture of the primary model and a point mass at zero (Lambert, 1992) and the other is to use a hurdle model which contains a model for zero counts (the hurdle component) and a separate model for the strictly positive counts (once the hurdle, a batsman playing a scoring stroke for instance, has been cleared). Hurdle models are particularly popular in the economics literature; see, for example, Gurmu (1997, 1998). They are the natural choice when the zeroes are entirely structural, such as in a biological process (Ridout et al., 1998) or a weather pattern (Scheel et al., 2013). We favour the mixture representation as this can be interpreted as the number of ducks being a mixture of (the Poisson random effects) model based zeros and a component representing the increased vulnerability of a batsman early in an innings. This representation has the additional advantage (not followed up here) of providing a framework for generalising the model to inflate other scores, such as four or six, that may occur more frequently due to being achievable with a single scoring stroke, that is, via a ‘four’ or a ‘six’.

The excess zeroes are assumed to be unrelated to the other effects and so we model the probability of getting a (completed) duck for player i as

$$Pr(X_{ijk} = 0) = \pi_i + (1 - \pi_i)/(1 + \beta_{ijk})^{\eta_i}.$$

Note that as the player-specific parameter $\pi_i \rightarrow 0$, the zero-inflated component diminishes and the number of (completed) ducks is well described by an orthodox Poisson random effects model. Thus, denoting a batsman with a (completed) duck by the binary variable d , the likelihood contribution from player i , for the k th innings of the j th year of his career, is amended from that in (2) to

$$\begin{aligned} & \{ \pi_i + (1 - \pi_i)/(1 + \beta_{ijk})^{\eta_i} \}^{(1 - c_{ijk})d_{ijk}} \\ & \times \left[(1 - \pi_i) \left\{ \binom{x_{ijk} + \eta_i - 1}{x_{ijk}} \beta_{ijk}^{x_{ijk}} / (1 + \beta_{ijk})^{\eta_i + x_{ijk}} \right\}^{1 - c_{ijk}} P(X \geq x_{ijk})^{c_{ijk}} \right]^{1 - d_{ijk}}. \end{aligned} \quad (3)$$

Introducing a zero-inflation effect also reduces the expected number of runs scored by a factor of $(1 - \pi_i)$.

4. The prior distribution

We need to construct a joint prior distribution for the many parameters in this model. In general, we have chosen to describe our prior beliefs by taking fairly weak independent priors for each parameter component. This has the benefit of “letting the data speak” and gives our results a reasonable level of robustness against our choice of prior.

We adopt a random effects style (or hierarchical) prior for the player-specific ability parameters in which ability varies between batsmen by taking

$$\theta_i | \mu_\theta, \sigma_\theta \stackrel{indep}{\sim} N(\mu_\theta, \sigma_\theta^2).$$

We also take semi-conjugate prior distributions for the ability parameters, with $\mu_\theta \sim N(m_\mu, s_\mu^2)$ and $\sigma_\theta^2 \sim IG(a_\sigma, b_\sigma)$, where $IG(a, b)$ denotes the Inverse Gamma distribution with mean $b/(a - 1)$. It was felt that the median number of runs scored across all innings (including not-outs) would be around 20 and so we take $m_\mu = \log 20$. Also the variability between decades of runs scored was likely to be within a 60%-fold increase or decrease and so we take $s_\mu = 0.25$ (as $e^{0.5} \simeq 1.6$). Variation of player ability was thought to be typically about a four fold increase/decrease around the decade mean, giving σ_θ^2 a mean of around 0.5, and that the probability that this fold increase/decrease would exceed ten was around 5%. Together these requirements give a prior distribution with (roughly) $a_\sigma = 3$ and $b_\sigma = 1$.

It was felt that the year effects δ_ℓ should vary fairly smoothly in time and that prior beliefs were less certain for years going further and further into the past. Therefore, together with the identifiability constraint $\delta_{141} = 0$, we use the (backwards) simple random walk $\delta_\ell = \delta_{\ell+1} + \sigma_\delta \epsilon_\ell$, $\ell = 140, \dots, 1$, where the ϵ_ℓ are independent standard normal quantities. To see its smoothing role, it is useful to note that this random walk induces

$$\delta_\ell | \delta_{(\ell)}, \sigma_\delta \sim N \left(\frac{\delta_{\ell-1} + \delta_{\ell+1}}{2}, \frac{\sigma_\delta^2}{2} \right), \quad \text{for } \ell = 2, \dots, 140,$$

with $\delta_1 | \boldsymbol{\delta}_{(1)}, \sigma_\delta \sim N(\delta_2, \sigma_\delta^2)$, where $\boldsymbol{\delta}_{(\ell)} = (\delta_i, i \neq \ell)$ represents all of the year effects except year ℓ . For notational convenience we write $\boldsymbol{\delta}$ for the year effects $\boldsymbol{\delta}_{(141)}$. These descriptions lead to the prior distribution of the year effects being $\boldsymbol{\delta} | \sigma_\delta \sim N_{140}(\mathbf{0}, \sigma_\delta^2 Q^{-1})$ where the inverse correlation matrix Q has the tri-diagonal structure

$$Q = \begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}.$$

The parameter σ_δ describes the smoothness of the year effects and, as this impacts player ability on an exponential scale, it was felt that σ_δ^2 should have an $IG(a_\delta, b_\delta)$ prior distribution with mean 0.01 and only a 5% probability of exceeding 0.03. This leads (roughly) to a choice of prior parameters $a_\delta = 2$ and $b_\delta = 0.01$.

