

# Investment decisions in a liberalised energy market with generation and hydrogen-based vector coupling storage in Integrated Energy System: A game-theoretic model-based approach

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## ABSTRACT

Meeting carbon reduction targets and enhancing energy supply flexibility necessitate the integration of natural gas and electricity networks, coupled with increased adoption of renewable energy. Bidirectional hydrogen-based Vector-Coupling Storage (VCS) offers a promising avenue for efficiently utilising surplus power from renewables, linking hydrogen as an energy carrier and storage with the Integrated Energy System (IES). This paper introduces a game-theoretic planning model for IES, encompassing natural gas, electricity, and independent VCS participants in a liberalised market. A game-theoretic model for capacity investment under an oligopolistic market structure in the liberalised energy market context is developed to capture the strategic behaviour of market participants. An annual investment model and an hourly operation simulation model are used to evaluate the value of hydrogen production, coupling components, and vector coupling storage in long-term investment decisions. The model, applied to the North of Tyne region in the UK, employs a scaled-down Future Energy Scenario dataset, reflecting a regional trajectory towards a net-zero emission target by 2050. Simulation results highlight market liberalisation's crucial role in attracting investments in renewable energy and hydrogen systems. Conversion efficiencies of electrolyzers and fuel cells emerge as key profitability determinants, emphasising the significance of achieving at least 50% round trip efficiency for profitable vector coupling storage. The findings quantify the advantages of large-scale VCS investments over Li-ion battery storage.

## 1. Introduction

### 1.1. Motivation

The motivation for integrating energy networks across vectors has strengthened in recent years, offering benefits such as enhanced flexibility and security of energy supply [1], reduced carbon emissions, optimised efficiency, and decreased operational costs [2]. Various coupling components, including utility-scale electrolysis, fuel cell stacks, gas turbines (GT), power-to-gas (P2G), and combined heat and power (CHP), facilitate this integration [3–6].

Achieving net-zero emissions by 2050 necessitates high penetrations of renewable energy sources into the energy system. Furthermore, in future integrated energy systems (IESs), hydrogen is anticipated to play a significant role, contributing to enhanced flexibility and security of the energy supply and replacing the other energy carriers which lead to producing high carbon emissions. The integration of

energy storage systems (ESS) into energy networks provides the system flexibility to compensate for carrier deficits and other supply constraints [4,5]. Fig. 1 illustrates the future IES where electrical energy can be generated through renewable energy sources, combined cycle gas turbines (CCGT), CHP, and other generation technologies. In this system, ESSs are divided into two different categories: single-vector storage (SVS) and vector-coupling storage (VCS). As can be seen, SVS refers to a storage device that is charged and discharged through one single vector. However, the VCS refers to a storage device that is charged by one of the vectors and discharges into the other vector. The VCS systems perform the intricate ESS coupling interface between electricity, gas, and hydrogen networks [5,7]. In Fig. 1, the battery energy storage system (BESS) is an SVS system, and the combination of grid-scale electrolyser, grid-scale fuel cells, and hydrogen storage represents a bidirectional hydrogen-based VCS between electricity and

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**Nomenclature****Indices**

$N_c$	Set of components in the gas mixture
$T, t$	Set and indices of time
$Z, z$	Set and indices of generation and storage technologies

**Parameters**

$c_{emis}$	Emission cost by the technology (£/tCO <sub>2</sub> )
$h^z$	Emission intensity by the technology $z$ (kg/kWh)
$p_t^g$	Price of gas at time $t$ (£/kWh)
$\eta_{P2G}$	Efficiency of P2G unit (%)
$\lambda_z$	Economic lifetime of the electricity technology $z$ (year)
$\rho_{air}$	Density of air at Standard Temperature and Pressure (STP) condition (kg/m <sup>3</sup> )
$\epsilon_z$	Specific capital expenditure for technology $z$ (£/MW)
$\varphi_z$	Fixed cost fraction of the technology $z$ (%)
$A_t, B_t$	Electricity demand scaling parameters
$AF$	Annuity factor
$Cap_{vcs}$	Capacity of the vector coupling storage (m <sup>3</sup> )
$D$	Pipe diameter (m)
$f$	Friction factor
$I_{max}^z$	Maximum investment limit of technology $z$ (£)
$K_t^z$	Initial capacity of the technology $z$ at time horizon $t$ (MW)
$L$	Length of the pipe (m)
$LoG_{vcs}^{max}$	Maximum level-of-gas defined by vector coupling storage(%)
$LoG_{vcs}^{min}$	Minimum level-of-gas defined by vector coupling storage (%)
$M_{air}$	Molar mass of air
$M_i$	Molar mass of the component $i$ in the gas mixture
$n$	Number of years in the planning horizon (years)
$p_t^e$	Price of electricity at time $t$ (£/kWh)
$p_t^h$	Price of hydrogen at time $t$ (£/kWh)
$p_n$	Standard Pressure (MPa)
$Q_t^e$	Electricity demand at time $t$ (kWh)
$Q_t^g$	Gas demand at time $t$ (kWh)
$Q_t^h$	Hydrogen demand at time $t$ (kWh)
$r$	Interest rate (%)
$R_{air}$	Air constant
$T$	Temperature of the gas (K)
$T_n$	Standard temperature at node $n$ (K)
$U_t, W_t$	Gas demand scaling parameters
$v_{max}^b$	Maximum voltage at bus $b$ , (p.u.)
$v_{min}^b$	Minimum voltage at bus $b$ , (p.u.)
$X_i$	Summation factor

**Variables**

$q_t^{col}$	Electricity generated by the coal power plant at time $t$ (kWh)
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$q_t^{nc}$	Electricity generated by the nuclear power plant at time $t$ (kWh)
$q_t^{wind}$	Electricity generated by the wind electric turbine at time $t$ (kWh)
$\alpha^e$	Shadow price of electricity network (£)
$\alpha^g$	Shadow price of gas network (£)
$\alpha^h$	Shadow price of hydrogen system (£)
$\delta^z$	Shadow price of investment constraint of technology $z$ (£)
$\eta_{fc}$	Conversion efficiency of fuel cells (%)
$\eta_h$	Storage efficiency of hydrogen storage (%)
$\mu_i$	Dynamic viscosity of the component $i$ in the gas mixture.
$\mu_{mix}$	Dynamic viscosity of the gas mixture (Pa s)
$\rho_{mix}$	Density of the gas mixture (kg/m <sup>3</sup> )
$\sigma^z$	Shadow price of capacity constraint of technology $z$ (%)
$c^z$	Cost function for the technology $z$ (£)
$C_t$	Cost of operations at time $t$ (£)
$CO_t$	Cost of emissions at time $t$ (£)
$FC$	Fixed cost of the operations (£)
$GCV_i$	Gross calorific value of component $i$ (kJ/m <sup>3</sup> )
$GCV_{mix}$	Gross calorific value of gas mixture (kJ/m <sup>3</sup> )
$I^z$	Capacity investment of the technology $z$ (MW)
$LoG_{vcs}$	Level of gas at the storage (%)
$NI$	Net investment outlays (£)
$NPV$	Net Present Value (£)
$p_i$	Pressure at node $i$ (Pa)
$q_t^z$	Net production by the technology $z$ at time $t$ (kWh)
$q_t^{gt}$	Electricity generated by the gas turbine at time $t$ (kWh)
$q_t^h$	Hydrogen supplied/stored by the hydrogen storage facility at time $t$ (kWh)
$q_t^{ez}$	Hydrogen produced by the electrolyser at time $t$ (kWh)
$q_t^{meth}$	Methane gas supplied by the supplier at time $t$ (kWh)
$q_t^{P2G}$	Gas produced by the power-to-gas facility at time $t$ (kWh)
$q_{(i,j),t}^{elec,active}$	Active power flow between node $i$ and $j$ at time $t$ , (MW)
$q_{(i,j),t}^{elec,reactive}$	Reactive power flow between node $i$ and $j$ at time $t$ , (MVAR)
$q_{i,j}^{elec,active,max}$	Maximum active power flow between node $i$ and $j$ , (MW)
$q_{i,j}^{elec,reactive,max}$	Maximum reactive power flow between node $i$ and $j$ , (MVAR)
$q_{i,j}^{gas,max}$	Maximum gas flow in the pipeline between node $i$ and $j$ , (m <sup>3</sup> )
$q_{j,in}$	Gas flow entering node $j$ from the branches connected to it (m <sup>3</sup> )
$q_{j,out}$	Gas flow leaving node $j$ from the branches connected to it (m <sup>3</sup> )
$q_k$	Gas flow in branch $k$
$QL_{j,cggt}$	Gas load at node $j$ (m <sup>3</sup> )
$q_{max}$	Maximum power generated from the CCGT generator, (MW)

$q_{max}^{ez}$	Maximum input power to the electrolyser, (MW)
$q_{max}^{p2g}$	Maximum input power at time to the P2G, (MW)
$q_{max}^z$	Maximum power generated from technology z at time t (MW)
$q_{min}^{ccgt}$	Minimum power generated from the CCGT generator, (MW)
$q_{min}^{ez}$	Minimum input power input to the electrolyser, (MW)
$q_{min}^{p2g}$	Minimum input power input to the P2G, (MW)
$q_{min}^z$	Minimum power generated from technology z at time t (MW)
$R_t$	Operational revenue at time t (£)
$Re$	Reynolds number
$S_{mix}$	Specific gravity of the gas mixture
$v^b$	Voltage at bus b at time t (p.u.)
$V_{ucs}^{available}$	Available volume of storage at the vector coupling storage (m <sup>3</sup> )
$Z_{air}$	Compressibility factor of air at STP condition
$Z_{mix}$	Compressibility factor of the gas mixture

1.2. Literature review and related works

This paper addresses IES's investment and expansion problem, focusing on how investors compete for maximum profit in a liberalised market. In this competitive environment, while the energy price is fixed and agreed upon by all parties involved, investors seek to maximise their production or storage capacities to increase profits. Consequently, an optimisation method is required to effectively represent the competition and conflicts of interest between these investors. There are five main categories of optimisation methods applicable in such contexts: Pareto optimisation, bi-level optimisation, multi-period frameworks, different stages optimisation, and game theory.

The Pareto optimisation method is a well-known approach for solving multi-objective function problems [9]. It has been applied in numerous areas, as demonstrated in [10], where it was used to optimise diesel generation performance and minimise NOx emissions. Similarly, in [11], the Pareto method was employed for optimising electrical hubs through a multi-criteria assessment. Further applications include multi-objective power flow optimisation considering fuel cost and emissions [12], hybrid energy storage system optimisation [13], and energy supply and demand scheduling [14]. In [15], Pareto optimisation was utilised for waste heat recovery, considering factors such as efficiency, economics, and environmental impact. Despite variations in objective functions and solution methods, the commonality in research using Pareto optimisation lies in the trade-off between the objectives. However, Pareto optimisation falls short in scenarios involving competition and conflicting interests between investors, and consequently, investor behaviour cannot be controlled. Hence, Pareto optimisation is suitable for regulated markets but inadequate for deregulated ones.

Multi-level optimisation, particularly bi-level optimisation, addresses problems with hierarchical and diverse objectives. In bi-level optimisation, decisions are made at two levels: the lower level is embedded within the upper level [16]. For instance, in [17], bi-level optimisation was used for optimal voltage and reactive power scheduling. A more complex model is presented in [18], where a stochastic expansion planning framework is proposed for regional IES, aiming to minimise investment costs and pipeline risks. Similarly, a risk-constrained bidding and offering strategy for integrated electricity-hydrogen energy systems is developed in [19], focusing on balancing economic viability and operational flexibility in day-ahead energy and reserve markets. This model can be considered a tri-level optimisation method, addressing decisions across three layers: day-ahead market decisions, reserve deployment, and wind power production. Another bi-level approach is found in [20], which proposes a robust generation expansion planning framework for regional IES with carbon growth constraints and multiple uncertainties.

