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Condition monitoring for the quadruple water tank system using H-infinity Kalman Filtering

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Abstract. The problem of statistical fault diagnosis for the quadruple water-tanks system is examined. The solution of the fault diagnosis problem for the dynamic model of the four-water tanks system is a non-trivial case, due to non-linearities and the system's multivariable structure. In the article's approach, the system's dynamic model undergoes first approximate linearization around a temporary operating point which is recomputed at each sampling period. The linearization procedure relies on Taylor series expansion and on the computation of the Jacobian matrices of the state-space description of the system. The H-infinity Kalman Filter is used as a robust state estimator for the approximately linearized model of the quadruple water tanks system. By comparing the outputs of the H-infinity Kalman Filter against the outputs measured from the real water tanks system the residuals sequence is generated. It is concluded that the sum of the squares of the residuals' vectors, being weighted by the inverse of the associated covariance matrix, stands for a stochastic variable that follows the χ^2 distribution. As a consequence, a statistical method for condition monitoring of the quadruple water tanks system is drawn, by using the properties of the χ^2 distribution and the related confidence intervals. Actually, normal functioning can be ensured as long as the value of the aforementioned stochastic variable stays within the previously noted confidence intervals. On the other side, one can infer the malfunctioning of the quadruple water tanks system with a high level of certainty (e.g. of the order of 96% to 98%), when these confidence intervals are exceeded. The article's method allows also for fault isolation, that is for identifying the specific component of the quadruple water tanks system that has been subject to fault or cyber-attack.

1 Introduction

Assuring the secure functioning for coupled water-tanks systems and the detection of cyber-attacks in the associated control loops, remain challenging but nontrivial problems [1]-[3]. Due to the nonlinear and multivariable structure of the state-space model of the quadruple

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water-tanks system the solution of the related filtering problem is difficult [4]-[5]. This also inhibits the development of efficient model-based fault detection and isolation approaches [6]-[7]. Methods which allow for early fault detection and isolation in the model of the coupled water-tanks system can be also extended towards tools for condition monitoring of urban water distribution networks [8]-[9]. Moreover, methods which allow for cyber-attacks detection in the model of the coupled water-tanks system can be extended towards tools that identify malicious human intervention to the control loops of water distribution networks [10]-[11].

The present article proposes a scheme for model-based condition monitoring and cyber-attacks detection in the quadruple water-tanks system. This scheme relies on the H-infinity Kalman Filter and on statistical fault diagnosis criteria. Although the H-infinity Kalman Filter is acknowledged to be a robust state estimator, it remains a filter primarily designed for linear dynamical systems subject to model uncertainty [12]. The filter cannot be directly applied to state estimation problems of nonlinear dynamical systems, as for instance the quadruple water tanks model. To extend the filter's use to the case of nonlinear dynamics, the article proposes first an approximate linearization method of such systems' state-space description [13]-[14]. The linearization makes use of Taylor series expansion and of the computation of the quadruple water tanks system Jacobian matrices around a temporary operating point which is recomputed at each iteration of the estimation method [15]-[17]. At a second stage the measured outputs of the H-infinity Kalman Filter are subtracted from the real outputs of the quadruple water tanks system, where the latter are measured through dedicated sensors. In this manner the residuals' sequence is generated [18].

Through the processing of the residuals a statistical criterion is formulated allowing for detecting and isolating incipient faults and cyber-attacks in the four coupled water-tanks system. Actually, it is shown that the sum of the squares of the residuals' vector, being weighted by the inverse of the associated covariance matrix, stands for a stochastic variable that follows the χ^2 distribution [19]-[20]. Next, by exploiting the statistical properties of this distribution and the related confidence intervals one can infer the normal function or the malfunction of four coupled water tanks system. As long the value of the above noted stochastic variable remains within the confidence intervals of the χ^2 distribution, then it can be concluded that the functioning of the quadruple water tanks system is normal. On the other side, when the value of the aforementioned stochastic variable exceeds persistently these confidence intervals it can be concluded with a certainty level of the order of 96% or 98% that the quadruple water tanks system has been subject to a fault or has undergone a cyber-attack. Additionally, by applying the statistical test to sub-spaces of the state-space description of the coupled water tanks system, faults and cyber-attacks isolation can be achieved.

