

A Dual Frailty Model for Lifetime Analysis in Maritime Transportation

Robin Henderson · Ralitsa Mihaylova ·
Paul Oman

Received: date / Accepted: date

Abstract We consider changes in ownership of commercial shipping vessels from an event history perspective. Each change in ownership can be influenced by the properties of the vessel itself, its age and history to date, the characteristics of both the seller and the buyer, and time-varying market conditions. Similar factors can affect the process of deciding when to scrap the vessel as no longer being economically viable. We consider a multi-state approach in which states are defined by the owning companies, a sale marks a transition, and scrapping of the vessel corresponds to moving to an absorbing state. We propose a dual frailty model that attempts to capture unexplained heterogeneity in the data, with one frailty term for the seller and one for the buyer. We describe a Monte Carlo Markov chain estimation procedure and verify its accuracy through simulations. We investigate the consequences of mistakenly ignoring frailty in these circumstances. We compare results with and without the inclusion of frailty.

Keywords Bayes · Clarksea index · Ownership duration · Partial likelihood · Proportional intensity · Random effects · Sentiment

R. Henderson
School of Mathematics, Statistics and Physics
Newcastle University, UK
E-mail: Robin.Henderson@ncl.ac.uk

R. Mihaylova
School of Engineering
Newcastle University, UK
E-mail: r.mihaylova@ncl.ac.uk

J.P. Oman
Department of Mathematics, Physics and Electrical Engineering
Northumbria University, UK
E-mail: paul.oman@northumbria.ac.uk

1 Introduction

We consider an event history approach to the analysis of ownership duration data, with focus on maritime transport. A change of ownership of an item - in our case an ocean-going vessel - can be considered as an event in time with the interesting feature that each event has a dual interpretation: as a *sale* by one company and a *purchase* by another. We will explore and apply methods for the analysis of such data in this paper.

Examples of event history analyses of ownership duration to date include studies of the housing market (Archer et al. 2010); corporate ownership and equity duration (Bøhren et al. 2005); foreign owners and plant survival for companies (Kronborg & Thomsen 2009) and, in the transport sector, automotive vehicle ownership (Iwata & Matsumoto 2016). The use of event history analysis in the maritime related literature, however, is very limited, though some notable examples include work on survival of Norwegian shipping companies (Tenold & Aarbu 2011) and an investigation into the effectiveness of ship inspections (Bijwaard & Knapp 2009). It is somewhat surprising that research on length and pattern of ownership in shipping is so scarce (Stott 2014), given that the majority of world trade is transported by sea (UNCTAD 2017). We, therefore, suggest that an examination of how event history analysis could be applied in this field is long overdue and that any such analysis should accommodate the dual interpretation of an event, as explained in the first paragraph.

A first approach to such an event history analysis of vessel ownership is usually to examine covariates in an attempt to explain the variation in the time taken until the specified event occurs. Such times are defined as the length of time from some start event (when the vessel is bought) until the time to some stop event (when the vessel is sold or scrapped) but with ownership times being censored for those vessels that are owned by a company at the end of the observation period.

One drawback is that not all possible reasons for the variation in the intensity function may have been included in the model as there are many factors that could affect the length of ownership of a vessel (Stopford 2009). In Figure 1, for example, we show a transition diagram for sales between the first and second owners of the vessels in our data set. The diagram will be explained and discussed further in the next section but it is clear that the relationship between buyers and sellers is complex and there is potential for significant heterogeneity in behaviour.

We can model such heterogeneity with *random effects*, an approach that was first introduced by Beard (1959) with the alternative term *frailty* coined by Vaupel et al. (1979) 20 years later. Much work on frailty in a variety of settings soon followed, particularly important examples being Oakes (1982) and Hougaard (1986). This growing body of work was enhanced when Aalen demonstrated clearly how heterogeneity could be modelled (Aalen 1987), while warning that ignoring frailty could have a significant impact on estimating effects. In a series of further papers Aalen went on to provide the theoretical

underpinning for modelling the population hazard for the univariate frailty model, making the inclusion of frailty more accessible and routine (Aalen 1988, 1992, 1994, Aalen & Tretli 1999, Aalen et al. 2014).

It is recommended that an attempt should always be made to include frailty if it is possible that heterogeneity is present. We therefore look to include sources of possible unexplained variation in our current shipping scenario in a novel approach that contrasts the frailty values of buyers and sellers. More explicitly, one possible source of heterogeneity involves the companies who own the vessels as they may all be susceptible to sales at different rates. In addition, companies looking to buy a vessel may also influence the rate of sales as each purchase also needs a willing buyer and all companies are willing to varying extents. It, therefore, seems sensible to combine in the intensity the willingness of companies to sell with the willingness of other companies to buy. This can be partially accommodated by including covariates for the vessel, the company that owns the vessel and the company that wants to buy the vessel. We cannot, however, be confident that we have captured all of the effects that may influence the sale of a vessel so we look to include separate frailty terms for both the selling company and the buying company, an approach we call *dual frailty* and explain as follows.

Let us first define ‘sentiment’ S as a measure of whether a company is looking to sell a vessel or buy a vessel. We shall assume that the intensity function for a sale is in the familiar Cox proportional intensity form with our sentiment value influencing the intensity from within the exponential term (a full expression for the intensity will be given in Section 3). We assume that positive values of S are assigned to companies willing to sell (thereby increasing the intensity function for a sale) and negative values assigned to companies willing to buy (thereby decreasing the intensity for a sale). For mathematical expedience, we choose to amend the form of the intensity with the frailty term $Z = \exp(S)$ so that it appears outside of the exponential and we can assign it the familiar gamma distribution to aid our analysis.

After describing the data and problem in Section 2, we will develop our model and estimation procedure in Section 3. We shall then explore with simulations in Section 4, report our results in Section 5 and discuss possible extensions in Section 6.

2 Transactions data

The ownership structure in shipping is fragmented, with various different definitions, the two most important of which are *registered* and *beneficial* owner respectively (Veenstra & Bergantino 2000). The registered owner of a vessel is the legal title appearing on the vessel’s registration but this often changes in shipping for various reasons, including tax and liability. The beneficial owner, also referred to as ultimate owner (Kang & Kim 2012), is the entity that gains “the ultimate financial benefit from a vessel’s operation” (Fox 2005).

Table 1 Transactions data summary

Type	Number				Total owners					
	Vessels	Censored	Scrapped	Sales	1	2	3	4	5	6
Dry bulk	797	624	173	883	289	258	156	68	16	10
Tanker	585	468	117	450	284	191	77	27	6	0
Container	617	481	136	337	372	171	59	12	3	0
All	1999	1573	426	1670	936	620	292	107	25	10

We consider the beneficial ownership records of 1999 commercial vessels built between 1987 and 2007 and followed until 2015. The data were collected by Ralitsa Mihaylova as part of her PhD programme, and were obtained via individual inspection of records made available by two leading shipping data providers: Sea-Web (a joint venture between IHS Maritime and Trade and Lloyds Register) and Clarksons Research Services Ltd (CRSL). Mihaylova (2017) provides full information.