Optimisation methods based on multi-period frameworks are crucial for long-term planning, ensuring sustainable and optimal solutions over multiple periods. For example, in [21], a multi-period framework was proposed for power plant expansion, considering uncertainties in the electricity market and employing a bi-level optimisation approach.

Two-stage optimisation methods, often used in stochastic programming, involve sequential decision-making. The first stage makes initial decisions without knowing future uncertainties, while the second stage adjusts these decisions once uncertainties are realised. In [22], a two-stage stochastic planning framework is presented for integrating solar photovoltaics, BESS, and gas microturbines in coupled microenergy grids.

Game theory has also been applied to generation expansion problems, particularly in considering the influence of transmission lines and investment cost competition [23]. The work in [23] uses a static model without accounting for uncertainty, while [24] adopts a probabilistic dynamic programming approach to handle investment cost uncertainty due to fluctuating electricity demand. In [25], a Cournot game model is used for power plant investment, representing competition among independent producers in an auction. This model combines bi-level and

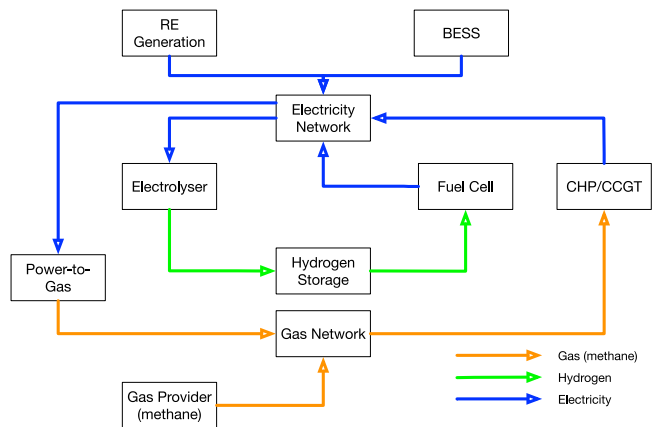


Fig. 1. Interaction within the future energy system.

hydrogen vectors. The energy flow from electricity to gas networks through the P2G represents a unidirectional VCS.

In the past, many countries had energy sectors owned and operated by a single organisation, with one entity making decisions across various aspects of the industry. However, many of these sectors have transitioned to a liberalised model aimed at fostering competition and reducing prices [8]. The key features of this liberalised structure include a competitive wholesale energy market, the elimination of supply monopolies, the separation of network operation from energy generation, importing, production, and storage, the distinction between energy generation and supply, and the shift from public to private infrastructure. This shift has led to the emergence of multiple interacting investors, such as generation and storage companies and retail suppliers. Understanding the dynamics among these players is essential to predict how the energy system may evolve. This underscores the importance of using game theory models to study the strategic interactions between these various investors.

multi-period frameworks, incorporating reliability and economic functions into two layers, with reliability at the upper level and economics at the lower level. In the lower level, each player decides on the quantity to produce to minimise costs, considering the reliability constraints from the higher level. Similarly, in [26], a coordinated bidding strategy for wind farms and P2G facilities is proposed using a cooperative game approach to maximise joint profits. The wind farms participate in day-ahead, real-time, and reserve markets, while P2G facilities participate in day-ahead and reserve markets for arbitrage opportunities. The nucleolus and Shapley-value methods ensure fair profit allocation, increasing profitability for both parties while enhancing operational economics. Another game theory-based model [27] involves multi-agent capacity optimisation in IES with compressed air energy storage (CAES), using Nash equilibrium for non-cooperative games and Shapley value for cooperative games. This study highlights the economic and environmental benefits of CAES in IES. Additionally, optimal capacities for generation and storage, along with market-clearing prices, were determined for a group of microgrids incorporating solar PV, wind, fuel cells, and electric vehicles in [28]. Each microgrid independently decides the amount of energy to buy or sell and at what price. The decisions are made independently by each microgrid based on their marginal costs, and the market clearing price is iteratively calculated by the Distribution Network Operator until equilibrium is achieved. In [29], a game-theory-based approach is used to analyse the impact of policy instruments on energy system actors in a Nordic municipality, focusing on emission reductions and cost allocation. Various policy instruments like carbon tax, electricity certificates, and investment subsidies are tested to see their impact on CO<sub>2</sub> emissions and the actors' strategies. A cooperative game model is explored, with the Nash bargaining solution ensuring fair benefit allocation.

The game-theoretic model developed in [30] represents a Cournot oligopoly competition model investigating the impact of hydrogen storage on production and investment decisions in the context of the German electricity market. Results indicate that integrating energy storage positively affects the energy system, particularly in markets with high renewable energy penetration and inelastic demand. Although the model simulates annual investment decisions, it does not account for system operability through hourly operational simulations. A local energy market framework for electricity and hydrogen is established in [31], with a decentralised clearance mechanism that preserves privacy. In [32], a short-term electricity and hydrogen market partial equilibrium model is developed, using a welfare-economic framework to assess market outcomes. It concludes that P2G investments are profitable from a social welfare perspective, although the economic performance of P2G depends on factors such as carbon price and installation costs. However, this model does not consider the short-term interaction between the electricity and gas networks when simulating gas flow during investment decisions, focusing primarily on the relevance of P2G technology. A similar study in [31] formulates a unit commitment problem to analyse green hydrogen injections and gas composition tracking in an integrated electricity and gas system. While hydrogen composition in the network is tracked, the study does not assess how electrolyser and fuel cell integration affects the energy system. Additionally, [33] employs the Aumann-Shapley approach to fairly allocate the benefits of network expansion projects. In [34], a cooperative planning model for IES, including P2G plants, natural gas, and electricity systems, is proposed. This model demonstrates the efficacy of cooperative IES and P2G planning in deregulated competitive environments, outperforming centralised and sequential planning models.

In addition to these optimisation methods, previous research has often addressed the planning of distributed generators (DGs), SVS, or P2G plants in isolation, without accounting for their interdependencies within IES [22,35]. For instance, in [35], the optimal sizing of a grid balancing system combining gas turbines and P2G plants is determined using a scenario-based statistical approach, with key

performance indicators such as operating hours, carbon emissions, and wind power curtailment. Similarly, [22] compares the use of electrolysers for hydrogen production from surplus renewable energy with traditional network expansion, concluding that while electrolysers have higher costs, profitability is possible depending on their full load hours. However, the findings from such scenario-based approaches may not be generalised due to their reliance on specific scenarios.

Despite this extensive body of work, the following gaps and contributions have been identified:

- Short-term and long-term integration in IES investment planning: Many of the existing works focus either on long-term investment or short-term operational planning, but not both in an integrated manner. For example, models like those in [30,32] simulate long-term investment decisions, but they do not consider system operability at an hourly level, which is essential for understanding the dynamics and integration of renewables, hydrogen, and storage. The proposed paper fills this gap by integrating short-term operations (power, gas, and hydrogen flow) with long-term investment planning, providing a more comprehensive view of the energy system. The integration is formulated under a bi-level and multi-period optimisation formulation; the high level considers the long-term investment decisions, and the low level considers the impact of long-term investment on the short-term operation of IES, and this will be over multiple periods until 2025.
- Incorporation of Hydrogen and VCS: Most previous works that incorporate storage solutions tend to focus on specific storage types, such as BESS or CAES [22,27,35], or on hydrogen production alone without fully exploring the role of integrated storage solutions like VCS in deregulated markets. The paper addresses this by modelling both VCS and renewable hydrogen production. The VCS, as a storage facility, can provide flexibility in the operation of the IES. This paper also examines the profitability requirements of VCS, given the concerns related to the low efficiencies of electrolysers and fuel cells. These requirements can be defined from the short-term operational models. Additionally, previous literature, like in [22], primarily focuses on small-scale or specific technologies such as BESS or hydrogen production from surplus renewable energy. There is limited exploration of large-scale VCS in a liberalised energy market, particularly when compared to conventional SVS technologies. The proposed paper quantifies the benefits of large-scale VCS investments and compares them to SVS like BESS, providing new insights into large-scale energy storage solutions. Incorporating VCS and its bidirectional functionality into the operational model is a novel consideration in this context. It bridges the gap between operational feasibility and strategic investment in VCS technologies to achieve decarbonisation goals.
- Game-theoretic modelling in a decentralised market: Several studies utilise game theory to analyse energy markets but often in different contexts or with a focus on cooperative frameworks (e.g., [26] for cooperative bidding in wind and P2G markets). However, many do not account for a Cournot oligopoly competition model for integrated electricity, gas, and hydrogen resources, particularly in deregulated energy markets. The work uses a Cournot competition model to simulate real-world market conditions, where each player (generator or storage investor) independently maximises their profit without coordination. This better reflects the decentralised nature of modern energy markets. In the Cournot competition model, each participant independently determines output or capacity levels, guided by a profit-maximising strategy.
- Interaction of policy instruments and market dynamics: Although some works, such as [29], explore the impact of policy instruments (e.g., carbon taxes, investment subsidies) on energy actors, they do so in the context of cooperative strategies or regulated market scenarios. The proposed paper extends this analysis by

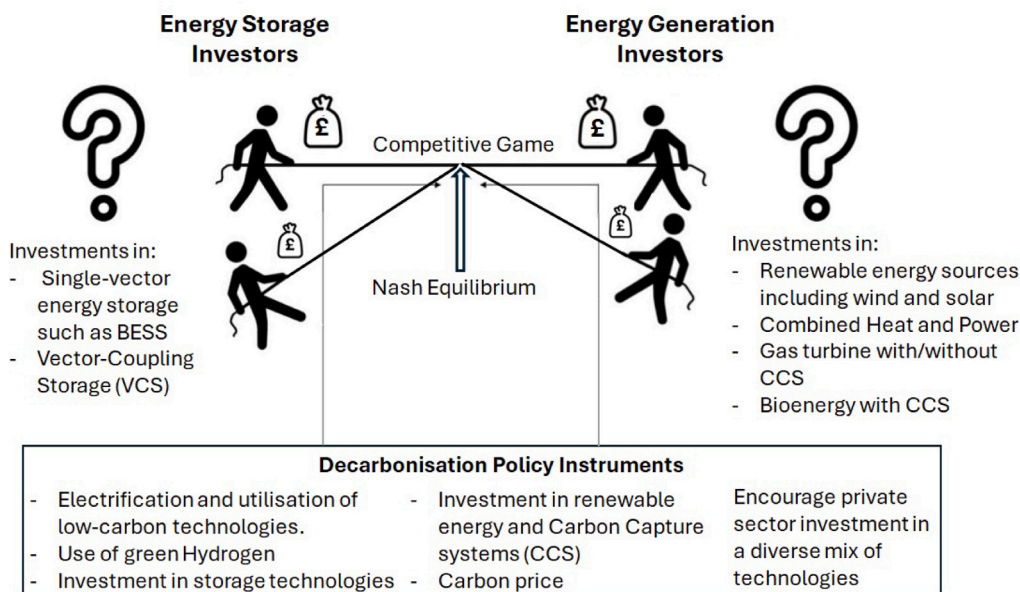


Fig. 2. Graphical representation of the game theory approach for solving the energy system optimisation problem in a liberalised market, highlighting the players, their decision options, and the policy instruments for energy system decarbonisation.

evaluating policy-driven competition in deregulated markets and its effect on long-term investments in hydrogen and renewable generation, offering new insights into how policy instruments influence the behaviour of market players. The paper considers the impact of policy on the energy and technology mix, the carbon price, technology costs, and the energy demand.

By addressing these gaps, the proposed paper contributes to a deeper understanding of IES planning, focusing on the intersection of renewable generation, hydrogen production, large-scale storage, and competitive market dynamics.