2 Dynamic model of the quadruple tank system

The quadruple water-tanks system is depicted in Fig. 1. The level of the water in the lower two tanks is controlled, using two pumps. The control inputs are u_1, u_2, that is the voltage signals applied to the motors of the pumps. The outputs are the sensor measurements (voltages) y_1 and y_2 about the level of the water in the two lower-level tanks. Mass-balance equations and Bernoulli equations yield the dynamic model of the system [3]. These are:

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \\
\frac{dh_4}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{(1-\gamma_1)k_1}{A_4}u_1
\end{aligned} \tag{1}$$

where $A_i, i = 1, \cdots, 4$ is the cross-section of tank i, $a_i,\ i = 1, \cdots, 4$ is the cross-section of the i-th outlet hole, and $h_i, i = 1, \cdots, 4$ is the water level at the i-th tank. The input voltage applied to pump i is $u_i,\ i = 1, 2$ and the associated water flow through the pump is $k_i u_i$. Parameters $\gamma_i, i = 1, 2, \in (0, 1)$ are related with the valves' opening and are determined from how the valves are set prior to the experiment. The following parameters of the system are also defined: The inflow to tank 1 is $\gamma_1 k_1 v_1$, the inflow to tank 4 is $(1 - \gamma_1)k_1 v_1$, the inflow to tank 2 is $\gamma_2 k_2 u_2$ and finally the inflow to tank 3 is $(1 - \gamma_2)k_2 v_2$. The acceleration of gravity is denoted as g. The measured output signals at the water tanks are $k_c h_i$ for $i = 1, \cdots, 4$.

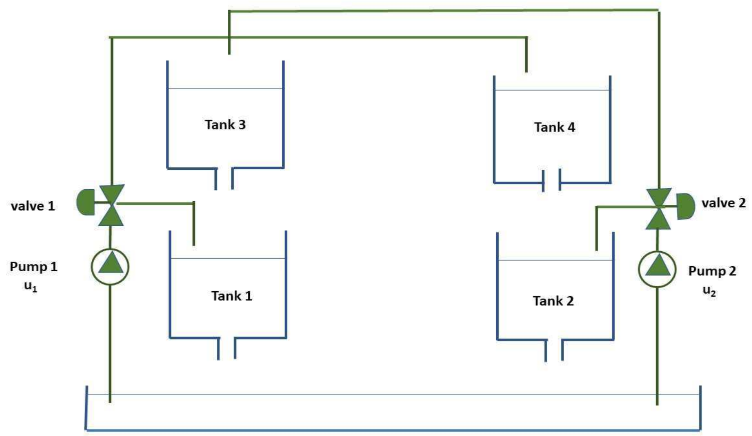


Figure 1. Diagram of the quadruple water-tanks system

By denoting the state vector of the system as $x = [x_1, x_2, x_3, x_4]^T = [h_1, h_2, h_3, h_4]^T$ the state-space equation of the quadruple tank system becomes [3]:

$$\begin{aligned}
\frac{dx_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3} + \frac{\gamma_1 k_1}{A_1}u_1 \\
\frac{dx_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4} + \frac{\gamma_2 k_2}{A_2}u_2 \\
\frac{dx_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gx_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \\
\frac{dx_4}{dt} &= -\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{(1-\gamma_1)k_1}{A_4}u_1
\end{aligned} \tag{2}$$

The state-space model of the quadruple tank system is also written in vector-fields form

$$\dot{x} = f(x) + G(x)u \tag{3}$$

where $x \in R^{4\times 1}$, $f(x) \in R^{4\times 1}$, $G(x) \in R^{4\times 2}$ and $u \in R^{2\times 1}$, or analytically

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix} = \begin{pmatrix}-\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3}\\ -\frac{a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4}\\ -\frac{a_3}{A_3}\sqrt{2gx_3}\\ -\frac{a_1}{A_1}\sqrt{2gx_1}\end{pmatrix} + \begin{pmatrix}\frac{\gamma_1 k_1}{A_1} & 0\\ 0 & \frac{\gamma_2 k_2}{A_2}\\ 0 & \frac{(1-\gamma_2)k_2}{A_3}\\ \frac{(1-\gamma_1)k_1}{A_4} & 0\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} \tag{4}$$

with measurement equation

$$\begin{pmatrix}y_1\\ y_2\\ y_3\\ y_4\end{pmatrix} = \begin{pmatrix}k_c & 0 & 0 & 0\\ 0 & k_c & 0 & 0\\ 0 & 0 & k_c & 0\\ 0 & 0 & 0 & k_c\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix} \tag{5}$$