Table 1 gives an overview of ownership and sales or scrap events for each vessel category. The entry point for a vessel is the date of the delivery to the first owner, which is the date the vessel enters into operation (Stott 2014). Vessels were followed until 2015 unless they were scrapped earlier. The shortest recorded lifetime was for a vessel that was scrapped at 13.5 years, and the longest was for a vessel that was still in use after 28.75 years. Working lifetimes were typically in the region of 25 years: Kaplan-Meier estimates are 80% still in use at 20 years but only 6% in use at 28.75 years. The maximum number of owners recorded in the data was six, with 47% of vessels not being sold at all during the follow-up period.

Mihaylova (2017) analysed ownership durations for these data, using standard survival analysis techniques rather than the dual frailty and multistate approach considered here, and without allowance for buyer effects. Our selection of covariates is based on the findings there.

We used four vessel-level covariates: type of vessel, deadweight (scaled to a multiple of 10,000 tonnes), speed (knots) and number of previous owners, the latter being time-varying. The vessel types are dry-bulk vessels, tankers and containers, but as dry-bulk vessels and tankers both represent the bulk market (dry and liquid respectively) in our analyses we simply use a binary indicator for container or otherwise. The deadweight reflects the cargo-carrying capacity of a vessel.

The number of companies involved in the commercial history of the vessels was 1,125. Not all were active at the same time. We identified 413 companies as being active at the beginning of the follow-up period. The number rose steadily to a stable level around 860 from 2005 to 2008, and then, following the financial crash, declined to 696 at the end of follow-up. We classified each company by type and nationality. We took four company types (private, public, financial and state-owned) and six nationality categories (China, Germany, Greece, Japan, other traditional maritime nations and emerging maritime na-

Table 2 Overview of categorical covariates. Italics indicate baseline levels.

Covariate	Levels	Counts
Vessel type	Container	617
	<i>Bulker/tanker</i>	1382
Company type	<i>Private</i>	895
	Financial	26
	Public	159
	State	45
Company nationality	China	117
	EMN	184
	<i>TMN</i>	380
	Germany	101
	Greece	245
	Japan	98

tions). Nationality is associated with beneficial ownership as it reflects the country where the primary economic contribution ends up, which may or not be where the company is registered or where the owner is based. The data on nationality was retrieved from Sea-Web. The categories “traditional maritime nations” (TMN) and “emerging maritime nations” (EMN) are based on the maritime traditions framework developed by Alderton & Winchester (2002). As company-level covariates we defined appropriate indicator variables for the classification, with baseline taken to be a privately-owned company associated with a traditional maritime nation.

There are many potential exogenous variables, as discussed in Mihaylova (2017). For simplicity we use just one, the logged Clarksea index, which is a good proxy for market conditions. It is a weighted average of the daily earnings of the main vessel types, where the weighting is based on the number of vessels in each fleet segment provided by CRSL. We chose the Clarksea index because freight earnings are believed to trigger activities within the shipping market, including the ordering and sale and purchase of vessels (Abouarghoub et al. 2012). In the analyses reported in Section 5 we used the Clarksea index at the calendar time of potential sales, and also lagged by six months. In addition we allowed interaction between the index and vessel type.

Most of the covariates are categorical: their frequencies are given in Table 2. There are four quantitative covariates: deadweight, speed, number of previous owners and the Clarksea index. Deadweight ranged from 1.2×10^4 tonnes to 44×10^4 tonnes, with mean 8×10^4 tonnes. Speeds ranged from 12 to 26.5 knots with mean 16.7 knots. Vessels had up to five previous owners in the time period. The logged Clarksea index varied between 8.970 and 10.789: its value over time is given later, in Figure 4.

Finally for this section, to illustrate some of the complexity of the transactions data, Figure 1 represents the transitions from first to second owner based on nationality. The number of vessels participating in the transition from first to second owner is represented by the outermost circle. The outgoing flows show the number of vessels sold (second circle) and therefore the colour of the flow corresponds to the colour of the owner nationality. The incoming

flows (third circle) represent the number of vessels that were bought. From the diagram, it is clear that Japanese and German owners are more involved in purchasing new vessels than acquiring second-hand tonnage, whereas Greek owners appear to be more active in the second-hand market.

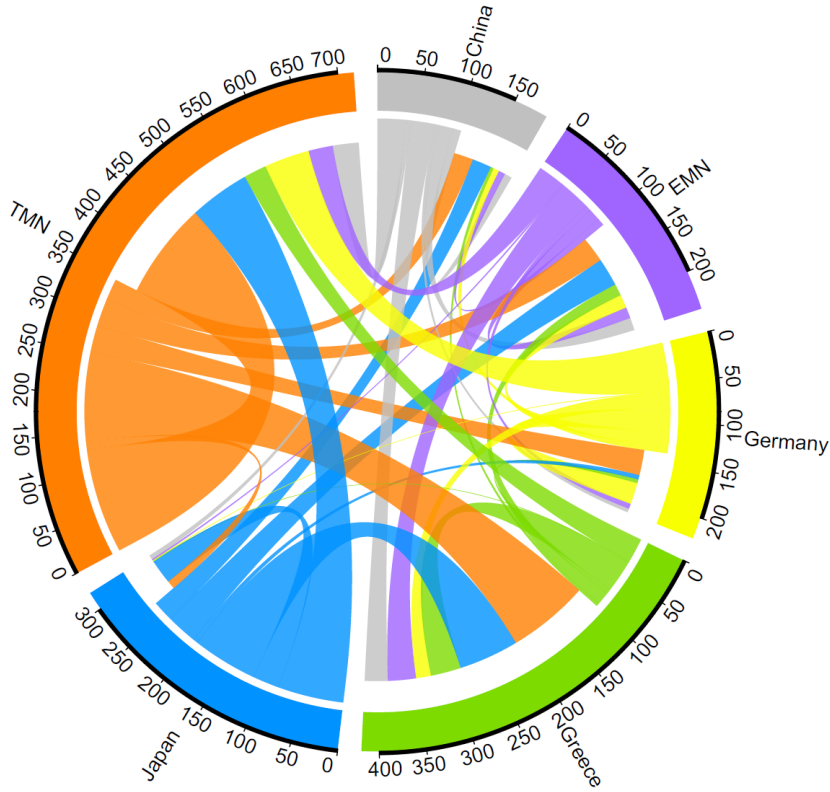


Fig. 1 First changes in ownership, classified by state-group of selling and buying companies. The start and end width of each line is proportional to the number of vessels. The outermost circle represents the number of transactions involving companies in the state-group. The next circle represents sales and the third represents purchases. The plot was produced using methodology developed by Sander et al. (2014). TMN is traditional maritime nations and EMN is emerging maritime nations.

3 Model and estimation

3.1 Preliminaries

The data consists of information on K vessels that are delivered in a calendar time interval (τ_1, τ_2) , with follow-up to time τ . For each vessel we have the delivery date, the dates, if any, at which the vessel is sold from one company

to another, and the date of scrapping if appropriate. The final time τ censors the observed history for vessels not yet scrapped.

There are N companies which are potential buyers or sellers of vessels for at least part of the observation interval. Events of interest are the sale of a vessel from one company to another or the scrapping of the vessel by the current owner. In each case the event intensity is expected to depend upon both calendar time t and vessel age a . Let $t_v(a)$ be the calendar time at which vessel v reaches age a .

First we consider covariates. These fall into three groups:

- $x_v^V(t)$ is a p_1 -vector of covariates associated with vessel v at calendar time t . Some of these may be static, such as deadweight for instance, whilst others may be dynamic summaries of the vessel's history, such as number of previous owners or duration of current ownership.
- $x_c^C(t)$ is a p_2 -vector of covariates associated with company c . In principle these can vary with calendar time although in our application they are time-fixed characteristics of the company.
- $x^E(t)$ is a p_3 -vector of exogenous covariates reflecting market conditions or time trends perhaps.