### 1.3. Paper contribution and organisation

This paper proposes a game-theoretic model for long-term investment and short-term operation planning in IES, focusing on achieving net-zero emissions by 2050. The model, based on Cournot’s Oligopoly competition, integrates electricity, gas, and hydrogen resources, enabling market participants to maximise their benefits. As shown in Fig. 2, two primary groups of investors are considered: generation companies and storage facility companies. While other entities, such as energy suppliers and aggregators, could also be modelled, this paper focuses on generation companies investing in technologies like renewable energy (solar and wind), gas generation with or without carbon capture systems (CCS), bioenergy with CCS, and CHP. On the storage side, investments in systems such as BESS and unidirectional or bidirectional VCS are analysed.

The model evaluates both the conditions under which gas-fired generation with CCS can compete with renewable energy and the potential of VCS to outperform BESS, considering concerns about the low efficiency of electrolysis and fuel cell technologies. The strategic behaviour of market participants is captured through game theory, with players making decisions on capacity expansion and short-term operational cycles to optimise their profits. The market is liberalised to encourage private sector investments in a diverse mix of technologies, supported by policy measures, regulatory frameworks, and carbon pricing mechanisms.

To determine long-term investments, the model simulates annual capacity investments in generation and storage technologies, alongside hourly optimal operational planning. This allows for a comprehensive assessment of the value of hydrogen production, energy coupling

components, and VCS within the broader context of energy network operability. By considering both the short-term operational needs and long-term capacity investments, the model offers a novel approach to planning in IES, with a particular focus on renewable energy, hydrogen production, and storage technologies as key contributors towards net-zero emissions.

The remainder of the paper is organised as follows. Section 2 describes the formulation of the game-theoretic model and the development of the mathematical model for the integrated energy network components. The case study and description of the developed scenarios are reported in Section 3. Results and discussions on the case studies are elaborated in Section 4. Finally, in Section 5 the conclusions of the paper are presented.

## 2. Game-theoretical model for competitive planning of the integrated energy system

### 2.1. Model structure

The model structure outlined in Fig. 3 is an illustration of the interaction and information flow between the annual investment and short-term operational model. Within each model, stakeholders involved in this interaction are identified and included. The identified stakeholders in the developed liberalised energy market model are the investors and operators of electricity generation and ESS. The ESS stakeholders include the SVS and bidirectional hydrogen-based VCS. Stakeholders involved in this model are rational entities to maximise the returns of their investments within the economic lifetime of their investments.

The model considers IES planning at two distinct time scales: short and long-term. Fig. 4 provides a visual representation of the interaction between the long-term investment model and the short-term operational model, illustrating how information flows and decisions are made across these time scales. The annual investment model which represents long-term planning takes decisions related to capacity planning and policies. The model is developed to maximise the investment by identifying investments that yield maximum net present value. The output of this model, such as annual energy demand, planned investments, and estimated carbon emissions, serves as input for the short-term operational model for each specific year.

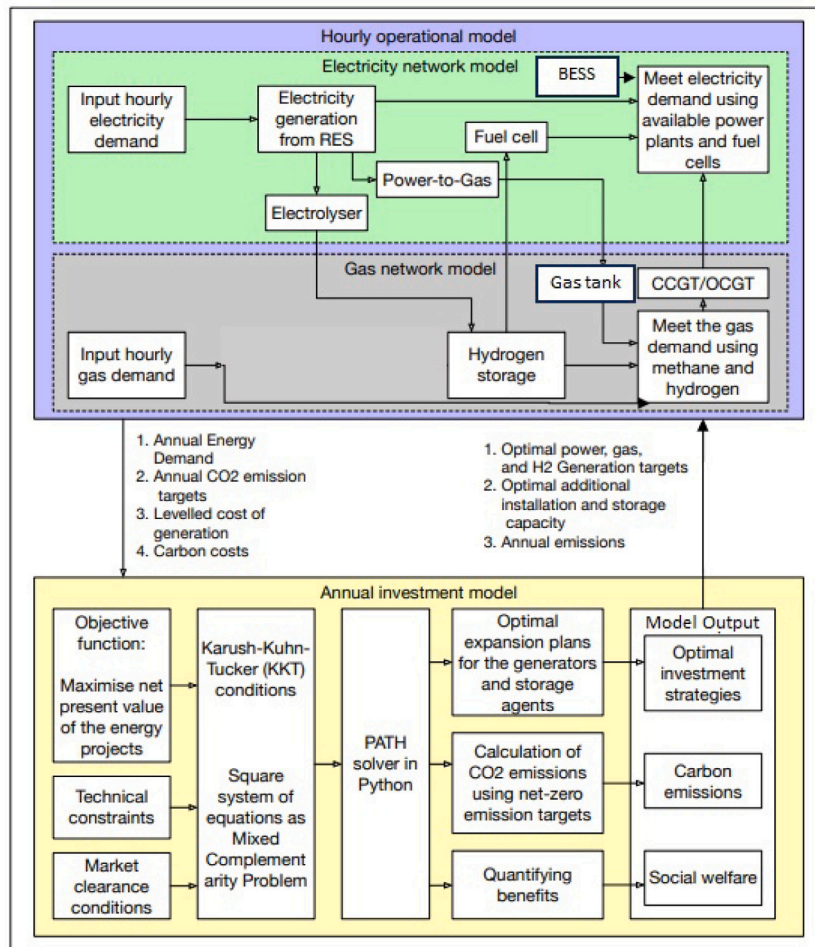


Fig. 3. Model structure.

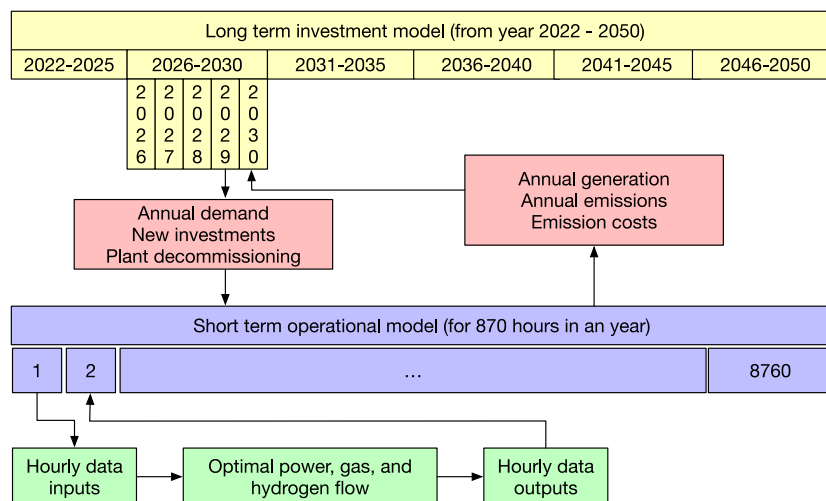


Fig. 4. Interaction of long-term and short-term models at different time-scales.

The long-term planning model incorporates policy measures by accounting for the types of generators, their contributions to the energy mix, and the maximum capacity of each generation technology required to achieve the desired energy mix. It also considers projections of energy market trends and carbon prices, alongside decarbonisation policies. This framework ensures that long-term planning decisions remain

grounded in operational feasibility and evolving market conditions, which are modelled in the short-term operation.

The hourly energy system operational model focuses on short-term planning, addressing both electricity and gas networks' operational aspects. Coupling devices, such as fuel cells, power-to-gas systems, electrolyzers, and VCS, are modelled to capture the intricate interactions

between these networks. This model provides detailed insights into annual operational performance by considering factors such as hourly energy demand variations, renewable energy availability, and fossil fuel constraints. The outputs from the short-term model, such as annual generation data, emissions levels, and emission costs, are fed back into the long-term investment model. This feedback loop allows the long-term model to refine its planning decisions for subsequent years, ensuring alignment with operational realities.

Fig. 4 demonstrates this iterative process, highlighting how the models' outputs influence each other. For example, during a planning year like 2029, the long-term model identifies new investments and decommissioned assets, which are input into the short-term operational model to determine optimal hourly power, gas, and hydrogen flows over 8760 h. The aggregated annual data, including emissions and generation costs, are then provided to the long-term model for planning the next year, such as 2030. By integrating these two perspectives, the framework achieves a balanced approach to IES planning, optimising both short-term operations and long-term investments to meet decarbonisation goals.

In the following sub-sections, the annual investment simulation model and hourly operational model illustrated in Fig. 3 are elaborated with mathematical formulations. Section 2.2 elaborates the formulation of the annual investment model optimised to maximise the net present value of the investors in the liberalised energy market. Relevant mathematical formulations which are necessary to model the interaction between electricity and gas networks are presented in Section 2.3.

## 2.2. Optimal investment model

The investors in the liberalised energy market maximise their Net Present Value (NPV), including the discounted annual operational cash flows in the planned horizon.

### 2.2.1. Objective function

The objective function is the representation of a typical investor's characteristics who tries to maximise the NPV of owned assets. The discounted cash flows serve as the remuneration for the net initial investment outlays (NI). The objective function is presented in Eq. (1).

$$\max NPV = AF \left[ \sum_{t \in T} (R_t - C_t - CO_t) - FC \right] - NI \quad (1)$$

A constant annual cash flow is assumed in this context. Thus, the operational profit is multiplied by the annuity factor (AF). AF is defined in Eq. (2).

$$AF = \frac{(1+r)^n - 1}{r(1+r)^n} \quad (2)$$

The annual revenues of the firm ( $R_t$ ) during period  $t$  are derived to represent the total electricity sales of the firm. Similarly, the cost of operation of the firm ( $C_t$ ) denotes the overall operational cost at time  $t$  for all technologies operated by the investor. Eqs. (3) and (4) represent the revenue and cost of operation, respectively.

$$R_t = p_t^e (q_t^{gt} + q_t^{nc} + q_t^{col} + q_t^{hydo} + q_t^{re}) + p_t^g (q_t^{P2G} + q_t^{meth}) + p_t^h (q_t^{ez} + q_t^h) \quad (3)$$

$$C_t = \sum_{z \in Z} c^z (q_t^z) \cdot q_t^z \quad (4)$$

In Eq. (4), the cost function  $c^z$  is a quadratic function with coefficients  $a_z$ ,  $b_z$ , and  $c_z$ . The cost function is represented in (5).

$$c^z (q_t^z) = a_z (q_t^z)^2 + b_z (q_t^z) + c_z \quad (5)$$

Eq. (6) represents the cost of emission.

$$CO_t = \sum_{z \in Z} q_t^z \cdot h^z \cdot c_{emis} \quad (6)$$

In addition, the annual fixed cost ( $FC$ ) as represented in (7) is calculated by multiplying the total capacity by the end of the year ( $K^z + I^z$ ) with specific capital expenditure ( $\epsilon_z$ ) and fixed cost fraction ( $\varphi_z$ ) of the given technology.

$$FC = \sum_{z \in Z} [\varphi_z \cdot \epsilon_z (K^z + I^z)] \quad (7)$$

$$NI = \sum_{z \in Z} \left[ \epsilon_z \cdot I^z \cdot \left( 1 - \frac{\lambda_z - n}{\lambda_z (1+r)^n} \right) \right] \quad (8)$$

### 2.2.2. Constraints and market clearing conditions

The technical constraints are formulated in this section. Eq. (9) represents the investment constraint with initial capacity and additional investment. The constraint applies to all technologies ( $\forall z \in Z$ ) where the value of the additional investment variable ( $I^z$ ) is bound between the limits. Eq. (10) restricts the investments in technology ( $I^z$ ) to its specified maximum investment limit ( $I_{max}^z$ ). Further, Eqs. (11)–(12) represent the non-negativity constraints for all the decision variables.