3 Approximate linearization of the quadruple water tanks system

The state-space model of the four water-tank system undergoes approximate linearization, at each sampling instance, around the temporal operating point (x^*, u^*) where x^* is the present value of the system's state vector and u^* is the last value of the control inputs vector that was exerted on it. This results into the linearized state-space description

$$\dot{x} = Ax + Bu + \tilde{d} \tag{6}$$

where $\tilde{d}$ is the linearization error and

$$\begin{aligned}
A = \nabla_x[f(x) + G(x)u]\mid_{(x^*,u^*)} &\Rightarrow A = \nabla_x f(x)\mid_{(x^*,u^*)}\\
B = \nabla_u[f(x) + G(x)u]\mid_{(x^*,u^*)} &\Rightarrow B = G(x)\mid_{(x^*,u^*)}
\end{aligned} \tag{7}$$

Thus, after linearization around its current operating point, the model of the quadruple water tanks system is written in the form of Eq (6), that is $\dot{x} = Ax + Bu + \tilde{d}$. For the approximately linearized model of the quadruple water tanks system an H-infinity controller is developed. The controller has the form

$$u(t) = -Kx(t) \tag{8}$$

with $K = \frac{1}{r}B^T P$ where P is a positive semi-definite symmetric matrix which is obtained from the solution of the Riccati equation

$$A^T P + PA + Q - P(\tfrac{1}{r}BB^T - \tfrac{1}{2\rho^2}LL^T)P = 0 \tag{9}$$

where Q is also a positive definite symmetric matrix.

4 Statistical fault diagnosis with the H-infinity Kalman Filter

4.1 The H-infinity Kalman Filter

To estimate missing information about the state vector of the four water-tanks system it is proposed to use a filtering scheme and based on it to apply state estimation-based control [18]. The recursion of the H-infinity Kalman Filter can be formulated again in terms of a *measurement update* and a *time update* part [12]

Measurement update:

$$\begin{aligned} D(k) &= [I - \theta W(k)P^{-}(k) + C^{T}(k)R(k)^{-1}C(k)P^{-}(k)]^{-1} \\ K(k) &= P^{-}(k)D(k)C^{T}(k)R(k)^{-1} \\ \hat{x}(k) &= \hat{x}^{-}(k) + K(k)[y(k) - C\hat{x}^{-}(k)] \end{aligned} \tag{10}$$

Time update:

$$\begin{aligned} \hat{x}^{-}(k+1) &= A(k)x(k) + B(k)u(k) \\ P^{-}(k+1) &= A(k)P^{-}(k)D(k)A^{T}(k) + Q(k) \end{aligned} \tag{11}$$

where it is assumed that parameter θ is sufficiently small to maintain

$$P^{-}(k)^{-1} - \theta W(k) + C^{T}(k)R(k)^{-1}C(k) \tag{12}$$

positive definite. When $\theta = 0$ the H_{∞} Kalman Filter becomes equivalent to the standard Kalman Filter. It is noted that apart from the process noise covariance matrix $Q(k)$ and the measurement noise covariance matrix $R(k)$ the H_{∞} Kalman filter requires tuning of the weight matrices L and S, as well as of parameter θ.

4.2 Fault detection

The residuals' sequence, that is the differences between (i) the real outputs of the quadruple water tanks system (ii) the outputs estimated by the H-infinity Kalman Filter, is a discrete error process e_k with dimension $m \times 1$ (here $m = N$ is the dimension of the output measurements vector). Actually, it is a zero-mean Gaussian white-noise process with covariance given by E_k. A conclusion can be stated based on a measure of certainty that the four water tanks system has neither be subject to a fault nor to a cyberattack. To this end, the following *normalized error square* (NES) is defined [18]

$$\epsilon_k = e_k^T E_k^{-1} e_k \tag{13}$$

The normalized error square follows a χ^2 distribution. An appropriate test for the normalized error sum is to numerically show that the following condition is met within a level of confidence (according to the properties of the χ^2 distribution)

$$E\{\epsilon_k\} = m \tag{14}$$

This can be achieved using statistical hypothesis testing, which is associated with confidence intervals. A 95% confidence interval is frequently applied, which is specified using $100(1-a)$ with $a = 0.05$. Actually, a two-sided probability region is considered cutting-off two end tails of 2.5% each. For M runs the normalized error square that is obtained is given by

$$\bar{\epsilon}_k = \frac{1}{M}\sum_{i=1}^{M}\epsilon_k(i) = \frac{1}{M}\sum_{i=1}^{M}e_k^T(i)E_k^{-1}(i)e_k(i) \tag{15}$$

where ϵ_i stands for the i-th run at time t_k. Then $M\bar{\epsilon}_k$ will follow a χ^2 density with Mm degrees of freedom. This condition can be checked using a χ^2 test. The hypothesis holds, if the following condition is satisfied

$$\bar{\epsilon}_k \in [\zeta_1, \zeta_2] \tag{16}$$

where ζ_1 and ζ_2 are derived from the tail probabilities of the χ^2 density. For example, for $m = 20$ (dimension of the measurements vector) and $M = 100$ (total number of the output vector's samples) one has $\chi^2_{Mm}(0.025) = 1878$ and $\chi^2_{Mm}(0.975) = 2126$. Using that $M = 100$ one obtains $\zeta_1 = \chi^2_{Mm}(0.025)/M = 18.78$ and $\zeta_2 = \chi^2_{Mm}(0.975)/M = 21.26$.