Next we have two at-risk indicators. Since companies may not be active throughout the full observation window, the first indicator $Y_c^C(t)$ is taken to be one if company c is known to be active at calendar time t and is zero otherwise. The second at-risk indicator $Y_v^V(a)$ is defined to be one if vessel v is still in use and hence liable to be sold at age a , and is taken to be zero if the vessel has been scrapped or censored.

Further, we associate with each company c an unobserved random variable $S_c(t)$ that captures the *sentiment* of the company, with positive values implying a willingness to capitalise on assets and sell vessel holdings, and negative values implying an unwillingness to capitalise and a preference for investment in new vessels. It will be convenient to denote $Z_c(t) = \exp\{S_c(t)\}$. To be consistent with biostatistical terminology we will refer to $Z_c(t)$ as “frailty” and reserve “sentiment” for $S_c(t)$ as convenient. For simplicity, we will assume for our first analyses that the random effects are time-constant, $Z_c = Z_c(t)$ and $S_c = S_c(t)$. We will come back to this issue later.

Finally, let \mathcal{F}_t denote the observed history of events and covariate evolution up to calendar time t and let Z be the N -vector of frailties.

3.2 Dual frailty model

At age a a generic vessel v may be sold or scrapped. If sold, then any of the companies that are active at the relevant calendar time may be the purchaser. Hence we have a multistate problem and our approach will be based on modelling the transition intensities (eg Andersen & Keiding 2002).

We assume the frailty effects Z_c are independent gamma random variables with mean one and variance ξ . Conditioning on the combined vector of frailties

Z and prior history, let $\alpha_v(a, b | \mathcal{F}_{t_v(a)}, Z)$ be the transition intensity for a sale of vessel v to buying company b at vessel age a , which occurs at calendar time $t = t_v(a)$. We will use $s = s(a, v)$ to indicate the selling company, ie the current owner. We assume a semi-parametric multiplicative model

$$\begin{aligned} \alpha_v(a, b | \mathcal{F}_{t_v(a)}, Z) &= \exp\{S_s(t) - S_b(t)\} \\ &\quad \times \alpha_0(a) \exp\{\beta_V x_v^V(a) + \beta_S x_s^C(t) + \beta_B x_b^C(t) + \beta_E x^E(t)\} \\ &= Z_s Z_b^{-1} \alpha_0(a) \exp\{\beta_V x_v^V(a) + \beta_S x_s^C(t) + \beta_B x_b^C(t) + \beta_E x^E(t)\} \\ &= Z_s Z_b^{-1} \alpha_0(a) R_v(a, b; \beta), \end{aligned} \quad (1)$$

say, where $\beta = (\beta_V, \beta_S, \beta_B, \beta_E)$, $R_v(a, b; \beta)$ is the relative risk determined by covariates and $\alpha_0(t)$ is the baseline intensity. The overall sales intensity for the vessel is then

$$\alpha_v(a | \mathcal{F}_{t_v(a)}, Z) = \sum_{b \neq s(a, v)} Y_b^C \{t_v(a)\} \alpha_v(a, b | \mathcal{F}_{t_v(a)}, Z), \quad (2)$$

and the cumulative sales intensity of vessel v to age u is

$$A_v(u | \mathcal{F}_u, Z) = \int_0^u Y_v^V(a) \alpha_v(a | \mathcal{F}_{t_v(a)}, Z) da. \quad (3)$$

In (1), β_V measures how vessel-level characteristics affect the intensity of sales, β_S measures how the owning company characteristics influence the likelihood of selling, and β_B is similarly used to measure how the characteristics of potential buyers affect the intensity of sales. The final regression coefficient β_E is used in an attempt to capture exogenous market conditions or time trends. There are two frailty terms in (1), one for the selling company and one for the buying company. A sale from company s to company b is more likely, given covariates, when Z_s is high and Z_b is low, and less likely if the circumstances are reversed. We are not aware of any previous work on dual frailty effects of this type.

We make no assumptions about $\alpha_0(a)$. Thus we do not specify the form of the vessel age effect, but within (1) we have fully specified the form of the calendar time effect by allowing time trends in $x^E(t)$. Our reason is that this form allows us to include economic indicators in $x^E(t)$, whose effects would not be identifiable if we had a nonparametric baseline in calendar time t rather than age a .

Vessels can be scrapped as well as sold. Our model for the corresponding cause-specific hazard is

$$\begin{aligned} \lambda_v(a | \mathcal{F}_{t_v(a)}, Z) &= Z_s \lambda_0(a) \exp\{\theta_V x_v^V(a) + \theta_S x_s^C(t) + \theta_E x^E(t)\} \\ &= Z_s \lambda_0(a) Q_v(a; \theta), \end{aligned} \quad (4)$$

say, where $\theta = (\theta_V, \theta_S, \theta_E)$ and $Q_v(a; \theta)$ is the non-frailty relative risk. There is no buying company effect for scrap, which is of course an absorbing state. We use

$$\Lambda_v(u | \mathcal{F}_u, Z) = \int_0^u Y_v^V(a) \lambda_v(a | \mathcal{F}_{t_v(a)}, Z) da \quad (5)$$

to denote the cumulative scrap hazard of vessel v up to age u .

3.3 Estimation

Given Z , partial likelihood methods are available for estimation of β and θ . How best to deal with Z is not so obvious however. A variety of methods can be used for standard frailty models for survival data, including, inter alia, EM (eg Barker & Henderson 2005), penalised likelihood (eg Androukalis et al. 2012), h-likelihood (eg Ha et al. 2001), or marginal approaches after integrating out the frailties. None of these lend themselves to our dual frailty problem.

Instead, we will take a Bayesian approach, estimating Z , β and θ using Markov chain Monte Carlo and using versions of the Breslow estimator for baselines. We suggest standard Metropolis-Hastings for the regression parameters β and θ and the log of the frailty variance ξ , with random walk proposals. For Z , a Gibbs sampling approach is more efficient and is feasible given our construction.

To see this, consider the contribution of vessel v to the full-data likelihood. In short and loose notation, this is

$$L_v = \left(\prod_{sales} \alpha_v \right) e^{-A_v} \lambda_v^\delta e^{-\Lambda_v},$$

where δ is an indicator of the vessel being scrapped, α_v and λ_v are the sales intensity and scrap hazard, and A_v and Λ_v are the cumulative intensities. From (1)–(5) each intensity α_v is proportional to $Z_s Z_b^{-1}$ for some Z_s and Z_b and each λ is a multiple of the frailty Z_c of the final owning company. The cumulative intensity A_v , given at (3), is the integral of the sales intensity (2). The integral can be broken into disjoint segments, one per ownership period, within each of which the integrand is a sum of terms involving products like $Z_s Z_b^{-1}$ for the selling and potential buying companies. The cumulative scrap intensity Λ_v can similarly be broken into disjoint segments, in each of which the integrand is proportional to the frailty of the current owner.