We now consider the prior distributions for the remaining parameters, beginning with the game-specific parameters: the effect of playing away ζ_2 , the innings effects $\nu_{2:4}$, the quality of the opposition $\xi_{2:10}$ and the opposition-era interactions $\omega_{2:10,1:13}$ (recall that $\zeta_1 = \nu_1 = \xi_1 = \omega_{1:10,14} = \omega_{1,1:13} = 0$ for identifiability). The strength of our opinion on their potential size is quite weak and so we give these parameters zero mean normal prior distributions with standard deviation 0.5, this taken to equate to a 95% prior credible interval for these effects spanning an increase/decrease of around 2.7-fold on the runs scored. Our prior beliefs about the player-specific ageing function are that the peak age is around 30 years old and that the rate of maturity/decline of players at seven years before (after) their peak is roughly $2/3$ ($-2/3$). We represent our fairly weak prior beliefs by taking $\alpha_{1i} \sim N(30, 4)$ and $\alpha_{2i} \sim LN(-3, 9)$.

Previous studies have considered a geometric random effects distribution for runs scored and so we give the individual random effects heterogeneity parameters η_i a log-normal prior with unit prior median, but also make this prior fairly weak by taking $\eta_i \sim LN(0, 1)$. Our prior beliefs about the individual zero-inflation parameters π_i are captured by a $Beta(a_\pi, b_\pi)$ distribution with mean 0.1 and only a 5% probability of π_i exceeding 0.3. This leads (roughly) to a choice of prior parameters $a_\pi = 1$ and $b_\pi = 9$.

5. The posterior distribution

The posterior density can be factorised as

$$\pi(\boldsymbol{\kappa}, \boldsymbol{\eta}, \boldsymbol{\pi} | \boldsymbol{x}, \boldsymbol{c}, \boldsymbol{d}) \propto \pi(\boldsymbol{x}, \boldsymbol{c}, \boldsymbol{d} | \boldsymbol{\kappa}, \boldsymbol{\eta}, \boldsymbol{\pi}) \pi(\boldsymbol{\kappa}) \pi(\boldsymbol{\eta}) \pi(\boldsymbol{\pi})$$

with $\boldsymbol{\lambda} = \boldsymbol{\lambda}(\boldsymbol{\kappa})$, where \boldsymbol{x} , \boldsymbol{c} and \boldsymbol{d} are the vectors of runs scored and associated censoring and duck indicators respectively, and $\boldsymbol{\kappa} = (\boldsymbol{\theta}, \boldsymbol{\delta}, \sigma_\delta, \boldsymbol{\alpha}, \zeta_2, \boldsymbol{\nu}, \boldsymbol{\xi}, \boldsymbol{\omega})$ contains the remaining parameters in the model, with $\boldsymbol{\nu} = (\nu_{2:4})$, $\boldsymbol{\xi} = (\xi_{2:10})$, and $\boldsymbol{\omega} = (\omega_{2:10,1:13})$. This posterior distribution is analytically intractable and we therefore turn to a sampling-based approach and make inferences via the use of Markov chain Monte Carlo (MCMC) methods.

In our MCMC scheme we generally use Metropolis-Hastings steps with symmetric normal random walk proposals on an appropriate scale and centred on the current value; for example, on the log scale for positive quantities or the logit scale for quantities restricted to $(0, 1)$. Overall we have found this strategy to work well except for updates to the year

effects δ . Here Gibbs updates are available for each component δ_ℓ but their full conditional distributions depend strongly on the values taken by the year effects on either side, that is, $\pi(\delta_1|\cdot) = \pi(\delta_1|\delta_2, \cdot)$ and $\pi(\delta_\ell|\cdot) = \pi(\delta_\ell|\delta_{\ell-1}, \delta_{\ell+1}, \cdot)$, $\ell \neq 1$. This is not surprising given the dependence structure in the prior distribution for δ . It is well known that such strong dependence can lead to poor mixing such as that in, for example, the distribution of hidden states in hidden Markov models. Also this strong dependence prohibits using software such as JAGS (Plummer, 2004) to obtain posterior realisations in a timely manner. Instead we follow Gamerman (1997) and construct a normal proposal distribution for δ via a Taylor series approximation to its full conditional distribution; see the supplementary materials for further details. We have found this strategy to greatly improve the mixing of the scheme.