$$q_t^z - K_t^z - I^z \leq 0 \quad \forall z \in Z \quad (9)$$

$$I^z - I_{max}^z \leq 0 \quad \forall z \in Z \quad (10)$$

$$q_t^z \geq 0 \quad \forall z \in Z \quad (11)$$

$$I^z \geq 0 \quad \forall z \in Z \quad (12)$$

The market clearing condition defined in this study is a linear inverse demand function as shown in Eqs. (13) and (14). The coefficients  $A_t$ ,  $B_t$  denote the demand scaling parameters for the electricity demand. Eqs. (16)–(18) denotes the balance of demand and supply of power, gas, and hydrogen in the system. The balancing equations balance the generation/production from the sources with the energy demand from the consumers and other generation/production sources. In this study, it is assumed that there are no losses in the electricity, gas, and hydrogen networks in the investment model, as these losses will be accounted for in the short-term model.

$$p_t^e = A_t - B_t * Q_t^e \quad \forall t \in T \quad (13)$$

$$p_t^g = U_t - W_t * Q_t^g \quad \forall t \in T \quad (14)$$

$$p_t^h = U_t - W_t * Q_t^h \quad \forall t \in T \quad (15)$$

$$q_t^{wind} + q_t^{col} + q_t^{cgt} + q_t^{fc} - q_t^{p2g} - q_t^{ez} - Q_t^e = 0 \quad \forall t \in T \quad (16)$$

$$q_t^{p2g} + q_t^{meth} - q_t^{cgt} - Q_t^g = 0 \quad \forall t \in T \quad (17)$$

$$q_t^{ez} + q_t^h - q_t^{fc} - Q_t^h = 0 \quad \forall t \in T \quad (18)$$

## 2.3. Short-term operational model considering the complex interaction between the electricity and gas networks

A gas and electricity load flow engine is developed to simulate the short-term operation of the energy system. The steady-state representation of the AC power flow in the electricity network and gas flow in the gas network are formulated and solved using the Newton method.

The objective function of the short-term operational model is presented in Eq. (19). The equation is similar to the cost expression we use in our investment model (Eq. (4)). The costs of energy technologies ( $c^z(q_t^z)$ ) are quadratic functions defined similar to Eq. (5).

$$\min C_t = \sum_{z \in Z} c^z (q_t^z) \cdot q_t^z \quad \forall z \in Z \quad (19)$$

The following subsections present the mathematical model of the electricity network, gas network and vector coupling components.

### 2.3.1. Electricity network model

Eqs. (20) and (21) represent the active and reactive power flow at each bus. The study uses the Pandapower Python package to model the electricity network.

$$P_{G_i} - P_{L_i} - \sum_i |V_i| |V_j| (G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)) = 0 \quad (20)$$

$$Q_{G_i} - Q_{L_i} - \sum_i |V_i| |V_j| (G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)) = 0 \quad (21)$$

Constraints that are defined to represent the limits are presented in Eqs. (22)–(24). Eq. (22) represents the minimum and maximum technical limits of technology  $z$ . It should be noted that the value of  $q_{max}^z$  is the optimal generation capacity of the technology  $z$  obtained from the investment model. The short-term operational model defines a value for  $q_t^z$  after running the optimal power, gas, and hydrogen flow (short-term operational model).

$$q_{min}^z \leq q_t^z \leq q_{max}^z \quad \forall z \in Z \quad (22)$$

For example, Eqs. (23)–(25) define the limits of electrolyser, CCGT, and P2G technologies respectively.

$$q_{min}^{ez} \leq q_t^{ez} \leq q_{max}^{ez} \quad (23)$$

$$q_{min}^{ccgt} \leq q_t^{ccgt} \leq q_{max}^{ccgt} \quad (24)$$

$$q_{min}^{p2g} \leq q_t^{p2g} \leq q_{max}^{p2g} \quad (25)$$

Eqs. (26) and (27) limit the maximum active and reactive power flows between bus  $i$  and  $j$ .

$$0 \leq q_{(i,j),t}^{elec,active} \leq q_{i,j}^{elec,active,max} \quad (26)$$

$$0 \leq q_{(i,j),t}^{elec,reactive} \leq q_{i,j}^{elec,reactive,max} \quad (27)$$

Voltage limits at the electricity network bus  $b$  are defined as follows:

$$v_{min}^b \leq v_t^b \leq v_{max}^b \quad (28)$$

### 2.3.2. Gas network

The equation for the balance of gas flow at the nodes of the gas network is as follows:

$$\sum_j q_{j,in} - \sum_j q_{j,out} - q_{L,j} = 0 \quad \forall j \quad (29)$$

The general flow equation used for calculating the gas flow in the branches of the gas network is presented in Eq. (30). The value of the standard temperature ( $T_n$ ) is assumed to be 288.15 K and the standard pressure is assumed to be  $p_n \simeq 0.1$  MPa. Other assumptions and simplifications are summarised in [36].

$$q_k = \pi \sqrt{\frac{R_{air}}{8}} \times \frac{T_n}{p_n} \times \sqrt{\frac{(p_i^2 - p_j^2) \times D^5}{f \cdot S_{mix} \cdot L \cdot T \cdot Z_{mix}}} \quad (30)$$

Friction factor  $f$  is calculated based on the value of the Reynolds number ( $Re$ ) of gas flow:

$$Re = \frac{D \cdot v \cdot \rho_{mix}}{\mu_{mix}} \quad (31)$$

where the value of the velocity of the gas flow ( $v$ ) is calculated using the pipe cross-sectional area ( $A$ ) as follows:

$$v = \frac{q}{A} = \frac{q}{(\pi/4) \cdot D^2} \quad (32)$$

and the value of density of the gas mixture ( $\rho_{mix}$ ) is calculated using:

$$\rho_{mix} = S_{mix} \times \rho_{air} \quad (33)$$

where the specific gravity of the gas mixture ( $S_{mix}$ ) is as follows [37]:

$$S_{mix} = \frac{Z_{air} \cdot \sum_{i=1}^{N_c} (X_i \cdot M_i)}{M_{air} \cdot Z_{mix}} \quad (34)$$

and the value of compressibility factor of the gas mixture ( $Z_{mix}$ ) is obtained by [37]:

$$Z_{mix} = 1 - \left( \sum_{i=1}^{N_c} (X_i \cdot c_i) \right)^2 \quad (35)$$

Also, the value of dynamic viscosity of the gas mixture ( $\mu_{mix}$ ) is computed as given in Eq. (36) [38]. The value of the summation factor ( $X_i$ ) is obtained by referring to page 11 of the standard ISO 6976:1995 [37].

$$\mu_{mix} = \frac{\sum_{i=1}^{N_c} (X_i \cdot \mu_i \cdot M_i^{0.5})}{\sum_{i=1}^{N_c} (X_i \cdot M_i^{0.5})} \quad (36)$$

Substituting Eq. (35) in (34), then (34) in (33) and replacing the Eqs. (32), (33) and (36) in (31), the equation for calculation of Reynolds number ( $Re$ ), after some simplifications, will be:

$$Re = \frac{\rho_{air}}{(\pi/4)} \cdot \frac{S_{mix}}{D} \cdot \frac{\sum_{i=1}^{N_c} (X_i \cdot M_i^{0.5})}{\sum_{i=1}^{N_c} (X_i \cdot \mu_i \cdot M_i^{0.5})} \times q \quad (37)$$

Once the value of the Reynolds number is calculated, the value of the friction factor ( $f$ ) can be calculated based on the regime of the flow as follows: - Laminar flow ( $Re < 2300$ ):

$$f = \frac{64}{Re} \quad (38)$$

- Turbulent flow ( $Re > 4000$ ):

In this case, which frequently happens in gas networks, the friction factor is calculated using Colebrook's equation:

$$\frac{1}{\sqrt{f}} = -2 \times \log_{10} \left( \frac{\epsilon/D}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}} \right) \quad (39)$$

in which  $\epsilon$  is the roughness of the internal surface of the pipe. As can be seen, the equations for the calculation of friction factor, i.e. Eqs. (38) and (39), depend on the value of the Reynolds number. On the other hand, the equation for the calculation of the Reynolds number, i.e. Eq. (37), depends on  $q$ , the value of flow. Also, the equation for the calculation of flow, i.e. Eq. (30), depends on the value of  $f$ , friction factor. Therefore, these values need to be calculated through iterations. For this purpose, a guess value for  $q$  is considered. Then, the value of  $Re$  is calculated using Eq. (37). Afterwards, the value of  $f$  is calculated using Eq. (38) or (39) depending on the flow regime. Having the value of friction factor, in the next step, the value of  $q$  is updated using Eq. (30). This process is iterated until convergence.

The gas network is subject to a set of limits which are:

$$0 \leq q_{i,j}^{gas} \leq q_{i,j}^{gas,max} \quad (40)$$

Eq. (40) limits the gas flow in the pipeline between nodes  $i$  and  $j$ .

### 2.3.3. Model of the vector coupling components

Energy system components such as Power-to-Gas and VCS couple the electricity and gas networks. The regular operation follows charging while surplus generation and discharging while deficit generation (especially from RES) pattern from one vector to another. Eq. (41) is used to correlate the gas production by the P2G unit based on the power consumption.

$$q_{P2G} = \frac{\eta_{P2G} * P_{P2G}}{GCV_{mix}} \quad (41)$$

where  $GCV_{mix}$  is Gross calorific value of gas mixture in  $\text{kJ/m}^3$ .  $GCV_{mix}$  is calculated using the gross calorific values ( $GCV_i$ ) and molar fractions ( $X_i$ ) of the components of natural gas mixture as follows [37]:

$$GCV_{mix} = \frac{\sum_{i=1}^{N_c} (X_i \times GCV_i)}{Z_{mix}} \quad (42)$$



In the case of a VCS, the state of stored energy in the VCS is represented by the term level-of-gas (LoG). Eq. (43) represents the LoG.

$$LoG_{vcs} = \frac{V_{vcs}^{available}}{Cap_{vcs}} * 100 \quad (43)$$

The value of level-of-gas by a VCS ( $LoG_{vcs}$ ) is subject to the following constraint in Eq. (44) that defines its limits between its minimum ( $LoG_{vcs}^{min}$ ) and maximum capacity ( $LoG_{vcs}^{max}$ ).

$$LoG_{vcs}^{min} \leq LoG_{vcs} \leq LoG_{vcs}^{max} \quad (44)$$

#### 2.4. Problem solution

This section focuses on solutions to the mathematical formulations discussed in the previous subsections. The long-term investment model (presented in Section 2.2) and the short-term operational (presented in Section 2.3) model are distinctive mathematical models that are in different time frames as shown in Fig. 3. Thus, both models need to be solved with separate solvers. An algorithm that connects these models is presented later in this section, along with the flow of information.

##### 2.4.1. Long term investment model

The long-term investment model proposed in this study is part of a new paradigm that follows a deregulated structure characterised by oligopolistic competition within a liberalised energy market. The presented energy market model assumes a small number of firms that possess significant market power and strategically interact in determining their output. These firms compete based on a Cournot framework, where each firm considers the production decisions of its competitors and the resulting impact on market prices. In our paper, the firms that own power plants are not price-takers, as their production decisions influence the electricity price. Instead, these firms strategically determine their output quantities to maximise their profits while accounting for the market-clearing price that balances total supply and demand. This approach accurately reflects the dynamics of an oligopolistic market within a liberalised energy system. While this study focuses on modelling competition in an oligopolistic market using the Cournot framework, the extension of the model to include additional dynamics, such as market power asymmetry or differentiated strategies among dominant firms, is identified as a potential area for future work. These extensions would further enhance the representation of imperfect competition and align the model with real-world scenarios where dominant players exert greater influence over the market.