Taken over all vessels, the above means that the full-data likelihood considered as a function of a frailty Z_c for a generic company c , is proportional to

$$Z_c^M \exp\{-D_1 Z_c - D_2 Z_c^{-1}\},$$

for some M and non-negative D_1 and D_2 . The value of M is simply the difference between the total number of vessels sold or scrapped by company c and the total number vessels purchased by that company. The value of D_1 depends upon the cumulative sales and scrap intensities for vessels owned by

the company, while D_2 is the cumulative intensity for purchases. Expressions for D_1 and D_2 are unwieldy to write down and hence omitted, but they are straightforward to program. Note that D_1 and D_2 both involve frailty terms for other companies.

When the likelihood is combined with an independent gamma $\Gamma(1/\xi, 1/\xi)$ prior for Z_c , we see that the posterior distribution of Z_c , given the data, the parameters, and the frailty values for all other companies, is proportional to

$$Z_c^{M+1/\xi-1} \exp\{-(D_1 + 1/\xi)Z_c - D_2 Z_c^{-1}\}.$$

This is a generalised inverse gamma distribution (Hougaard 2000, page 508). The R package `GIGrvg` includes a routine `rgig` to simulate from this distribution and hence Gibbs sampling can be implemented.

This leaves the baseline sales intensity and scrap hazard, $\alpha_0(a)$ and $\lambda_0(a)$ respectively. Given our use of MCMC, we take the common approach of profiling out the baseline intensity and hazard functions at each iteration. For fixed frailties Z and regression parameters $\beta = (\beta_V, \beta_S, \beta_B, \beta_E)$ and $\theta = (\theta_V, \theta_S, \theta_E)$, versions of the Breslow estimators of the cumulative baseline sales intensity

$$A_0(u) = \int_0^u \alpha_0(a) da$$

and cumulative baseline scrap hazard

$$\Lambda_0(u) = \int_0^u \lambda_0(a) da$$

are available. Letting $dN_{sale}(a)$ and $dN_{scrap}(a)$ be the total numbers of sales and scraps of vessels at age a respectively, the estimators are

$$\hat{A}_0(u; \beta, Z) = \sum_{a \leq u} \frac{dN_{sale}(a)}{\sum_v \sum_{b \neq s(a,v)} Y_v^V(a) Y_b^C(t_v(a)) Z_{s(a,v)} Z_b^{-1} R_v(a, b; \beta)} \quad (6)$$

and

$$\hat{\Lambda}_0(u; \theta, Z) = \sum_{a \leq u} \frac{dN_{scrap}(a)}{\sum_v Y_v^V(a) Z_{s(a,v)} Q_v(a; \theta)} \quad (7)$$

respectively.

Finally, we note that an alternative hybrid approach is to use partial likelihood for estimation of the regression parameters β and θ at each iteration of an MCMC for ξ and Z . The final variance estimate needs to combine the within-chain variation arising through variability in Z with the information-based estimate conditional upon Z .

4 Simulations

To investigate performance of the estimation procedure we undertook a small simulation study. For simplicity we did not consider vessel scrapping, we assumed there were no exogenous covariates $x^E(t)$ and we assumed that all companies were active throughout the study period.

We took $N = 500$ companies and $K = 1000$ vessels. We assumed two time-constant vessel-level covariates X^V . The first was binary and the second uniform on ± 20 , loosely representing vessel category and centred deadweight respectively. We also assumed two time-constant company level covariates X^C , in this case both binary. All covariates were independent of the others.

We set the vessel regression parameters to be $\beta_V = (0.3, -0.01)$, the selling company parameters to be $\beta_S = (0.4, -0.2)$ and the buying company parameters to be $\beta_B = (-0.2, 0.1)$. We simulated in discrete time, with baseline intensity 1×10^{-6} per month and follow-up truncated at 500 months for all vessels. We considered two choices of frailty variance: $\xi = 0.2$ and $\xi = 0.4$. These parameter choices typically led to 3500-4000 transactions, which is comparable to the real data.

We used Metropolis-Hastings for the regression parameters β and for $\log(\xi)$, with random walk proposals and zero-mean Gaussian increments. After some trial and error, we selected the standard deviation for the increments to be 0.07 for all regression parameters other than the continuous vessel-level covariate (which is on a different scale), for which we used standard deviation 0.002. For $\log(\xi)$ the increment standard deviation was 0.02. Initial values for the regression parameters were taken from a no-frailty analysis, all frailties Z_c were initially set at one, and the frailty variance ξ was initialised at the low value of 0.01. For each value of ξ we took 1000 replications, each with 3000 iterations for burn-in and a further 10,000 iterations for estimation.

Table 3 summarises our results, for the full MCMC and also for standard proportional intensity analyses in which frailty is mistakenly ignored. In that case there is clear attenuation of the regression parameters for vessels and selling companies, β_V and β_S respectively, as might be expected when frailty is ignored (Henderson & Oman 1999). Interestingly, there is no evidence of attenuation for the buying-company parameters β_B . In an appendix we explore this issue further.

When frailty is properly accounted for our procedure seems to work well, as it should. All means are within simulation noise of the true parameter values, the estimated standard errors closely match the empirical values, and the number of iterations is adequate.

5 Application to vessel trading

Maximised log partial likelihoods under a no-frailty model are in Table 4 and coefficient estimates with and without frailty are in Tables 5-7. For the model including frailty, we ran the Monte Carlo chain for 23,000 iterations and dis-

Table 3 Simulation results. Estimates are from 1000 simulations, each involving 500 companies and 1000 vessels. Columns 4-6 are from (partial) likelihood analyses, ignoring frailty. Columns 7-9 are from MCMC analyses, properly allowing for frailty. ‘Est SE’ is based on the average within-run variance, and ‘Emp SE’ is based on the observed variance of mean values across replications.

		True	Ignored frailty			MCMC		
			Mean	Est SE	Emp SE	Mean	Est SE	Emp SE
$\xi = 0.2$	β_V	True	Mean	Est SE	Emp SE	Mean	Est SE	Emp SE
		-0.010	-0.009	0.002	0.002	-0.010	0.002	0.002
	β_S	0.300	0.273	0.048	0.070	0.299	0.068	0.066
		0.400	0.357	0.050	0.071	0.401	0.071	0.069
	β_B	-0.200	-0.201	0.049	0.070	-0.202	0.070	0.067
		0.100	0.104	0.049	0.072	0.104	0.071	0.068
ξ	0.200				0.199	0.020	0.019	
$\xi = 0.4$	β_V	0.300	0.218	0.050	0.050	0.298	0.057	0.052
		-0.010	-0.007	0.002	0.002	-0.010	0.002	0.002
	β_S	0.300	0.253	0.049	0.123	0.304	0.087	0.082
		0.400	0.301	0.051	0.119	0.398	0.089	0.081
	β_B	-0.200	-0.208	0.050	0.141	-0.205	0.092	0.088
		0.100	0.109	0.050	0.146	0.104	0.093	0.092
ξ	0.400				0.398	0.032	0.032	

Table 4 Maximised log partial likelihood for no-frailty model

	Sale data	Scrap data
Null	-21597.01	-2331.58
No vessel effect $\beta_V = \theta_V = 0$	-21287.56	-2256.48
No seller effect $\beta_S = 0$	-21417.00	-2250.97
No exogenous effect $\beta_E = \theta_E = 0$	-21298.04	-2287.95
No buyer effect $\beta_B = 0$	-21353.07	NA
Full	-21262.85	-2235.24

carded the first 3000 as burn-in. Just as for the simulations we initialised all frailties at the value one and took the initial value of ξ to be 0.01. The trace plot in Figure 2 shows how the estimate of ξ quickly increased during the burn-in and then converged to a distribution centred just below 0.2. Trace plots for other parameters all also indicated convergence after burn-in. There are no maximised log likelihoods for the frailty model, given our use of MCMC estimation.