6. Results

An implementation of the MCMC scheme in R (R Core Team, 2014) is available from <https://github.com/petephippison/Ranking-Test-batsmen> together with the data. We report here results from a typical run of the MCMC scheme which used a burn-in of 5K iterations and was then run for a further 200K iterations, with this (converged) output thinned by taking every 20th iterate. This gave a posterior sample of $N = 10\text{K}$ (almost) un-autocorrelated values for analysis. Convergence was assessed through a variety of graphical and numerical diagnostics via the R package coda (Plummer et al., 2006).

6.1. Random effects distribution for player ability

The (marginal) posterior distributions for the mean and standard deviation (μ_θ and σ_θ) of the random effects distribution for player ability are shown in Fig. 2. Clearly the data have been quite informative. We can get a quick understanding of this posterior distribution by looking at its implication for the (random effects) distribution of the number of runs scored (by players at their peak age, playing at home against Australia, and in the first innings of a Test match taking place in 2017). Ignoring the (player-specific) zero-inflation effect, the five number summary (Min–LQ–Med–UQ–Max) for the median number of runs scored ($\exp(\mu_\theta)$) is 24.7 – 26.4 – 27.3 – 28.3 – 30.2, and that for the average number of runs scored ($\exp(\mu_\theta + \sigma_\theta^2/2)$) is 25.2 – 26.9 – 27.9 – 28.8 – 30.8. These distributions seem reasonable after taking into account that the zero-inflation parameters π are around 8% (see section 6.4).

6.2. Year effects

The posterior distribution for the year effects is summarised in the upper panel of Fig. 3. The effects are shown on an exponential scale and so represent the multiplicative effect on run scoring for each year, relative to playing against Australia in the most recent year (2017), here shown by the horizontal dashed line. It is clear that there are very few important year effects, with the main (and negative) deviation being 1887–1891, a period when it is widely acknowledged that bowling conditions were favourable. The next strongest (and positive) deviation occurred in 2009, a year featuring four of the sixteen highest team scores of all time. The bottom-left panel of Fig. 3 shows the prior and posterior distribution of the smoothing parameter σ_δ^2 for the year effects. The slight shift in the posterior towards lower values suggests that the prior distribution did not over smooth.

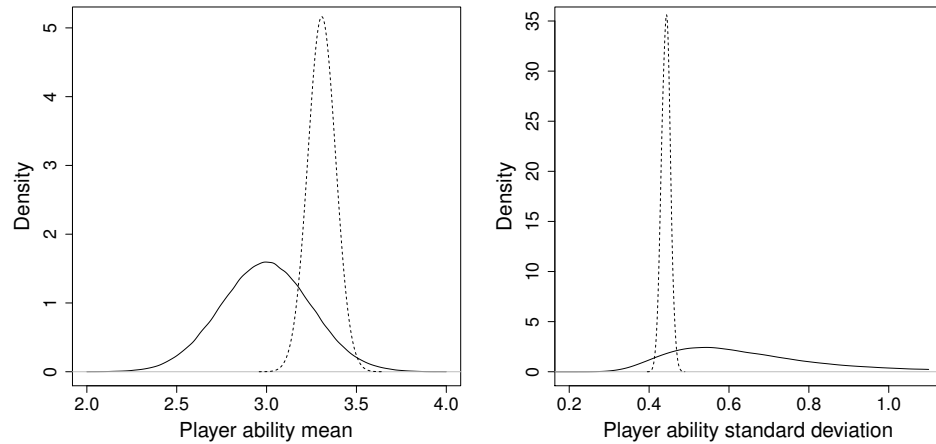


Fig. 2. Prior (solid line) and posterior (dashed line) density plots for ability parameters μ_θ (left plot) and σ_θ (right plot).

6.3. Home advantage, innings and opposition effects

The bottom-right panel of Fig. 3 provides a visual comparison of the size of the multiplicative effect on run-scoring when batting in different innings and playing away from home. Note that these effects are all relative to playing at home against Australia in the first innings in 2017, represented by the dashed horizontal line. The effect of playing away from home on runs scored, $\exp(\zeta_2)$, has posterior mean 0.90 and 95% confidence interval (0.89, 0.92). Thus, there is a pronounced detrimental effect of playing away from home, leading to batsmen scoring 10% fewer runs. This finding is consistent with that found for ‘home advantage’ in other sports (Pollard and Pollard, 2005; Jones, 2007; Baio and Blangiardo, 2010). The posterior means (with 95% confidence intervals) for the second, third and fourth innings effects ($\exp(\nu_{2:4})$) are 0.95 (0.93, 0.97), 0.90 (0.88, 0.92) and 0.84 (0.81, 0.86) respectively, with the reference value of one for the first innings. These effects act multiplicatively on run-scoring. Hence, performance decreases as the match goes on, with the innings effect at its strongest in the final innings of the match, as cricketing folklore would have predicted. The second innings of a Test match is tougher than the first innings with a reduction of 5% in runs scored, but the effect increases to a 10% reduction in runs scored in the third innings and a 16% reduction in the final innings (compared to the first innings). It’s interesting to see that the effect of batting in the third innings and that of playing away from home are very similar.