A suitable mathematical representation of the proposed model is the Nash–Cournot equilibrium of the production-investment game [39]. In Nash–Cournot models, multiple market players who participate in the liberalised energy market optimise their respective payoffs simultaneously. This is where complementarity models are introduced to generalise linear or non-linear programming problems [40]. To find the optimal solution for the market participants, Karush–Kuhn–Tucker (KKT) optimality conditions are derived [41]. The technical constraints and market clearing conditions form a mixed complementarity problem (MCP) presented later in this subsection. The MCP, consisting of a square system of equations, is solved using the PATH algorithm [42]. In this study, PATHAMPL is the solver used with the Python Pyomo package. The objective function and constraints in the proposed investment model are assumed to be strictly convex. In our study, we have considered a quadratic function for the cost, which has positive coefficients to ensure convexity. Additionally, all linear constraints have lower limits set at zero. Convexity can be ensured by analysing the Hessian matrix [43–45].

The KKT conditions with respect to the decision variables and constraints form a mixed complementarity problem. The  $\perp$  symbol denotes the complementarity. Further examples on the formulation of complementarity problems, as applied to energy market problems, are elaborated in [40].

Further, Eqs. (45)–(69) represents the complementarity condition of a market player for a specific time horizon. The size of the overall problem varies depending on the number of players in the market.

The conditions that we use in the mathematical solver is elaborated in the remain part of this section. The solver finds solution for  $q^{wind}$ ,  $q^{col}$ ,  $q^{ccgt}$ ,  $q^{fc}$ ,  $q^{ez}$ ,  $q^{p2g}$ ,  $q^{meth}$ ,  $q^h$ ,  $I^{wind}$ ,  $I^{col}$ ,  $I^h$ ,  $\sigma^{wind}$ ,  $\sigma^{col}$ ,  $\sigma^{ccgt}$ ,  $\sigma^{fc}$ ,  $\sigma^{ez}$ ,  $\sigma^{p2g}$ ,  $\sigma^{meth}$ ,  $\sigma^h$ ,  $\delta^{wind}$ ,  $\delta^{col}$ ,  $\delta^h$ ,  $\alpha^e$ ,  $\alpha^g$ ,  $\alpha^h$  satisfying the following conditions from Eqs. (45)–(69).

Eqs. (45)–(52) represent the complementarity conditions that represent the objective function with respect to electricity, gas, and hydrogen production.

$$q^{wind} \geq 0 \perp AF [p^e - c^{wind}] - \sigma^{wind} - \alpha^e \leq 0 \quad (45)$$

$$q^{col} \geq 0 \perp AF [p^e - c^{col}] - \sigma^{col} - \alpha^e \leq 0 \quad (46)$$

$$q^{ccgt} \geq 0 \perp AF [p^e - c^{ccgt}] - \sigma^{ccgt} - \alpha^e \leq 0 \quad (47)$$

$$q^{fc} \geq 0 \perp AF [p^e - c^{fc}] - \sigma^{fc} + \frac{1}{\eta_{fc}} \cdot \sigma^h - \alpha^e \leq 0 \quad (48)$$

$$q^{ez} \geq 0 \perp AF [p^h - c^{ez}] - \sigma^{ez} - \eta_{ez} \cdot \sigma^h - \alpha^h \leq 0 \quad (49)$$

$$q^{p2g} \geq 0 \perp AF [p^g - c^{p2g}] - \sigma^{p2g} - \alpha^g \leq 0 \quad (50)$$

$$q^{meth} \geq 0 \perp AF [p^g - c^{meth}] - \sigma^{meth} - \alpha^g \leq 0 \quad (51)$$

$$q^h \geq 0 \perp AF [p^h - c^h] + \sigma^h \cdot \frac{1}{\eta_h} - \alpha^h \leq 0 \quad (52)$$

Further, Eqs. (53)–(55) represent the complementarity conditions of the objective function with respect to the investment variable.

$$I^{wind} \geq 0 \perp -AF \cdot \varphi^{wind} \cdot \epsilon^{wind} - \epsilon^{wind} \cdot \left(1 - \frac{\lambda_{wind} - n}{\lambda_{wind}(1+r)^n}\right) + \sigma^{wind} - \delta^{wind} \quad (53)$$

$$I^{col} \geq 0 \perp -AF \cdot \varphi^{col} \cdot \epsilon^{col} - \epsilon^{col} \cdot \left(1 - \frac{\lambda_{col} - n}{\lambda_{col}(1+r)^n}\right) + \sigma^{col} - \delta^{col} \quad (54)$$

$$I^h \geq 0 \perp -AF \cdot \varphi^h \cdot \epsilon^h - \epsilon^h \cdot \left(1 - \frac{\lambda_h - n}{\lambda_h(1+r)^n}\right) + \sigma^h - \delta^h \quad (55)$$

Eqs. (56)–(63) represent the complementarity conditions that is derived from the production investment constraint that is defined in Eq. (9).

$$\sigma^{wind} \geq 0 \perp q^{wind} - K^{wind} - I^{wind} \leq 0 \quad (56)$$

$$\sigma^{col} \geq 0 \perp q^{col} - K^{col} - I^{col} \leq 0 \quad (57)$$

$$\sigma^{ccgt} \geq 0 \perp q^{ccgt} - K^{ccgt} \leq 0 \quad (58)$$

$$\sigma^{fc} \geq 0 \perp q^{fc} - K^{fc} \leq 0 \quad (59)$$

$$\sigma^{ez} \geq 0 \perp q^{ez} - K^{ez} \leq 0 \quad (60)$$

$$\sigma^{p2g} \geq 0 \perp q^{p2g} - K^{p2g} \leq 0 \quad (61)$$

$$\sigma^{meth} \geq 0 \perp q^{meth} - K^{meth} \leq 0 \quad (62)$$

$$\sigma^h \geq 0 \perp \eta_{ez} \cdot q^{ez} - \frac{1}{\eta_{fc}} \cdot q_{fc} - \frac{1}{\eta_h} \cdot q^h - K^h - I^h \leq 0 \quad (63)$$

Constraint defined in Eq. (10) is represented as complementarity conditions in Eqs. (64)–(66).

$$\delta^{wind} \geq 0 \perp I^{wind} - I_{max}^{wind} \leq 0 \quad (64)$$

$$\delta^{col} \geq 0 \perp I^{col} - I_{max}^{col} \leq 0 \quad (65)$$

$$\delta^h \geq 0 \perp I^h - I_{max}^h \leq 0 \quad (66)$$

Market clearing conditions are defined as complementarity conditions from Eqs. (67)–(69). In this model,  $\alpha^e$ ,  $\alpha^g$ , and  $\alpha^h$  variables are free and represents the marginal price of electricity, gas, and hydrogen in their respective markets.

$$q^{wind} + q^{col} + q^{cgt} + q^{fc} - Q^e = 0 \quad (\alpha^e \text{ unrestricted}) \quad (67)$$

$$q^{p2g} + q^{meth} - Q^g = 0 \quad (\alpha^g \text{ unrestricted}) \quad (68)$$

$$q^{ez} + q^h - Q^h = 0 \quad (\alpha^h \text{ unrestricted}) \quad (69)$$

#### 2.4.2. Short term operational model

The short-term operational model of the integrated electricity and gas network is modelled using the Pandapower [46] and Pandapipes [47] Python packages. Pandapower is used to model the electricity network, ensuring that power system components and their constraints are satisfied. The gas network is modelled using Pandapipes. Running an optimal power and gas flow as a multi-network utilises the constraints related to both power flow and gas flow. For power flow, Eqs. (20)–(21) and for gas flow, Eqs. (29)–(42) are built into their respective packages.

Additionally, using Pandapipes, the hydrogen vector coupling storage connected to the electricity network has been modelled and simulated using existing functions within the package. The combination of these packages provides the flexibility to model the interaction between the electricity and gas networks through vector coupling components such as P2G plants. Network models developed using these packages are provided with time-series data to perform optimal power, gas, and hydrogen flow (OPGHF). Solving an OPGHF is a non-linear programming (NLP) problem that is addressed using the solver provided within the packages.

#### 2.4.3. Algorithm

An algorithm is followed to perform the long-term investment analysis using short-term operations. Fig. 5 represents the algorithm developed to solve the mathematical framework. As shown in the algorithm, the process starts with performing an annual investment simulation for the first planning horizon using the base year as the initial condition. Results from the annual investment model are used to calculate the total operation cost of the model ( $Cost_{GT}$ ) which can be defined as sum of Eqs. (4) and (6).

$$Cost_{GT} = \sum_{i \in T} C_i + CO_i \quad \forall i \in T \quad (70)$$

Results from the game-theoretic model are used in the short-term operational simulation model to perform the optimal power, gas, and hydrogen flow (OPGHF). After the successful convergence of OPGHF, the total cost of operation ( $Cost_{OPGHF}$ ) of the energy system would be calculated with the results. Here, value of  $q_i^z$  is obtained from the OPGHF.

$$Cost_{OPGHF} = \sum_{i \in T} \sum_{z \in Z} q_i^z \cdot c^z + CO_i^z \quad \forall i \in T, z \in Z \quad (71)$$

Total operational costs of both game-theoretic and OPGHF would be compared to a threshold value ( $\epsilon$ ) to decide the convergence of the solution. The closer the values better the solution. The iteration is repeated by adjusting the maximum investment limits of the technologies until the point at which the simulation results converge.

### 3. Case study definition

In 2019, the United Kingdom (UK) became the first major economy in the world to legislate the target of achieving net-zero emissions by 2050 [48]. Binding to such a target would be achievable only through a profound policy framework for the key sectors contributing to the greenhouse gas emissions in the UK. The electricity sector in the UK is ambitious, aiming to be powered by clean energy technologies by 2035 [49]. Industries are determined to capture 20–30 MtCO<sub>2</sub> per year by 2030. Similarly, the hydrogen production capacity is set to deliver 5 GW by 2030 whilst having emissions from gas and oil [50]. In addition, the building and transportation sectors have formulated their plans to reduce emissions by promoting zero-emission heating solutions and electric vehicles for transportation.

In this case study, the developed model will be applied to the North of Tyne region in the UK to determine the required investment in electricity generation and storage facilities, including hydrogen-based VCS, to decarbonise the heating demand in this region. The investment decisions will be investigated for the future energy scenarios (FES) formulated by the National Grid. The National Grid has formulated four scenarios [51] to quantitatively elucidate the challenges linked to heating decarbonisation in the UK. The decision to utilise these scenarios is grounded in the fact that they are crafted by the UK's electricity and gas transmission network's system operator. These scenarios integrate historical data on electricity and gas demand with projections related to economic output, energy prices, and the adoption rates of energy efficiency measures and end-use technologies. Employing regression analyses, these variables are harnessed to produce forecasts for energy demand across the residential, commercial, and industrial sectors of the UK economy. Furthermore, the FES scenarios encapsulate diverse changes in energy demand, varying rates of technology adoption for different technologies, and distinctive levels of flexibility in energy consumption [52]. The future heating energy demand for the NoT region will be calculated by scaling down the heating energy demand of the publicly available FESs. [53].

In the following subsections, the NoT region will be described and more information about the FES and data manipulation will be provided.

#### 3.1. Description of the case study area

The North of Tyne (NoT) region is composed of five local authorities in the northeast of England, UK. The region covers approximately 5277 km<sup>2</sup> and has an estimated population of 833,000 people living in more than 360,000 homes. The region faces challenges such as a high rate of fuel poverty, a significant percentage of off-gas houses, above-average household gas usage per meter, several district heating schemes, and a large capacity for expanding renewable energy resources. [54].

The domestic, commercial, residential, and transport sectors consumed 17.3 GWh in 2018. This consumption represents 1% of the total UK energy consumption. The commercial and industrial sector represents around 37% of the total consumption in the region. The domestic and transport sectors contribute 35% and 28% of the total consumption in the region, respectively. 75% of the domestic consumption is supplied by natural gas, and electricity covers only 20% of the total domestic energy demand [55].