Conclusions concerning the regression coefficients are essentially the same whether or not frailty is included in the model. From Table 4 there are highly significant effects on sales intensity for covariates associated with the vessel and the buying and selling companies, and also relating to market conditions. The same is true for the intensity of vessel scrapping, except of course there is no buying company in that case.

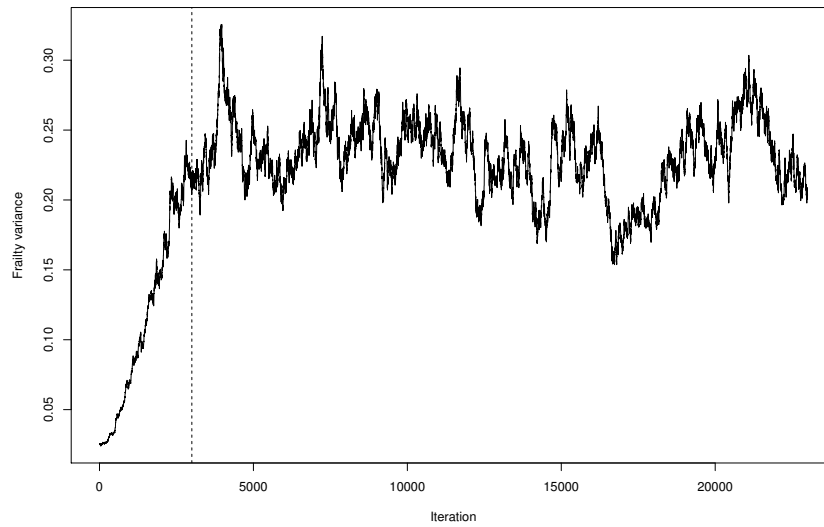


Fig. 2 MCMC frailty variance trace plot. Values to the left of the vertical line at iteration 3000 were discarded as burn-in.

Table 5 Vessel and company covariate effects for sales

		No frailty			Frailty		
		Est	SE	Wald	Est	SE	Wald
β_V	Deadweight	-0.010	0.004	-2.48	-0.006	0.004	-1.44
	Speed	-0.053	0.019	-2.75	-0.048	0.020	-2.35
	Container	3.974	1.316	3.02	3.884	1.324	2.93
	Previous owners	-0.195	0.039	-4.98	-0.122	0.045	-2.69
β_S	Financial	0.437	0.129	3.39	0.263	0.165	1.59
	Public	-0.100	0.063	-1.59	-0.042	0.083	-0.50
	State	-1.454	0.175	-8.30	-1.544	0.211	-7.31
	China	-0.420	0.098	-4.28	-0.550	0.117	-4.68
	EMN	-0.339	0.087	-3.88	-0.548	0.103	-5.30
	Germany	-0.097	0.097	-1.00	-0.119	0.120	-1.00
	Greece	-0.349	0.077	-4.53	-0.305	0.091	-3.34
Japan	0.239	0.074	3.22	0.297	0.108	2.75	
β_E	Index	0.553	0.092	6.02	0.566	0.092	6.14
	Lagged index	-0.219	0.094	-2.33	-0.197	0.094	-2.09
	Index*Container	0.201	0.205	0.98	0.224	0.206	1.09
	Lagged index*Container	-0.639	0.212	-3.01	-0.649	0.212	-3.05

Table 6 Company covariate effects for purchases

β_B		No frailty			Frailty		
		Est	SE	Wald	Est	SE	Wald
	Financial	0.375	0.148	2.53	0.450	0.175	2.57
	Public	0.098	0.073	1.33	0.114	0.092	1.24
	State	-0.048	0.133	-0.36	-0.114	0.172	-0.66
	China	-0.098	0.087	-1.12	-0.171	0.107	-1.59
	EMN	-0.275	0.077	-3.55	-0.270	0.095	-2.86
	Germany	-0.655	0.111	-5.92	-0.641	0.130	-4.94
	Greece	0.256	0.063	4.09	0.206	0.078	2.63
	Japan	-1.124	0.137	-8.22	-1.086	0.159	-6.82

Table 7 Vessel and company covariate effects for scrap

θ_V		No frailty			Frailty		
		Est	SE	Wald	Est	SE	Wald
	Deadweight	0.038	0.008	4.90	0.041	0.009	4.83
	Speed	0.111	0.032	3.44	0.131	0.034	3.82
	Container	7.977	4.642	1.72	8.607	4.675	1.84
	Previous owners	-0.138	0.054	-2.56	-0.063	0.060	-1.06
θ_S	Financial	0.481	0.417	1.15	0.276	0.438	0.63
	Public	-0.136	0.136	-1.00	-0.306	0.154	-1.98
	State	0.216	0.201	1.08	0.003	0.240	0.01
	China	0.277	0.166	1.67	0.362	0.188	1.93
	EMN	-0.126	0.169	-0.75	-0.283	0.183	-1.55
	Germany	0.533	0.192	2.78	0.486	0.229	2.12
	Greece	0.239	0.152	1.57	0.265	0.167	1.59
	Japan	0.861	0.204	4.21	1.045	0.242	4.31
θ_E	Index	-0.726	0.279	-2.60	-0.697	0.279	-2.49
	Lagged index	-0.992	0.265	-3.74	-1.007	0.266	-3.79
	Index*Container	-0.413	0.500	-0.82	-0.441	0.502	-0.88
	Lagged index*Container	-0.452	0.483	-0.94	-0.482	0.484	-0.99

The company-level effects are perhaps of most interest. State-owned companies are highly reluctant to sell, while Japanese and German companies are highly unlikely to buy vessels that are not new. Companies based in emerging maritime nations tend to either buy or sell relatively rarely. Greek companies, which are usually small and privately owned, are lively purchasers of second-hand vessels. The comments on Figure 1 in Section 2 are consistent with these results. Interestingly, financial companies are likely to both sell and buy, which is in accord with suggestions that such companies are in a position to benefit from the volatile nature of the shipping industry due to availability of capital (Thanopoulou & Strandenes 2017).

Turning briefly to other effects, vessels are more likely to be traded when the Clarksea index is high, and more likely to be scrapped when it is low. The effect on containers is lagged compared with other vessel types. This may be because of the nature of the container sector where competition is linked to availability of capacity whose purpose is to provide efficient service

to network strings. The container liner market is defined by high levels of cooperation and concentration (Sys 2009), which suggests that any capacity-related investment decisions need to be in alignment with the interests of strategic partners (Rau & Pinler 2016). If a liner operator believes that the cost of supply shortage is greater than the cost of excess capacity, they are likely to be optimistic when purchasing additional capacity (Fusillo 2003). Fusillo (2003) also provides evidence that: (i) capacity additions in the liner sector are large in comparison to demand and (ii) that partially this is due to a strategic entry-detering behaviour adopted by large operators where they use capacity expansion as means to limit competition. Therefore, the lagged effect might be a product of the resilience of liner operators due to cooperation and concentration of the market, the willingness to withstand market fluctuations rather than risk losing market share, the technical difficulties associated with rescheduling services and the time it takes to discuss capacity-related decisions with all strategic partners.