Fig. 4 displays the posterior means and associated 95% intervals of $\exp(\xi_q + \omega_{qd})$ for the ten Test playing countries ($q = 1, \dots, 10$) over the fifteen decades ($d = 1, \dots, 15$) during which Test cricket has been played. As mentioned earlier, fewer countries played Test cricket when it commenced as an international sport. The estimates in each case are relative to the strength of the current Australian Test team (represented by the horizontal dotted line on each plot). There are twenty instances of opposition effects that show appreciable deviation from that of Australia in the most recent decade: these are split as six instances of an opposition being significantly more difficult to score runs against than the current Australia team and fourteen cases where the opposition are easier to score runs against.

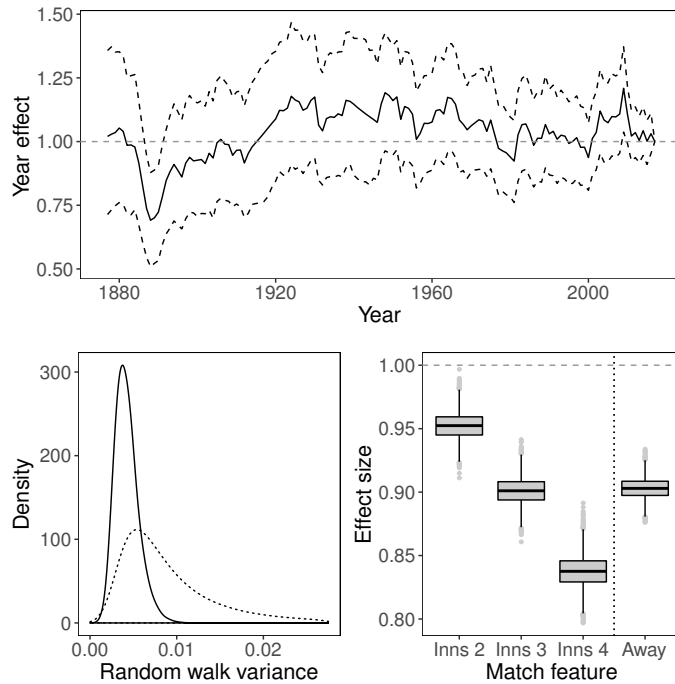


Fig. 3. Top: posterior mean and central 95% bands for the multiplicative year effects $\exp(\delta_\ell)$. Bottom left: posterior (solid) and prior (dashed) density plots for the smoothing parameter σ_δ^2 . Bottom right: boxplots of the posterior distributions for the multiplicative match innings effects $\exp(\nu_m)$ and playing away effect $\exp(\zeta_2)$.

The largest deviations (and with the lowest posterior means) were England in the 1880s and 1950s, and the West Indies in the 1980s, each causing a 20-25% reduction in average runs scored.

The two newest Test playing nations, Bangladesh and Zimbabwe, have struggled at times to be competitive and the three largest (significant) posterior means are for these two countries. Batsmen have preyed on this weakness, scoring on average over 50% more runs against Bangladesh and over 30% more runs against Zimbabwe. New Zealand were also relatively weak when they first played Test cricket (in the 1930s) with batsmen scoring on average 30% more runs. Other noteworthy examples of weaker opposition were India in the 1950s, India and New Zealand in the 1970s and England in the 1980s. In each case batsmen scored on average around 20% more runs against these countries in these decades.

We investigated the sensitivity of our conclusions on opposition effects to using five year time periods rather than decades and found very little difference. Also there is no need to standardise opposition scores against the current Australia side and it is straightforward to standardise scores against any opposition team in any decade. We provide the results for any choice of team and decade via an RShiny application, available at <https://petephippison.shinyapps.io/opposition/>

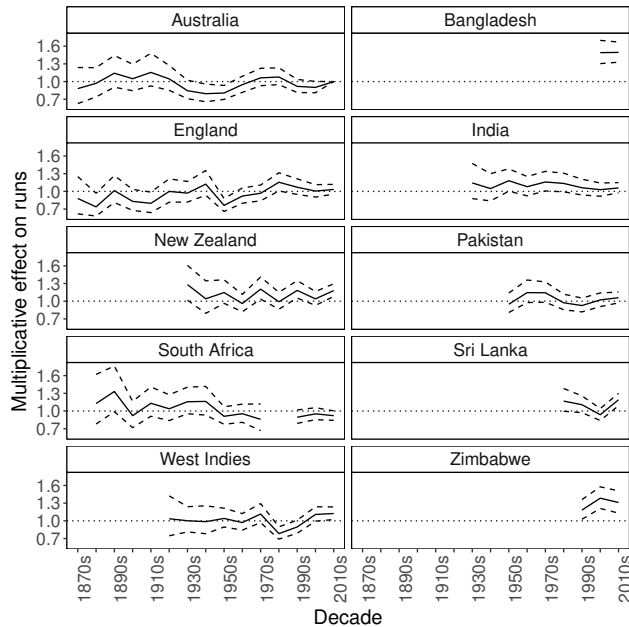


Fig. 4. Posterior mean and central 95% bands for the multiplicative opposition effects by decade, $\exp(\xi_q + \omega_{qd})$.