4.3 Fault isolation

By applying the statistical test into the individual components of the four coupled water tanks system, it is also possible to find out the specific component that has been subject to a fault or cyberattack [18]. For a water tanks system that comprises n tanks, one has to carry out n χ^2 statistical change detection tests, where each test is applied to the subset that comprises components $i-1$, i and $i+1$, $i = 2, \cdots, n-1$. Actually, out of the n χ^2 statistical change detection tests, the one that exhibits the highest score are those that identify the component that has been subjected to failure (the failure can be either a fault related with a change in the parameters of the quadruple water tanks system, or can be the result of a cyberattack).

In the case of multiple components one can identify the subset of components that have been subject to parametric change by applying the χ^2 statistical change detection test according to a combinatorial sequence. This means that

$$\binom{n}{k} = \frac{n}{k!(n-k)!} \tag{17}$$

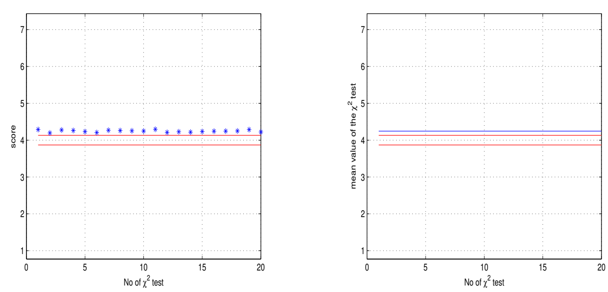
tests have to take place, for all clusters in the monitored power system, that finally comprise $n, n-1, n-2, \cdots, 2, 1$ components. Again the χ^2 tests that give the highest scores indicate the components which are most likely to have been subject to damage.

5 Simulation tests

To perform the faults detection and isolation tests with the previously analyzed χ^2 statistical criterion one can define the output measurements vector $y_m = [x_1, x_2, x_3, x_4]$ where x_i is the level of the water at the individual water tanks. Thus the dimension of the outputs measurements vector is $m = 4$. Considering that the number of output vector samples is $M = 2000$ and using a 98% confidence interval for the χ^2 distribution the fault thresholds can be as $L = 3.87$ and $U = 4.13$. In an equivalent manner when the statistical test is applied exclusively to the individual water tanks are defined as $L = 0.90$ and $U = 1.10$.

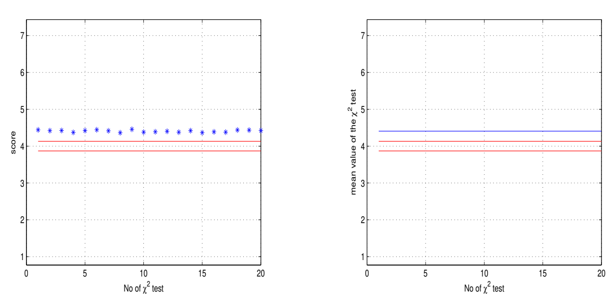
Indicative simulation results are presented about the following cases: (i) in Fig. 2 diagrams are given about the value of the χ^2 test in case that a parametric change has taken place in the input gain matrix elements water tank 1, (ii) in Fig. 3 diagrams are given about the value of the χ^2 test in case that a parametric change has taken place in the input gain matrix elements affecting water tank 2, (iii) in Fig. 4 diagrams are given about the value of the χ^2 test in case that a parametric change has taken place in the input gain matrix elements water tank 3, (iv) in Fig. 5 diagrams are given about the value of the fault isolation test performed at water tank 1 in case that an additive input disturbance affects input (pump) 2, and (v) in Fig. 6 diagrams are given about the value of the fault isolation test performed at water tank 2 in case that an additive input disturbance affects input (pump) 2

The simulation experiments have confirmed the ability of the proposed method to perform detection and isolation of incipient faults, that is of small parametric changes in the state-space model of the quadruple water tank system. These changes are related with failure of the electromechanical equipment of the four water-tanks system. Moreover, it has been confirmed that the method is capable for cyber-attacks detection, that is for identifying malicious human intrusion in the control and state estimation software of the coupled water-tanks system. Distortions of the outputs sensor measurements of the system, as well as of the inputs and outputs of the Kalman Filter, being related with unauthorized human intervention in the control loop of the water tanks system are classified as cyber-attacks.