The mean of the frailty variance was $\hat{\xi} = 0.231$ with MCMC standard deviation 0.027 after burn in. The distribution for individual company estimates \hat{Z}_c was concentrated about one, as expected. Almost 60% of estimates were in the range (0.8,1.2) and 94% in the range (0.5,1.5). There were however some quite extreme values, with the smallest being 0.08 and the largest being 2.24. The smallest frailty estimate was for a large, state-owned Chinese company. Consistent with low frailty implying a preponderance of buying over selling, this company bought 25 vessels in our observation period and sold just one. At the other end of the scale the largest frailty estimate was for a medium sized privately owned Chinese company that sold eight vessels and bought none.

Finally, Figure 3 shows the effect of vessel age on intensity of sales or scrapping, evaluated for a vessel with median values of all covariates. Sales intensity is fairly constant until the vessel is around 25 years old, after which there are no sales. Vessels are not likely to be scrapped until they are around 20 years old, after which time the cumulative hazard for scrapping increases steeply year by year.

6 Discussion

We have described a dual frailty model for ownership duration data and given an application on the buying, selling and scrapping of ocean-going vessels. The act of ordering new vessels was not considered due to the specific nature of newbuilding contracts in shipping. For example, the decision to order a vessel can be made years before the vessel is delivered to the buyer and, in addition, it is not uncommon for ships that have been ordered by one company to end up being delivered to another company. Subsequent purchases have a clearer process and have been, therefore, our focus in the paper.

We assumed the company-level frailty variables are time-constant and a natural extension would be to relax that assumption. A piecewise-constant model would seem to be appropriate, with change-points at some natural way-

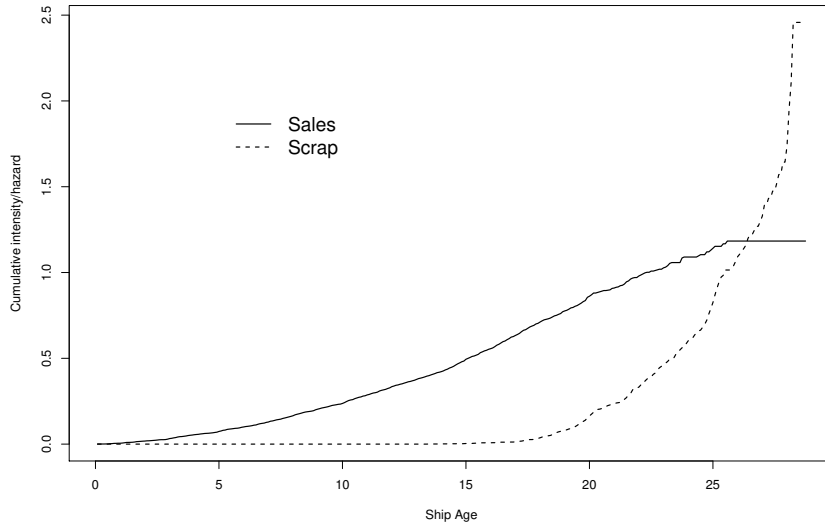


Fig. 3 Estimated cumulative intensity for sales and cumulative hazard for scraps, evaluated at median values of covariates

marks. In Figure 4 for instance we show the logged Clarksea index together with the relative risk of sale for containers compared with other vessel types. We also indicate three events that could have serious effects on the market for vessels: the 1997 Asian financial crisis, the 2000 dot.com bubble and its demise, and the 2008 financial crash. After each of these there seems to be a change in slope for the relative risk. Allowing a company - if active - to have a different frailty for each of the periods between these events is a reasonable next step. Clearly the values are likely to be correlated, which could be accommodated in principle through the correlated gamma frailty models of Henderson & Shimakura (2003) or Fiocco et al. (2009). There is no simple form for the density of these multivariate gamma distributions and hence the use of MCMC would not be straightforward. Instead we might consider modelling the sentiments $S_c(t)$ directly, most obviously using a multivariate Gaussian distribution. In this case the price to be paid is the lack of a closed form conditional distribution, given the data and other frailties, which precludes the use of Gibbs sampling.

Another assumption is that the same sentiment or frailty affects buying, selling and scrapping. A potentially interesting extension would be to have three frailties per company, drawn from some trivariate distribution, with one acting on sales, one on purchases and one on scrapping. Another is to have a single frailty Z which acts directly on sales, say, but then to have powered forms Z^{γ_1} and Z^{γ_2} for the effects of sentiment on purchases and scrapping,

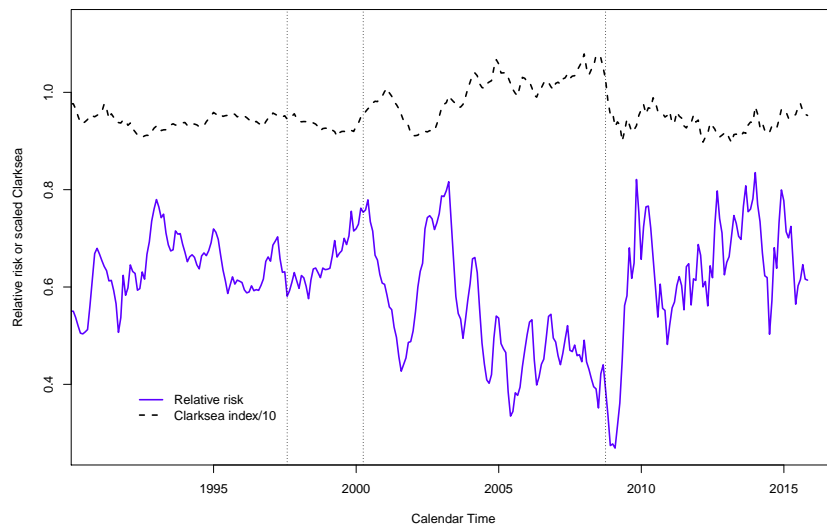


Fig. 4 Relative risk of sale for container compared with other vessel types, together with logged Clarksea index (scaled by 10). The first vertical line marks the Asian financial crisis of July 1997, the second the peak in March 2000 of the dot.com bubble, and the third the financial crash of September 2008.

respectively. A disadvantage of this model is that we would lose the generalised inverse gamma form for the posterior distributions of frailties. Further work on this would be worthwhile, and also for the time-varying situation. Clearly the available data would need to be rich in order for reliable inferences to be obtained from these more complex models.

Acknowledgements The research of Ralitsa Mihaylova was supported by the Prime Foundation. We thank the Prime Maritime Research Group for their support and CRSL for providing us with a bespoke vessel ownership history data.

Appendix

In the simulation results reported in Section 4 we noted attenuation towards zero of the vessel and selling company regression estimates when frailty was ignored, but no apparent attenuation for buying company coefficients. We will briefly investigate.

For simplicity we assume no vessel or exogenous covariates and a single time-constant binary company covariate X^C . We use vessel age as the only time scale, in effect assuming that all vessels are delivered at the same calendar time. We assume that all companies are active throughout the observation

Table 8 Estimated regression coefficients under ignored frailty. Results based on 1000 replications with $N=1000$, $K = 1000$ and $\alpha_0(a)=0.00001$.