6.4. Random effects heterogeneity and zero-inflation

Five number summaries of the posterior means and standard deviations for the player-specific random effects heterogeneity parameters (η_i) are $0.48 - 0.87 - 1.01 - 1.16 - 2.19$ and $0.07 - 0.24 - 0.40 - 0.54 - 0.89$ respectively. The posterior distributions for a number of batsmen show clear deviation from the geometric model ($\eta_i = 1$) for cricket scores postulated by Elderton (1945) and Wood (1945). We note that these authors did not account for zero-inflation (or censoring) but Wood did remark on a lack of fit for scores of zero.

Five number summaries of the posterior means and standard deviations for the player-specific zero-inflation parameters (π_i) are $0.01 - 0.06 - 0.08 - 0.11 - 0.34$ and $0.01 - 0.04 - 0.06 - 0.08 - 0.15$ respectively. The posterior distributions show clearly both evidence for zero inflation in Test match cricket scores and variation between players. The modal batsman's score in Test cricket is zero, and the commonly held belief that batsmen are at their most vulnerable at the onset of their innings is a plausible explanation here. Posterior means of the π_i for the top thirty ranked batsmen are included in Table 2. The excess of zeroes observed by Wood is a genuine feature of Test cricket scores. It is interesting to note the discussion on the use of the standard cricket batting average summary in Kimber and Hansford (1993): they point out that such a measure is only a consistent estimate if the scores follow a geometric distribution.

6.5. Individual ageing

We determine the ageing profile for a batsman by examining the posterior distribution of their expected runs scored at various ages a , that is, $(1 - \pi) \exp\{\theta + f(a)\}$. Fig. 5 shows

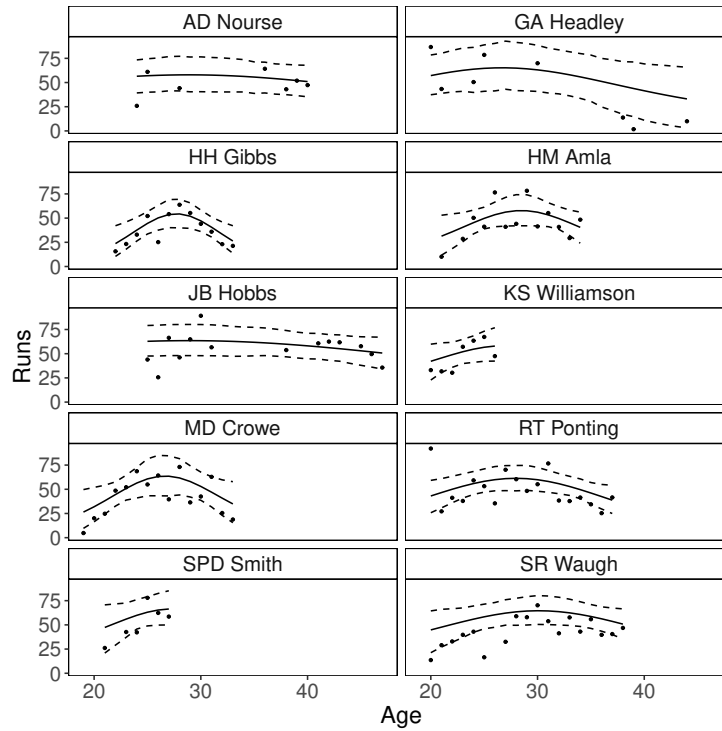


Fig. 5. Posterior mean and central 95% bands for the ageing profile $(1 - \pi_i) \exp\{\theta_i + f_i(a)\}$ of various batsmen together with their posterior mean adjusted runs scored.

posterior mean profiles (and central 95% bands) for a selection of players of broadly similar ability but with quite different ageing profiles. Also included in the plot are the posterior mean adjusted runs scored for each player, that is, the posterior mean of

$$\sum_{j,k:a_{ijk}=a} x_{ijk} \times \exp\{-(\delta_{y_{ijk}} + \zeta_{h_{ijk}} + \nu_{m_{ijk}} + \xi_{o_{ijk}} + \omega_{o_{ijk}, e_{ijk}})\} / n_{ia}$$

where n_{ia} is the number of completed innings played by player i at age a . The figure shows that the quadratic function largely captures the ageing profiles, particularly when taking account of the (posterior) uncertainty on the mean adjusted scores (not shown). The posterior mean of the peak ages (α_{1i}) is typically late twenties; see Table 2.