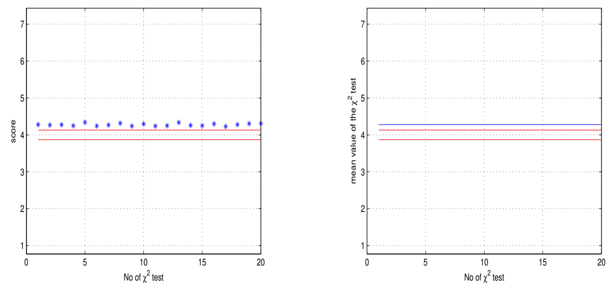
(a) **(b)**

Figure 2. Condition monitoring of the quadruple water tanks system with the use of the H-infinity Kalman Filter in case that a parametric change has taken place in the input gain matrix elements water tank 1: (a) successive χ^2 tests (b) mean value of the successive χ^2 tests



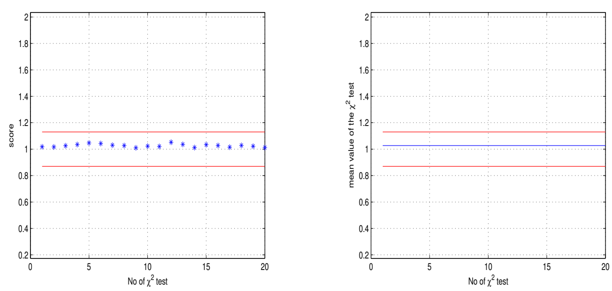
(a) **(b)**

Figure 3. Condition monitoring of the quadruple water tanks system with the use of the H-infinity Kalman Filter in case that a parametric change has taken place in the input gain matrix elements water tank 2: (a) successive χ^2 tests (b) mean value of the successive χ^2 tests



(a) **(b)**

Figure 4. Condition monitoring of the quadruple water tanks system with the use of the H-infinity Kalman Filter in case that a parametric change has taken place in the input gain matrix elements water tank 3: (a) successive χ^2 tests (b) mean value of the successive χ^2 tests

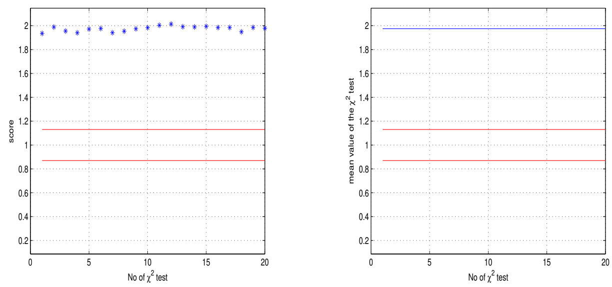


(a) **(b)**

Figure 5. Fault isolation tests for the quadruple water tanks system with the use of the H-infinity Kalman Filter in case that an additive disturbance has affected control input 2: (a) successive χ^2 tests performed at water tank 1 (b) mean value of the successive χ^2 tests performed at water tank 1

6 Conclusions

The H-infinity Kalman Filter is used as state estimator for the quadruple water tanks system. Since this filter is primarily designed for linear dynamical systems, to enable its use in the case of the four coupled water tanks system, the initial nonlinear state-space model undergoes approximate linearization around a temporary operating which is recomputed at each iteration of the estimation algorithm. The statistical processing of the H-infinity Kalman Filter's residuals allows for inferring with a high certainty level (of the order of 96% to 98%) if the four coupled water tanks system has undergone a fault or a cyber-attack. It is shown that the sum of the squares of the residuals' vector, weighted by the inverse of the associated covariance matrix, stands for a stochastic variable which follows the χ^2 distribution. The confidence intervals of the χ^2 distributed are considered to be fault thresholds. Thus, as long as the aforementioned stochastic variable falls within the confidence intervals it is concluded that the functioning of the quadruple water tanks system remains normal. On the other side,



(a) **(b)**

Figure 6. Fault isolation tests for the quadruple water tanks system with the use of the H-infinity Kalman Filter in case that an additive disturbance has affected control input 2: (a) successive χ^2 tests performed at water tank 2 (b) mean value of the successive χ^2 tests performed at water tank 2

when these fault thresholds are persistently exceeded it is concluded that the coupled water tanks system has either undergone a fault or a cyber-attack. Moreover, by applying the aforementioned statistical tests into subspaces of the state-space model of the system one can also perform fault and cyber-attacks isolation.

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