Scenario	True	$\xi = 0$		$\xi = 0.2$		$\xi = 0.4$	
		Mean	SE	Mean	SE	Mean	SE
1	$\beta_S=1$	1.000	0.029	0.933	0.047	0.860	0.058
	$\beta_B=0$	0.000	0.030	0.000	0.032	0.001	0.034
2	$\beta_S=0$	0.000	0.029	0.001	0.026	0.001	0.023
	$\beta_B=1$	1.002	0.031	1.002	0.045	1.004	0.083
3	$\beta_S=1$	1.000	0.023	0.923	0.063	0.774	0.162
	$\beta_B=1$	1.001	0.022	0.999	0.046	0.996	0.083

period, that there is no scrapping and that all vessels are monitored, and can potentially be sold, up to age τ . Hence there are no calendar time effects and no selection effects due to vessels being observed or not. Interest is in the selling company and buying company regression coefficients β_S and β_B .

We consider three scenarios.

1. Sales are driven only by the selling company covariate and frailty. The transition intensity for a sale of vessel v at age a from its owning company $s(a, v)$ to another company b is

$$\alpha_v(a, b | \mathcal{F}_a, Z) = Z_{s(a,v)} \alpha_0(a) \exp\{\beta_S x_{s(a,v)}^C\}.$$

2. Sales are driven by the frailty and covariate of the buying company only. The intensity is

$$\alpha_v(a, b | \mathcal{F}_a, Z) = Z_b^{-1} \alpha_0(a) \exp\{\beta_B x_b^C\}.$$

3. Characteristics of both the selling and buying companies affect the sales intensity:

$$\alpha_v(a, b | \mathcal{F}_a, Z) = Z_{s(a,v)} Z_b^{-1} \alpha_0(a) \exp\{\beta_S x_{s(a,v)}^C + \beta_B x_b^C\}.$$

The simulation results in Table 8 support the suggestion in Section 4 that the coefficient β_S associated with the selling company is attenuated when frailty is ignored, whereas the coefficient β_B associated with the buying company is not. In other simulations (not shown) we found the vessel-level regression coefficients are also attenuated under ignored frailty.

To explore a little, we consider estimation when both the number of vessels K and the number of companies N increase, not necessarily at same rate. Using similar methods to those of Henderson & Oman (1999), the estimators $\hat{\beta}_B$ and $\hat{\beta}_S$ can be shown to converge to so-called least-false values β_B^* and β_S^* respectively. These values are the solutions to the limiting score equations, with suitable scaling. We will concentrate first on Scenario 3, for which the

limiting score equations are

$$0 = \int_0^\tau \left\{ E_a [Z_{s(a)} Z_b^{-1} X_b^C \exp\{\beta_S X_{s(a)}^C + \beta_B X_b^C\}] \right. \quad (8)$$

$$\left. - \frac{E_a [X_b^C \exp\{\beta_B^* X_b^C\}]}{E_a [\exp\{\beta_B^* X_b^C\}]} E_a [Z_{s(a)} Z_b^{-1} \exp\{\beta_S X_{s(a)}^C + \beta_B X_b^C\}] \right\} \alpha_0(a) da$$

for β_B , and

$$0 = \int_0^\tau \left\{ E_a [Z_{s(a)} Z_b^{-1} X_{s(a)}^C \exp\{\beta_S X_{s(a)}^C + \beta_B X_b^C\}] \right. \quad (9)$$

$$\left. - \frac{E_a [X_{s(a)}^C \exp\{\beta_S^* X_{s(a)}^C\}]}{E_a [\exp\{\beta_S^* X_{s(a)}^C\}]} E_a [Z_{s(a)} Z_b^{-1} \exp\{\beta_S X_{s(a)}^C + \beta_B X_b^C\}] \right\} \alpha_0(a) da$$

for β_S . Here $Z_{s(a)}$ and $X_{s(a)}^C$ are the frailty and covariate for the company owning a randomly chosen vessel at age a , and Z_b and X_b^C are the frailty and covariate for the buying company. The expectations are over the distributions of vessels and companies in place at age/time a .

First consider (8). Noting that the company distribution is time-constant in our scenarios and that the selling company is independent of the buying company, we can simplify to

$$0 = \left\{ E [Z_b^{-1} X_b^C \exp\{\beta_B X_b^C\}] - \frac{E [X_b^C \exp\{\beta_B^* X_b^C\}]}{E [\exp\{\beta_B^* X_b^C\}]} E [Z_b^{-1} \exp\{\beta_B X_b^C\}] \right\}$$

$$\times \int_0^\tau E_a [Z_{s(a)} \exp\{\beta_S X_{s(a)}^C\}] \alpha_0(a) da.$$

Further, frailty Z_b is independent of X_b^C by assumption, and so (8) is equivalent to

$$0 = \left\{ E [X_b^C \exp\{\beta_B X_b^C\}] - \frac{E [X_b^C \exp\{\beta_B^* X_b^C\}]}{E [\exp\{\beta_B^* X_b^C\}]} E [\exp\{\beta_B X_b^C\}] \right\}$$

$$\times E [Z_b^{-1}] \int_0^\tau E_a [Z_{s(a)} \exp\{\beta_S X_{s(a)}^C\}] \alpha_0(a) da.$$

This is solved at $\beta_B^* = \beta_B$, the true value. Hence we expect consistent estimation of β_B in Scenario 3. The same can be shown for Scenario 2. For completeness, we can also show consistent estimation of $\beta_B = 0$ in Scenario 1.

In (9), the terms involving the buying company are time-constant and independent of the selling company. Hence they can be factored out and the limiting score equation becomes

$$0 = \int_0^\tau \left\{ E_a [Z_{s(a)} X_{s(a)}^C \exp\{\beta_S X_{s(a)}^C\}] \right. \quad (10)$$

$$\left. - \frac{E_a [X_{s(a)}^C \exp\{\beta_S^* X_{s(a)}^C\}]}{E_a [\exp\{\beta_S^* X_{s(a)}^C\}]} E_a [Z_{s(a)} \exp\{\beta_S X_{s(a)}^C\}] \right\} \alpha_0(a) da.$$

Table 9 Estimates of β_S for Scenario 1 when frailty is ignored. The true value is $\beta_S=1$. Results are based on batches of 100 simulations, using frailty variance $\xi = 0.4$ and $\alpha_0(a) = 0.000001$.

N	K	$\tau=300$		$\tau=1000$		$\tau=2000$	
		Mean	SE	Mean	SE	Mean	SE
1000	1000	0.860	0.059	0.903	0.059	0.940	0.056
10000	1000	0.865	0.040	0.912	0.029	0.941	0.026
1000	10000	0.950	0.066	0.981	0.071	0.995	0.084
10000	10000	0.951	0.019	0.978	0.021	0.990	0.025

This time the expectations are not time-constant: we have a selection effect which does not apply to vessels directly, but rather to the companies owning vessels. Intuitively, as age a increases the distributions of $Z_{s(a)}$ and $\beta_S X_{s(a)}^C$ will become concentrated on values that are associated with low intensity of sales, until an equilibrium is reached.