The generation from renewable energy sources has increased by approximately 121%, going from approximately 517 GWh in 2014 to approximately 1146 GWh in 2019, while the renewable energy capacity in the region has increased by approximately 240%, going from 245 MW in 2014 to 837 MW in 2019. As a direct consequence of this increase, carbon emissions in the area have experienced a reduction of around 15% between the years 2014 and 2018 [56].

The description and data of the energy networks in the region are given in [57]. To perform the simulation, the networks' model is created using pandapower [46] and pandapipes [58] python packages. The network parameters to create the networks' model are sourced from the energy networks operators in the region.

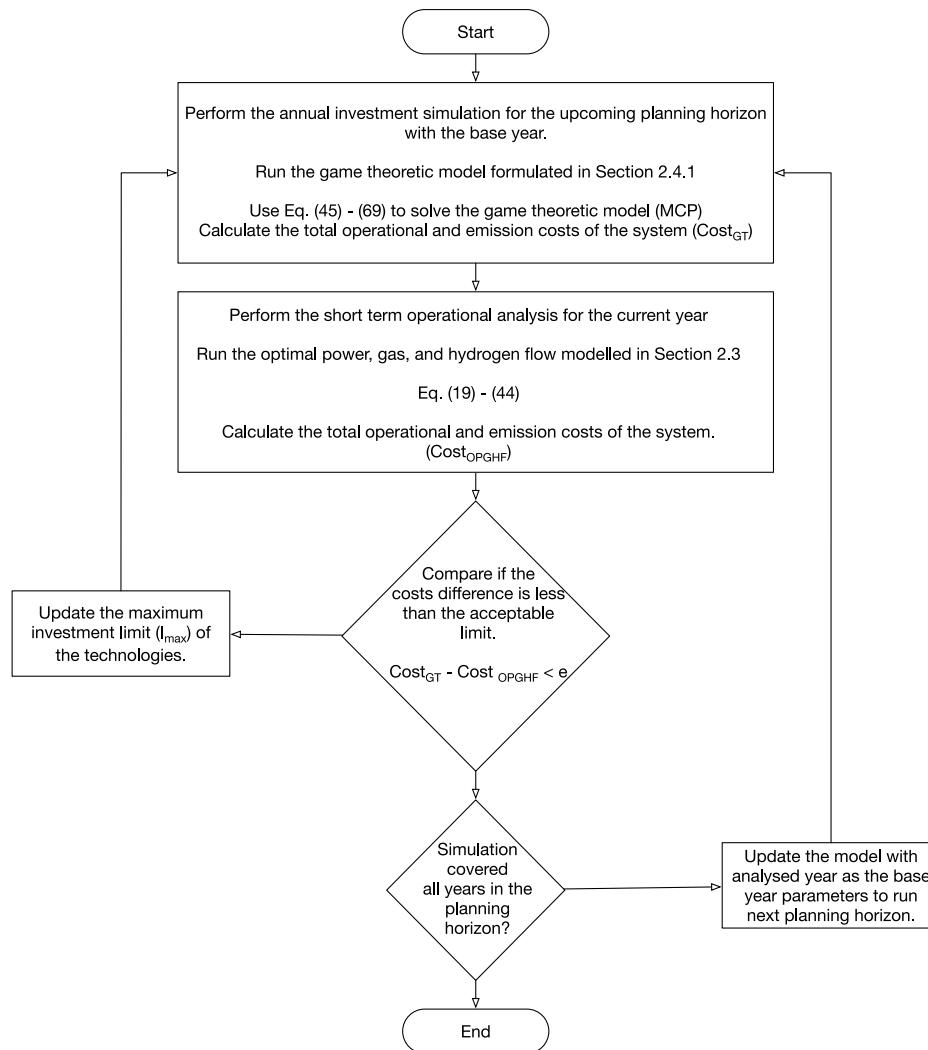


Fig. 5. Algorithm to perform the investment analysis.

### 3.2. Future energy scenarios developed by national grid

The UK's Future Energy Scenarios (FES) [53] highlight a variety of pathways to achieve Net Zero by 2050, each with distinct focuses on technology development, grid modernisation, and market liberalisation. In each scenario — Consumer Transformation, System Transformation, Leading the Way, and Falling Short — policies and investments shape the trajectory of energy system evolution, particularly in relation to hydrogen, Carbon Capture and Storage (CCS), and renewable technologies.

In the Consumer Transformation scenario, there is a strong emphasis on technology development and deployment, with significant investments in renewable energy technologies, energy storage, and electrification infrastructure. Key technologies include solar photovoltaics (PV), wind power, and BESS. Policy support is robust, featuring strong incentives and subsidies to accelerate the adoption of these technologies, particularly at the consumer level. Investments in grid modernisation are essential to integrate decentralised energy generation from distributed resources and to manage increased electricity demand from electrified heating and transport. This scenario also emphasises the benefits of a more liberalised market, with reforms that enhance consumer choice and encourage private investment in renewables and smart grid technologies.

In contrast, the System Transformation scenario balances investments between renewable energy and low-carbon technologies like

CCS, and both green and blue hydrogen. This scenario highlights the importance of grid modernisation, particularly through investments in large-scale renewable projects and infrastructure that supports hydrogen production. The grid must be resilient and capable of integrating both renewable energy and low-carbon technologies while ensuring energy security. Market liberalisation plays a critical role here by creating competitive markets that foster private sector investment in a diverse mix of technologies, driving a more centralised energy system with lower reliance on consumer-driven changes. Leading the Way represents the most ambitious decarbonisation pathway, with aggressive investments in cutting-edge technologies such as advanced renewables, and bioenergy with CCS. Extensive investments in grid modernisation are critical to support the high penetration of renewable energy, which requires advanced storage technologies. Robust market liberalisation policies, including strong regulatory frameworks and comprehensive carbon pricing, create an environment that drives private investment and fosters rapid decarbonisation.

The Falling Short scenario, by contrast, adopts a more gradual approach to decarbonisation. Investments focus on incremental improvements to existing infrastructure, with moderate support for renewable energy technologies. Fossil fuel technologies with CCS play a continued role, with gradual investments in renewables and energy efficiency measures. In this scenario, grid modernisation occurs at a slower pace, reflecting a focus on maintaining energy security and affordability. Market liberalisation is moderate, with reforms focused on stabilising

**Table 1**  
Parameters and source.

Parameter	Technology	Source
Levelised costs ( $c^z$ )	CCGT (with/without CCUS) Biomass (with/without CCUS) Solar PV Wind electric turbines CHPs	Electricity generation costs 2020 report and excel data set [59].
Levelised costs ( $c^z$ )	Electrolyser Fuel cells	Hydrogen production costs 2021 [60].
Specific capital expenditure( $\epsilon_z$ )	Electrolyser Fuel cells	Hydrogen production costs 2021 [60].
Specific capital expenditure ( $\epsilon_z$ )	CCGT (with/without CCUS) Biomass (with/without CCUS) Solar PV Wind electric turbines CHPs	Electricity generation costs 2020 [59].
Emission intensity ( $c_{emis}$ )	CCGT (with/without CCUS) Biomass (with/without CCUS) CHPs	Assessing the Cost Reduction Potential and Competitiveness of Novel (Next Generation) UK Carbon Capture Technology [61].

prices and ensuring a secure energy supply, but without the aggressive push for innovation seen in other scenarios.

Across all these scenarios, the future of the UK's energy system depends on the strategic integration of new technologies, the modernisation of grid infrastructure, and the reform of energy markets to create competitive and resilient energy systems. In this study, we use FES data as the reference dataset representing the pathways to the 2050 national decarbonisation target. The following section discusses the various ways the FES dataset was used to align the decarbonisation targets of the study region.

### 3.3. Data processing and manipulation

The energy demand profiles, generation profiles, and energy mix for each scenario until 2050, reflect the impact of policy instruments and decarbonisation measures. In the Consumer Transformation scenario, high electrification and renewable energy incentives lead to increased electricity demand and a decentralised generation mix dominated by solar and wind. The System Transformation scenario balances investments in renewables, and CCS, resulting in a diversified energy mix and stable energy prices. The Leading the Way scenario features rapid decarbonisation through extensive use of green hydrogen and advanced technologies, achieving deep emissions reductions with a high share of renewables and hydrogen in the energy mix. The Falling Short scenario reflects gradual policy support and moderate investments, maintaining a mix of fossil fuels with CCS and incremental renewable integration, leading to stable but slower progress towards decarbonisation.

In this study, energy demand and installed capacity data are scaled down from the FES to represent the NoT region. The scaling adjustments are based on the assumption that changes in installed capacity and power demand in NoT follow national trends. To scale the power demand, the peak electricity demand was first identified for all four scenarios up to 2050 from FES. Using the demand profile of 2020 as a base with a scale of 1, the variation in demand for subsequent years relative to this base year was calculated. For instance, if demand increased by 20%, the peak demand was multiplied by 1.2 to determine system demand. Similarly, demand growth was applied to load in short-term operational models.

For installed capacity, two factors were considered: the total installed capacity in the NoT region and the per-technology installed capacity. Since regional trends may differ from national trends due to existing infrastructure, we calculated the technology-specific capacity growth for each FES scenario at the national level. We also determined the total target installed capacity for NoT by following the national trend using the base year and scaling methodology. The optimisation process aimed to match national trends both in total capacity and technology-specific capacity, considering initial guesses for the solution. The optimisation determined the deployment of technologies

in five-year intervals, accounting for overall system capacity, plant decommissions, and technology-specific targets. For years not covered directly by FES (e.g., 2030, 2040, 2050), we interpolated the data to establish trends across all years. The same approach was applied to gas demand and capacity.

Additionally, the daily load profile was adjusted to account for decarbonisation measures. For example, to consider the penetration of heat pumps in the electricity demand profile, the electricity demand profile is modified according to the insights from [62].

Further, parameters required for the financial analysis are obtained from various reports published by the Department for Business, Energy & Industrial Strategy (BEIS). Such parameters are presented in Table 1. In this case study, the cost function  $c^z(q^z)$  in Eq. (4) is equated to the levelised cost of the generation and storage technologies corresponding to the planning horizon, given in Table 1 to reflect the impact of different policies embedded in the future energy scenarios. In this study, although the model generates values for  $\alpha^e$ ,  $\alpha^g$ , and  $\alpha^h$  (using Eqs. (69)–(67)), we choose to use the realistic price projections from the FES dataset to calculate revenue. This approach avoids making arbitrary guesses about inverse demand functions.

## 4. Results and discussion

In this section, simulations conducted are presented, and results are analysed to derive insights and inferences. Simulations are broadly categorised as follows.

**Baseline:** The evolution of the energy system until 2050 without vector coupling storage was simulated to establish a baseline. Among the FES scenarios, the 'Falling short' scenario was considered for the simulation. The earlier versions of the FES dataset were released with a scenario called steady progression, which was eventually replaced with 'falling short' to make it clear that the scenario does not lead to net-zero emissions by 2050. The baseline simulations broadly examined changes in the generation mix, capacity margin, and CO2 emissions between 2022 and 2050.

**Impact of vector coupling storage:** Analysing the impact of vector coupling storage involves running the simulations with three different scenarios developed by National Grid: Consumer transformation (CT), System transformation (ST), and leading the way (LW). Various aspects of the system are analysed to study the impact of VCS system in 2050.

**Investment plans by liberalised energy market participants:** The simulation investigated how liberalised energy market participants invest in the expansion of generation assets and vector coupling storage. Key aspects such as economic viability and return on investment are analysed by the energy market participants.