6.6. Player rankings

The posterior distribution of mean runs scored by the top thirty ranked players are shown as boxplots in Fig. 6, with numerical summaries in Table 2. Here the players are listed by their posterior mean of $(1 - \pi) \exp(\theta)$, that is, their expected runs scored at their peak age assuming the year of play is 2017 (no year effect) and batting at home in the first innings of a Test match against Australia. It is striking just how far Sir Donald Bradman is ahead of the other batsmen, in terms of posterior mass; his extraordinary average is well-known to cricket fans and the plot captures this clearly. The posterior distributions of

the players ranked from two to thirty are largely similar, with considerable overlap. After Bradman it is unclear who is the next ‘best’ batsman. This point is further underlined by the posterior distribution of each player’s rank, calculated across the MCMC samples. The figure also shows the median posterior rank, together with equi-tailed 95% confidence intervals. The numerical summaries for each batsman can be found in Table 2. Note that these are summaries of marginal distributions for each player and not, for example, the most probable joint ranking across all players. Therefore it is possible, and happens here, that no batsman has a posterior median rank of, say, two. However, given the level of variation in runs scored, it does not seem reasonable to rank batsman by a single number summary, be it mean score or median rank. Kimber and Hansford (1993) make a similar argument, stating that ‘it is clear that a one-number summary of the distribution of a batsman’s scores is not enough’. Our rank confidence intervals give a much more reasonable measure of rank position and its uncertainty.

The interval for Bradman’s rank is quite narrow, ranging from rank 1 to rank 14. There is little difference in the career batting averages of many players after Bradman and this is borne out in the spread of the confidence intervals for the rankings, which are largely similar and noticeably wide. It is interesting to see the level of (posterior) uncertainty on player rankings. Fig. 6 shows confidence intervals for the top twenty players along with players ranked 100th, 500th and every five-hundredth player thereafter up to the 2500th player and the final player, ranked 2855th. The high level of posterior uncertainty in these ranks chimes with a remark by Goldstein and Spiegelhalter (1996) when comparing institutional performances, that ‘such variability in rankings appears to be an inevitable consequence of attempting to rank individuals with broadly similar performances’. A full list of the ability scores and ranks for all 2855 batsmen can be found via the RShiny application at <https://petephilipson.shinyapps.io/BatsmenRankings/>

There are two established rankings lists with which we can compare our rankings. The first is the traditional rankings by career Test batting average and the second is the ‘Reliance ICC Best-Ever Test Championship Rating’ (ICC) list. These two differ in that the former is a single measure across a player’s entire career whereas the latter is the maximum of a dynamic index which places a greater emphasis on recent innings. Our approach could be considered to be a compromise between these two systems. Overall, of the top 30 in our rankings by posterior mean runs scored, we have 23 in common with all-time highest career batting average rankings, and 19 in common with the ICC rankings. Five batsmen in Table 2 do not appear in either of these established ranking lists; these batsmen (with our ranking by posterior mean runs and 95% confidence interval for their rank) are Waugh 6 (7, 168), Crowe 8 (4, 304), Border 19 (14, 180), Williamson 20 (4, 343) and Flower 28 (12, 274). This illustrates a central problem in ranking batsmen by a single number summary when there is a high level of innings-to-innings variation in runs scored by each batsman. In particular the traditional ranking does not adjust for any covariate information. The ICC rank does adjust for opposition/pitch effects but is empirical and has some other *ad hoc* adjustments. Neither system adjusts for the censoring (not-out) problem in a way that takes account of player ability.

6.7. Model fitting

We can study the ability of the model to predict ducks (zero scores) by looking at the (model-based) posterior predictive probability of a duck and seeing how this correlates with observed ducks. This predictive probability is calculated by averaging a typical model-

Table 2. Player rankings (ordered by posterior mean runs at peak age) together with posterior means for peak age and zero-inflation proportions, and summaries of player rank distributions.