Equation (10) is also the limiting score for Scenario 1, which we will turn to now because its equilibrium distribution is easily obtained. Let $p_{vc}(a)$ be the probability that vessel v is owned by company c at age a . In Scenario 1 there is no buyer effect and

$$p_{vc}(a + \delta a) = p_{vc}(a) \{1 - Z_c e^{\beta_S X_c^C} \alpha_0(a) \delta a\} + \frac{1}{N-1} \left(\sum_{c' \neq c} p_{vc'}(a) Z_{c'} e^{\beta_S X_{c'}^C} \alpha_0(a) \delta a \right) + o(\delta a).$$

Let π_{vc} be the equilibrium distribution. For global balance we must therefore have

$$\pi_{vc} \propto \frac{1}{Z_c e^{\beta_S X_c^C}} \quad \sum_c \pi_v = 1$$

for all v and c .

Interestingly, if the expectations in (10) are determined at the equilibrium distribution then the integrand is zero when $\beta_S^* = \beta_S$. Hence we expect the least-false values to converge to the true values if τ is allowed to increase indefinitely. How much bias there is for fixed τ will depend on how quickly the equilibrium distribution is reached, which depends upon the parameter values, including N and K . Table 9 gives further simulation results to help explore. The bias reduces with increasing τ and K but not N , though increasing N reduces the standard error. Further analytic investigation may be worthwhile.

References

- Aalen, O. (1988), ‘Heterogeneity in survival analysis’, *Statistics in Medicine* **7**, 1121–1137.
- Aalen, O. (1992), ‘Modelling heterogeneity in survival analysis by the compound poisson distribution’, *Annals of Applied Probability* **2**, 951–972.
- Aalen, O. (1994), ‘Effects of frailty in survival analysis’, *Statistical Methods in Medical Research* **3**, 227–243.
- Aalen, O. O. & Tretli, S. (1999), ‘Analyzing incidence of testis cancer by means of a frailty model’, *Cancer Causes and Control* **10**, 285–292.
- Aalen, O. O., Valberg, M., Grotol, T. & Tretli, S. (2014), ‘Understanding variation in disease risk: the elusive concept of frailty’, *International Journal of Epidemiology* **44**, 1408–1421.
- Abouarghoub, W., Biefang-Frisancho Mariscal, I. & Howells, P. (2012), ‘Dynamic earnings within tanker markets: An investigation of exogenous and endogenous structure breaks’, *American International Journal of Contemporary Research* **2**, 132–147.
- Alderton, T. & Winchester, N. (2002), ‘Flag states and safety: 1997-1999’, *Maritime Policy & Management* **29**, 151–162.
- Andersen, P. & Keiding, N. (2002), ‘Multi-state models for event history analysis’, *Statistical Methods in Medical Research* **11**, 91–115.
- Androukalis, E., Koukouvinos, C. & Vonta, F. (2012), ‘Estimation and variable selection via frailty models with penalized likelihood’, *Statistics in Medicine* **31**, 2223–2239.
- Archer, W., Ling, D. & Smith, B. (2010), ‘Ownership duration in the residential housing market: The influence of structure, tenure, household and neighborhood factors’, *The Journal of Real Estate Finance and Economics* **40**, 41–61.
- Barker, P. & Henderson, R. (2005), ‘Small-sample bias in the gamma frailty model for univariate survival’, *Lifetime Data Analysis* **11**, 265–284.
- Beard, R. (1959), ‘Note on some mathematical mortality models’, in G. Wolstenholme & M. O’Conner, eds, ‘Ciba Foundation Symposium: The Lifespan of Animals (Colloquia on Ageing)’, John Wiley and Sons, Chichester, pp. 302–311.
- Bijwaard, G. E. & Knapp, S. (2009), ‘Analysis of ship life cycles – The impact of economic cycles and ship inspections’, *Marine Policy* **33**, 350–369.
- Bøhren, Ø., Priestley, R. & Ødegaard, B. (2005), ‘The duration of equity ownership’, *EFA 2005 Moscow Meetings* .
- Fiocco, M., Putter, H. & Van Houwelingen, J. C. (2009), ‘A new serially correlated gamma-frailty process for longitudinal count data’, *Biostatistics* **10**, 245–57.
- Fox, J. (2005), ‘Vessel ownership and terrorism: Requiring disclosure of beneficial ownership is not the answer’, *Loyola Maritime Law Journal* **4**, 92–110.
- Fusillo, M. (2003), ‘Excess capacity and entry deterrence: The case of ocean liner shipping markets’, *Maritime Economics & Logistics* **5**, 100–115.

- Ha, I., Lee, Y. & Song, J. (2001), 'Hierarchical likelihood approach for frailty models', *Biometrika* **88**, 233–243.
- Henderson, R. & Oman, P. (1999), 'Effect of frailty on marginal regression estimates in survival analysis', *Journal of the Royal Statistical Society, Series B* **61**, 367–379.
- Henderson, R. & Shimakura, S. (2003), 'A serially correlated gamma frailty model for longitudinal count data', *Biometrika* **90**, 355–366.
- Hougaard, P. (1986), 'Survival models for heterogeneous populations derived from stable distributions', *Biometrika* **73**, 387–396.
- Hougaard, P. (2000), *Analysis of Multivariate Survival Data*, Springer, New York.
- Iwata, K. & Matsumoto, S. (2016), 'Use of hybrid vehicles in Japan: An analysis of used car market data', *Transportation Research Part D: Transport and Environment* **46**, 200–206.
- Kang, Y.-S. & Kim, B.-Y. (2012), 'Ownership structure and firm performance: Evidence from the chinese corporate reform', *China Economic Review* **23**, 471–481.
- Kronborg, D. & Thomsen, S. (2009), 'Foreign ownership and longterm survival', *Strategic Management Journal* **30**, 207–219.
- Mihaylova, R. (2017), *Periods of Ownership in Shipping: Patterns and Influences*, PhD Thesis, Newcastle University, UK.
- Oakes, D. (1982), 'A model for association in bivariate survival data', *Journal of the Royal Statistical Society, Series B* **44**, 414–422.
- Rau, P. & Pinler, S. (2016), 'Investment into container shipping capacity: A real options approach in oligopolistic competition', *Transportation Research Part E: Logistics and Transportation Review* **93**, 130 – 147.
- Sander, N., Abel, G., Bauer, R. & Schmidt, J. (2014), *Visualizing migration flow data with circular plots*, Vienna Institute of Demography Working Papers 2/2014.
- Stopford, M. (2009), *Maritime Economics*, 3 edn, Routledge, London.
- Stott, P. (2014), 'A retrospective review of the average period of ship ownership with implications for the potential payback period for retrofitted equipment', *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment* **228**, 249–261.
- Sys, C. (2009), 'Is the container liner shipping industry an oligopoly?', *Transport Policy* **16**, 259 – 270.
- Tenold, S. & Aarbu, K. (2011), 'Little man what now? Company deaths in Norwegian shipping 1960:1980', in L. Fischer & E. Lange, eds, 'New Directions in Norwegian Maritime History', International Maritime Economic History Association, St Johns, Newfoundland.
- Thanopoulou, H. & Strandenes, S. P. (2017), 'A theoretical framework for analysing long-term uncertainty in shipping', *Case Studies on Transport Policy* **5**, 325 – 331.
- UNCTAD (2017), *Review of Maritime Transport 2017*, United Nations Conference on Trade and Development.

-
- Vaupel, J., Manton, K. & Stallard, E. (1979), 'The impact of heterogeneity in individual frailty in the dynamics of mortality', *Demography* **16**, 439–454.
- Veenstra, A. & Bergantino, A. (2000), 'Changing ownership structures in the dutch fleet', *Maritime Policy and Management* **27**, 175–189.