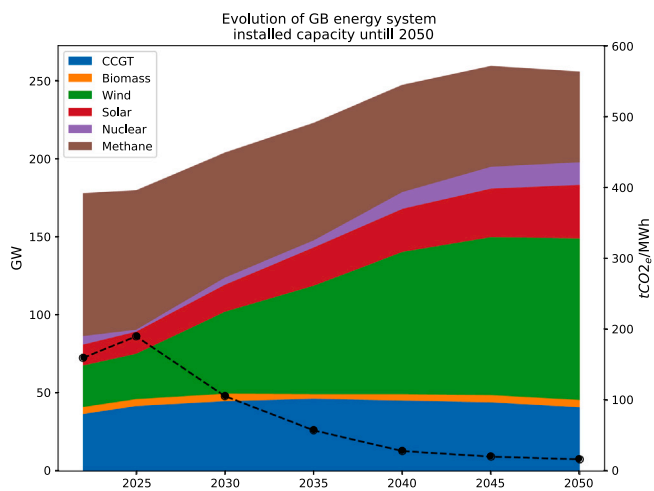


Fig. 6. Change in the installed capacity by 2050 by national electricity system.

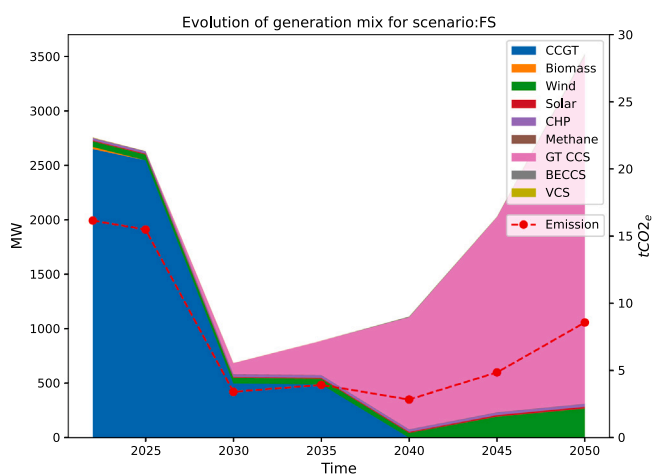


Fig. 7. CO<sub>2</sub> price vs. emission intensity of national system and North of Tyne region.

#### 4.1. Baseline

Baseline simulations are performed based on a ‘falling short’ scenario, which does not include hydrogen integration in the system. Fig. 6 presents the anticipated evolution of installed capacity in the national electricity system. With the falling short scenario, the national electricity system is poised to have approximately 58% installed capacity share by wind electric turbines.

Furthermore, Fig. 7 depicts the results from the investment simulation for the falling short scenario for the region under the study. The graph shows that, with the present trend and planned decommissioning, the regional electricity system may witness a steep decrease in carbon emissions. This decrease is due to the decommissioning of existing CCGT power plants. However, the commissioning of new CCGT and biomass plants with CCS technologies may result in an increase in emissions over the years. These emissions can go up to 50% of 2022 emission levels. By the year 2050, the majority of the investments will be in CCGT with CCS (about 90%) with a minor share in wind electric turbines(7%) in this region. The results differ from the national electricity generation mix. However, at the regional level, the investments follow a different pattern, which is highly influenced by the present electricity generation companies.

Fig. 8 represents the change in existing installed capacity over time for various technologies. The installed capacity drops to zero due to the decommissioning of existing power plants if there are no additional

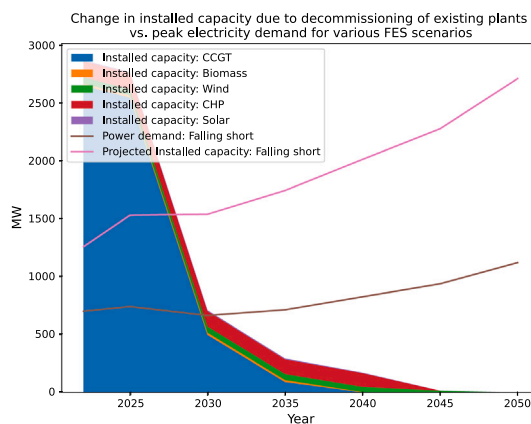


Fig. 8. Installed capacity existing plans of various technologies without any investments and planned decommission.

investments are made in the region. The projections of power demand and future capacity additions follow a trend based on the ‘falling short’ scenario. Thus, Fig. 8 depicts the projected pathways for power demand and installed capacity if the investment follows a ‘falling short’ trend. The lower line in the graph represents the projected peak electricity demand in the region from 2022–2050. The upper line in the graph shows the projected cumulative installed capacity trend until 2050 interpolated according to the national scenario.

#### 4.2. Impact of vector coupling storage

Two scenarios are chosen from the FES dataset to analyse the impact of vector coupling storage: steady transformation (ST) and leading the way (LW). To analyse the impact of vector coupling storage (VCS), these two scenarios are chosen because of their expected generation mix predictions at the national level by the FES dataset. Steady transformation is a hydrogen-intensive scenario that uses hydrogen technologies for heating and energy storage. The ‘leading the way’ scenario is a mix of electrification and hydrogen for heating and energy storage applications. The third scenario, which is not included in this study, is consumer transformation (CT), representing an equal blend of both LW and ST scenarios in terms of levels of societal change and the speed of decarbonisation. Thus, to reduce computational and analytical complexity, the CT scenario is not included in this study.

Simulations to analyse the impact of VCS are performed in two-time durations, namely long-term and short-term simulations. The long-term investment plans are analysed for a 5-year horizon from 2022 to 2050. The short-term simulations are performed for a representative day with a resolution of 1 h.

##### 4.2.1. Long-term investment analysis

The impact of VCS is investigated by analysing the change in installed capacity and emissions until 2050 for ST and LW scenarios. Fig. 9 represents the investment analysis results for the ST scenario. The overall installed capacity in the ST scenario drops in year 2040 after a rise due to the decommissioning of CCGT power plants. In addition, investments in hydrogen are stagnated and expected to become aggressive after 2040 due to a reduction in capital expenditure and levelised costs. Further, the carbon emission is expected to reduce by 88% compared to 2022 level.

On one hand, the ST scenario attracts 34.6% of CCGT with CCS investments in the region. On the other hand, the scenario attracts 10% wind electric turbines and 48% hydrogen system investments. There is a significant amount of additional investments in wind technologies compared to the present levels of the generation mix, which is at 2% in the region. Compared to the national scenario, the contribution of wind

**Table 2**  
NPV and RoI of various technologies for ST scenario.

2035				2050			
Generator type	Capacity (MW)	NPV (£ millions)	RoI (%)	Generator type	Capacity (MW)	NPV (£ million)	RoI (%)
GT CCS	100	264.99	16.74	GT CCS	1056	2985.26	23.64
CHP	8.25	21.95	1.82	CHP	9	27.68	4.54
Wind	0.1	0.15	13.38	Wind	170	364	31.32
Solar	No new investments		11.56	Solar	10	2.31	15.74
BECCS	3.18	2.57	0.00	BECCS	5.01	1.05	6.66

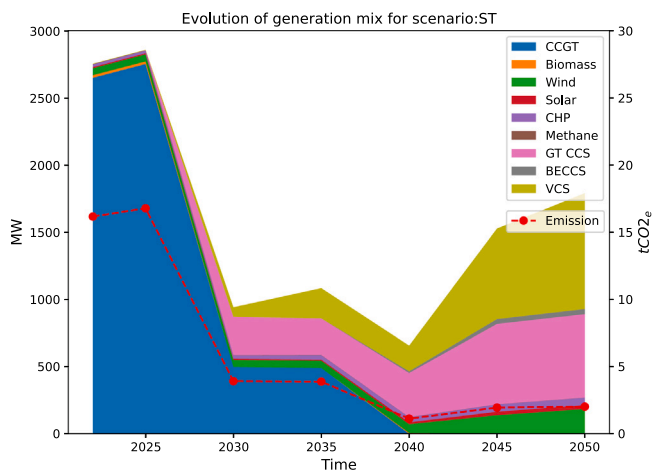


Fig. 9. Electricity generation mix until 2050 for ST scenario.

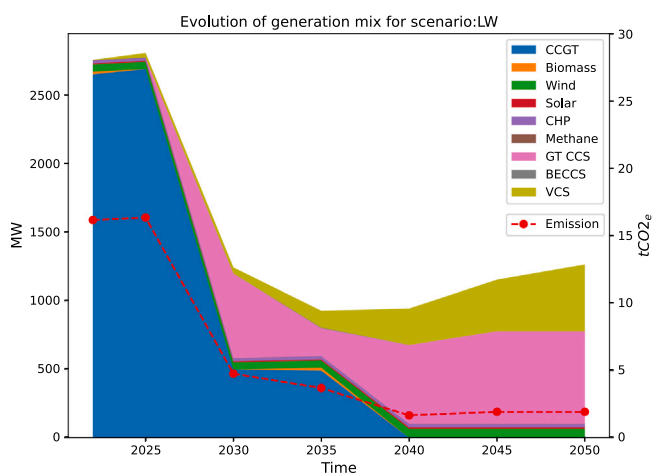


Fig. 10. Electricity generation mix until 2050 for LW scenario.

technology is less; however, the study focuses on a regional level where the present installed capacity mix affects future investments. Thus, the investments at a regional level may not follow the national trend.

The ST scenario attracts investments in BECCS. However, the LW scenario does not promote investments in BECCS due to its high capital expenditure (capex), levelised costs, and emissions. Fig. 10 depicts the results of the investments that follow LW scenario. The expected emission drop by the year 2050 in the LW scenario is approximately 88%. Moreover, the investments attracted to VCS are 38.5%. Other technologies, such as wind and CCGT CCS, represent 5% and 53% respectively. Thus, from both scenarios, it is evident that the future national electricity system might not be completely free of investments in gas and biomass fuels. As the technological and commercial efficiencies of CCS improve, more investments are expected in the region.

#### 4.2.2. Short-term operational analysis

The short-term operation of vector coupling storage is analysed in this section. Short-term simulations are performed to analyse the operation of fuel cells and electrolysers in the vector coupling storage. In this study, two major aspects of the VCS are analysed. Firstly, simulations are performed to investigate the operational pattern of the VCS in various study scenarios. Secondly, the impact of VCS is studied by analysing the carbon emission reduction potential of the technology.

Figs. 11(a) and 11(b) show the daily dispatch plot for the winter peak day in 2050. When cost minimisation is set as the key objective of the simulation, the operation of the VCS takes place during low electricity price hours to store hydrogen in the tank through electrolysis. During high electricity price hours, to take advantage, the fuel cell converts the hydrogen back to electricity. Thus, the coupling of VCS in the electricity system is expected to take advantage through price arbitrage. Such observations are made in this study due to the consideration of VCS as a technology for price arbitrage. The operational scenario will vary if the VCS is purposed for ancillary services.

In the NoT region, the installed capacity of wind technology is relatively low. However, due to its high availability, the contribution of the technology towards meeting the daily demand is high. Thus, future investments are expected to be promoted for wind-electric turbines.

The major assumption in the simulation model maintains level-of-hydrogen (LoH) limits during its operational hours. In this study, the operational range is specified between 20%–80%. In addition, the operation of the VCS is assumed to begin from 50% LoH and ends at 50% LoH. This is represented in Fig. 12. Thus, the VCS operation reshapes the generation profile of other technologies.

Further, by 2035, carbon capture and storage technologies will have significant investments. Such investments have a significant impact on the long-term generation as well as daily generation emissions. Especially when the penetration of heat pumps is expected to increase over the years. Fig. 13 presents the change in emissions during a peak winter day in 2035 and 2050. The results show that there is a visible transition in carbon emissions due to the engagement of more CCGT CCS technologies over the years. In 2030 LW scenario, the additional demand due to the heat pump is met using CCST CCS technology. In addition, the carbon pricing adopted in scenario ST has the maximum emissions reduction potential in 2050. The FS scenario assumes minimum penetration of heat pumps which resulted in lesser emissions from the system at the system level.