Rank	Name	Debut	Innings	Runs	SD	Peak age	Zero-inflation	Median rank (95% CI)
1	DG Bradman	1928	80	93.7	14.3	28.2	7%	2 (1-14)
2	SPD Smith	2010	100	66.3	10.1	27.9	2%	27 (3-158)
3	GS Sobers	1954	160	64.1	7.9	27.8	5%	33 (5-137)
4	GA Headley	1930	40	63.2	12.8	27.0	4%	40 (2-360)
5	CL Walcott	1948	74	63.2	9.6	28.4	2%	38 (4-206)
6	SR Waugh	1985	260	62.0	7.7	29.6	5%	43 (7-168)
7	H Sutcliffe	1924	84	61.9	9.0	28.2	3%	44 (5-218)
8	MD Crowe	1982	131	61.6	10.7	27.0	4%	48 (4-304)
9	JB Hobbs	1908	102	61.4	8.2	28.4	4%	45 (6-204)
10	JH Kallis	1995	280	61.3	6.8	28.6	3%	47 (8-144)
11	SR Tendulkar	1989	329	61.2	6.4	27.2	2%	46 (10-139)
12	ED Weekes	1948	81	60.5	8.6	27.8	5%	52 (6-233)
13	RT Ponting	1995	285	59.8	6.8	27.8	3%	55 (10-189)
14	WR Hammond	1927	140	59.4	7.4	28.1	2%	59 (9-212)
15	RG Pollock	1963	41	59.2	11.7	27.7	3%	69 (3-406)
16	KF Barrington	1955	131	58.9	7.1	28.6	2%	62 (11-217)
17	L Hutton	1937	138	58.7	7.3	28.1	2%	65 (10-224)
18	BC Lara	1990	232	58.6	5.8	28.3	3%	64 (16-173)
19	AR Border	1978	265	58.5	6.0	27.9	2%	64 (14-180)
20	KS Williamson	2010	110	58.0	10.4	27.7	3%	81 (4-343)
21	Y Khan	2000	213	57.7	6.6	28.7	5%	73 (13-219)
22	KC Sangakkara	2000	233	57.5	5.8	28.9	2%	72.5 (16-198)
23	R Dravid	1996	286	57.5	5.7	27.8	1%	73 (17-198)
24	GS Chappell	1970	151	57.4	6.7	28.2	5%	75 (12-242)
25	AC Voges	2015	31	57.2	14.2	28.5	4%	91 (3-644)
26	HM Amla	2004	183	57.2	8.5	28.5	2%	80 (9-329)
27	JE Root	2012	107	57.1	7.8	27.7	2%	79 (9-303)
28	A Flower	1992	112	56.9	7.3	28.5	2%	80 (12-274)
29	SM Gavaskar	1971	214	56.7	6.1	28.0	2%	81 (18-226)
30	M Yousuf	1998	156	56.6	7.2	28.6	5%	87 (11-267)

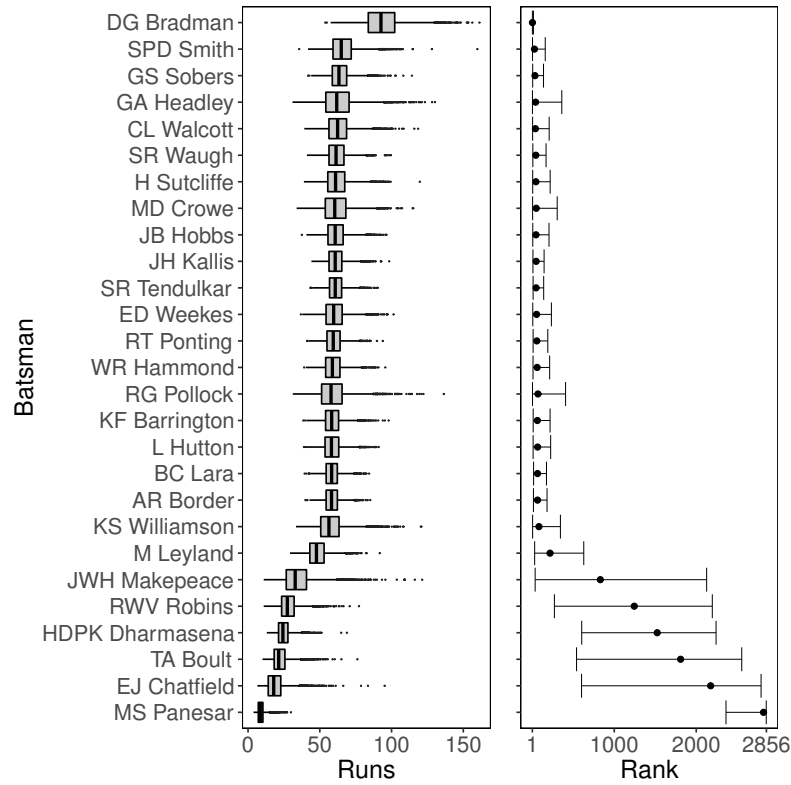


Fig. 6. Posterior distributions of some player abilities $\exp(\theta_i)$ and their ranking confidence intervals.

based probability $Pr(X_{ijk} = 0 | \boldsymbol{\kappa}, \eta_i, \pi_i)$ over the uncertainty in the posterior distribution. Therefore we estimate these predictive probabilities using

$$Pr(X_{ijk} = 0 | \mathbf{x}, \mathbf{c}, \mathbf{d}) = \frac{1}{N} \sum_{\ell=1}^N Pr(X_{ijk} = 0 | \boldsymbol{\kappa}^{(\ell)}, \eta_i^{(\ell)}, \pi_i^{(\ell)}),$$

where $\{\boldsymbol{\kappa}^{(\ell)}, \boldsymbol{\eta}^{(\ell)}, \boldsymbol{\pi}^{(\ell)} : \ell = 1, \dots, N\}$ is the posterior sample. Fig. 7 shows a summary of this information. The left hand plot shows the (posterior) predictive distribution for the total number of ducks in the dataset and confirms that this is consistent with its observed data value. In the right hand plot, the predictive probabilities have been first grouped into centiles and then the observed proportion of ducks in each centile has been plotted against the mid-point of each centile. The plot shows good agreement between the model predictions and observed proportions as there is little deviation from the 45° line. Fig. ?? in the supplementary materials shows plots similar to that in the right hand plot in Fig. 7 but gives a more comprehensive picture. Instead of just showing the calibration of duck predictions, this plot contains that for all numbers of runs scored (grouped into intervals, typically of size ten). Overall these plots show that, although the model does not provide a perfect calibration, it does give a fairly accurate description of runs scored.

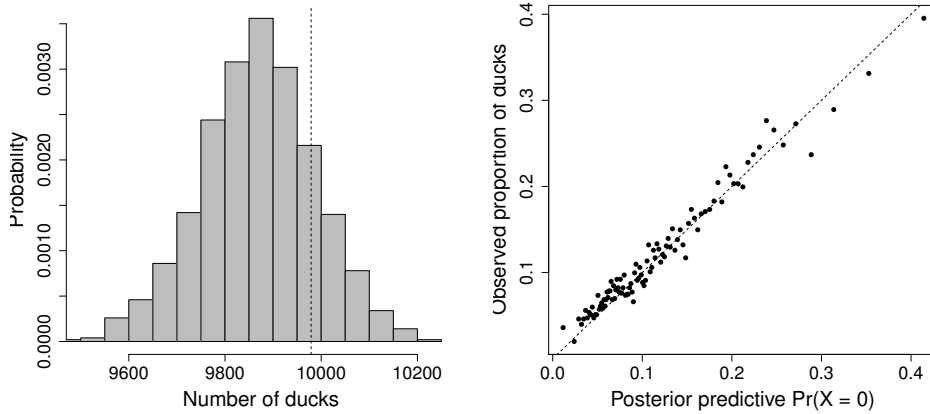


Fig. 7. Left: Posterior predictive distribution of total ducks (dashed line – observed number in the data); Right: Observed proportion of ducks against centiles of posterior predictive probabilities of a duck.

7. Discussion

The data clearly show that there is considerable within batsmen variability in cricket scores and there is demonstrable evidence that batsmen are especially vulnerable at the beginning of their innings. Also the standard cricket batting average measure makes the unreasonable assumption that run scoring follows a geometric distribution. Further the zero-inflated random effects Poisson model (with log-linear factors) gives a good description of the runs scored in Test matches. In terms of ranking players, we found that Sir Donald Bradman, unsurprisingly, was the best player (under the model) and there was relatively little uncertainty about his ranking. However, there was considerable uncertainty in the rankings of players lower down the list.

We compared our rankings with those of two established lists: one list by career Test batting average and the other, the ICC Best-Ever Test Championship Rating list. Not surprisingly we found disagreement between all three lists. This illustrates a central problem in ranking batsmen by a single number summary when there is a high level of innings-to-innings variation in runs scored by each batsman. In these circumstances it is more appropriate to summarise a career by a distribution which recognises the uncertainty in these single number summaries. In this paper we look at the player's overall ability within a model which accounts for the high level of innings-to-innings variation, various cricket-specific factors (not-outs, zero inflation) and adjusts for various important player-independent factors. Even without such adjustments, simple data averages can easily mislead as some batsmen play relatively few innings: the five number summary for career innings played is 1 – 4 – 12 – 35 – 329. We summarise our understanding of the player's ability by a distribution or an interval which accounts for uncertainty. These summaries are impacted less by circumstance and luck (such as when an lbw decision goes in the batsman's favour and he makes a big score) than any fundamental difference in ability.

The model represents the quality of the opposition through dynamic opposition-specific

parameters in order to capture potential changes in the performance of Test playing countries over time, such as periods of notable strength and weakness. Other factors were considered for inclusion in the model but omitted due to data limitations or in the interests of model parsimony. The effect of playing at home was explicitly accounted for, and this could be sub-categorised further into individual Test match grounds. However, although some grounds such as Lord's and the Sydney Cricket Ground have been staples on the Test match roster, there has been a lot of change in venues used in the sub-continent and so insufficient data is available to be able to account for individual stadium effects. Test matches are typically played as part of a series but data on the match number within a series was not available in our dataset. Similarly, we might take account of the position of the batsmen in the batting order. However, we believe that batting position is chosen to suit the individual characteristics of each player in order to maximise runs scored, and so leave out this factor from our model.

An obvious extension of this work would be to apply it to the performance of both batsmen and bowlers. The approach could be further extended to analyse data from one day internationals (ODIs), which, despite only being an international sport since 1971, has already seen around 3900 fixtures take place. This equates to almost the same amount of data as used here for Test matches since ODIs feature one innings per team per match. A larger meta-model simultaneously analysing the Test/ODI batsmen and bowlers could also help in addressing the issue of ranking players as such a model would have the potential to identify not only the quantity of the runs but also refine attempts to ascertain their 'quality' by explicitly factoring in more granular data relating to the opposition, such as the strength of the bowling attack in a given innings. Twenty20 cricket is another avenue for future work but currently there may be insufficient data for an analysis of the type used in this work.

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