#### 4.3. Investment plans by liberalised energy market participants

Analysing the investment plans by the liberalised energy market participants is investigated by analysing the economic viability of the electric generators and VCS through net present value (NPV) and return on investments (RoI). Further, the analysis is compared with investments in Li-ion batteries in the aspect of long-term energy storage. Finally, a sensitive analysis is performed to study the relation between the round-trip efficiency of the VCS storage with its profitability.

Table 2 shows the NPV and RoI of various investments for 2035 and 2050 based on the ST scenario. In the analysis, the revenues for the project are calculated based on the projected per-unit revenue for the investors based on the BEIS data. All the scenarios result in

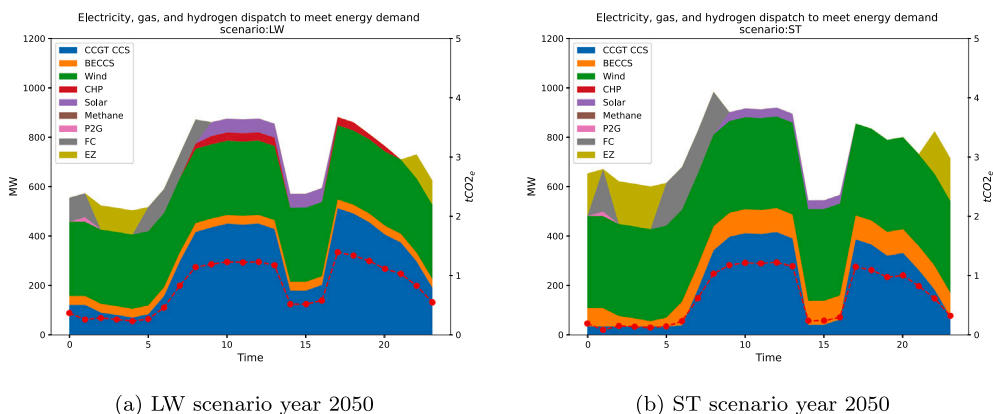


Fig. 11. Daily dispatch of the energy system for LW and ST scenarios.

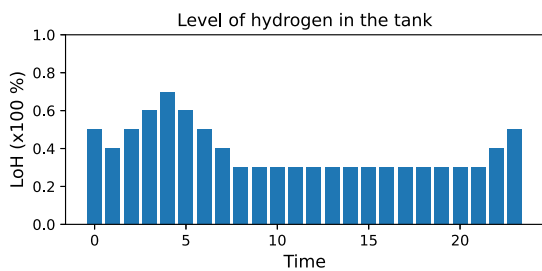


Fig. 12. Evolution of the hydrogen level in the tank.

positive NPV values for all technologies. However, when the RoI is considered, technologies such as solar PV and CHP are least performed. Such a difference is witnessed due to high capital expenditure and operational costs. Wind electric turbines have the maximum RoI due to relatively high availability among zero-emission technologies. In addition, the investment and operational costs are moderate compared to other investment options. Further, solar PV technologies are attractive in ST scenario in terms of RoI. However, the model predicts only 1.5% investments in the installed capacity due to its low availability (11%). Therefore, solar PV investments are expected to focus on off-grid applications in the future.

The economic viability of the VCS is compared with a same-size of Li-ion battery to investigate its difference in profitability at both ST and LW scenarios. Results are shown in Table 3. Results show that there exists close competition between VCS and Li-ion storage technologies in terms of RoI. In addition, the RoI is higher for Li-ion storage devices if it is used for 6 h for charging and discharging. However, though the round trip efficiency of the VCS is 36% which is lower compared to the Li-ion battery's 86%, the RoI is close for both technologies. Thus, the profitability of the technology relies on its availability and round-trip efficiency. In future, the capital costs of VCS are expected to decrease compared to the present level.

The relation between round-trip efficiency and return on investment is analysed using Table 4. At 50% round trip efficiency, the VCS become profitable compared to the Li-ion storage devices. That infers, in future, both electrolysers and fuel cells shall have at least 70% conversion efficiency to boost investments in the technologies.

Fig. 14 shows the point at which the VCS becomes profitable compared to Li-ion technology. The results indicate that the VCS becomes profitable if the round-trip efficiency of the VCS reaches approximately 45%. This means that, in the future, the electrolyser, hydrogen storage, and fuel-cell coupled arrangement should have at least a 70% conversion efficiency to boost investment in the technology.

**Table 3**  
Economic viability and difference in investing in VCS and Li-ion technologies.

Scenario	Year	Capacity	Type	NPV (£ million)	RoI (%)
ST	2035	225	VCS	177.85	50.02
	2050	862.86	VCS	838.66	39.61
LW	2035	122.78	VCS	97.05	50.02
	2050	487.65	VCS	473.98	39.61
ST	2035	225	Li-ion	75.31	42.03
	2050	862.86	Li-ion	566.36	44.88
LW	2035	122.78	Li-ion	41.09	42.03
	2050	487.65	Li-ion	320.08	44.88

**Table 4**  
Round trip efficiency and RoI.

Round trip efficiency	RoI 2035 (%)	RoI 2050 (%)
25	34.73	27.5
36	50	39.6
50	69.47	55
75	104.2	82.514
85	118.103	93.51

An important point to note here is that the area of investment considered in this study is to take advantage of price arbitrage benefits. VCS can be coupled with the grid for other purposes, such as ancillary services. Furthermore, the return on investment in VCS technology for both years is different due to factors such as levelised cost, changes in electricity prices, changes in capital investment costs, etc. The levelised cost of VCS is expected to increase by 10%, although the investment cost is expected to drop by approximately 11%. Thus, other cost parameters and revenue streams have a major impact on the future profitability and investment flow to the VCS.

4.4. Observations

The numerical results of this case study indicate the following observations.

1. An analysis based on scenarios generated using future energy scenarios provided guidance on the direction the energy system is likely to take in the next 27 years. The results indicate that the policies planned for the 'system transformation' scenario are the most suitable for the decarbonisation of the energy system by attracting investments in hydrogen production, storage, and conversion systems.
2. The predicted scenarios of the future anticipate a very low inclusion of conventional power plants. However, the results

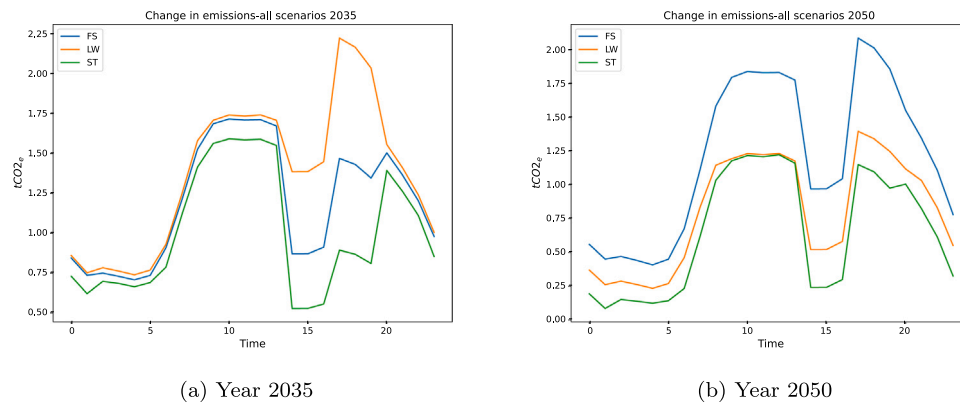


Fig. 13. Change in daily emissions for various scenarios.

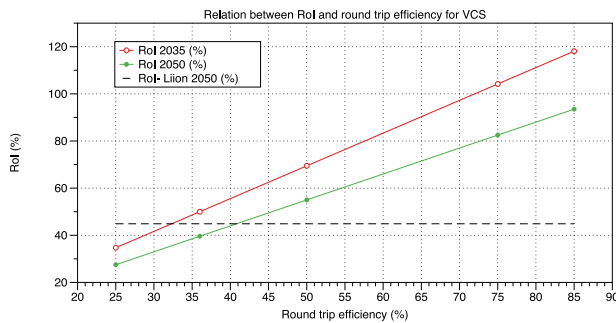


Fig. 14. Round trip efficiency vs. RoI for VCS and Li-ion battery.

show the coexistence of conventional power plants with the help of carbon capture and storage (CCS) technologies, at least at a regional level. The case study region is a clear example of this observation. In terms of power system stability, this coexistence is essential to provide inertia during unexpected events. A high share of inverter-based resources, such as renewable energy systems and battery energy storage systems (BESS), affects system inertia and short-circuit levels.

3. The major application of VCS considered in this study is to take advantage of price arbitrage through short-term energy storage. Other aspects of storage, such as providing support for ancillary services, are not considered in this analysis. To boost investor confidence and attract investments in VCS over large-scale battery technologies, the round-trip efficiency of the technology should be at least 50%. This means that electrolyzers and fuel cells should have at least a 70% conversion efficiency. The present technology is far from this figure; thus, rigorous research is required to improve the efficiencies of such systems.
4. The model supports the net zero emission target goals by effectively managing the interaction between VCS and attracting investments in zero and low emission technologies that replace existing high emission technologies. This is a predictive model that proposes future power system expansion plans based on existing power plants and their possible decommissioning years in the study region. The results of the model contribute to the planning of assets and other associated requirements, such as transmission infrastructure, power system protection, stability equipment, and more.

#### 4.5. Limitations

There are several limitations within the outlined research. The first identified limitation is that the application of VCS assumed in this paper

is to take advantage of price arbitrage on a short-term basis, specifically within a day. The study does not include aspects of long-term hydrogen storage through large-scale low-cost systems or other purposes of VCS, such as providing ancillary services.

Another limitation of this paper is the generalisation of electrolysis technology. Various electrolysis technologies are available; however, the study considers only one type of electrolyser.

Additionally, the energy market structure modelled in this paper assumes a perfect competition model. In reality, the energy market is imperfect due to the market power of various energy market players.

Finally, the case study used to carry out the analysis represents a geographical location that may not be the best representation of the national scenario. The study aims to understand investment behaviour from a regional geographical aspect. However, in this paper, the electricity and gas networks are not interacting with other regions through the import or export of energy.

#### 5. Conclusion

This paper introduces an innovative competitive planning model, utilising a Cournot oligopoly competition model for Integrated Energy Systems (IES). The model acknowledges the independence of decision-making entities within the electricity system, natural gas system, hydrogen production, and vector coupling storage in the IES, with each entity responsible for optimising its profits. The co-planning strategy involves integrating hydrogen-based Vector Coupling Storage (VCS) and electricity generation within the IES, considering short-term energy system operations to inform long-term investment plans.

The annual investment model, capturing long-term planning decisions on capacity and policies, is complemented by an hourly energy system model simulating the network's operational aspects for short-term planning. The effectiveness of this novel planning model is evaluated in an IES system located in the North of Tyne region in the UK. The application of the model focuses on planning generation and hydrogen-based vector coupling storage to facilitate heating decarbonisation, aligning with the future energy scenarios outlined by the National Grid. The contributions of this model include: (1) elucidating the structure of hydrogen-based bidirectional vector coupling storage in the IES; (2) fostering collaboration among VCS plants, the electricity system, and the natural gas system; (3) optimising the comprehensive utilisation of IES resources. Consequently, the proposed model exhibits significant potential for practical applications in IES planning.

Limitations discussed in the previous section pave the way for future research in this area. Subsequent studies could expand this work to a national scenario to examine the impact of investments in Vector Coupling Storage (VCS). The research can also delve into exploring other applications of VCS, such as providing ancillary services. The profitability and investment potential for these alternative applications



may differ and necessitate separate case studies. Additionally, investigating various blending configurations of hydrogen with natural gas systems for meeting heating demand could be another area of research. This approach would offer insights into the ongoing plans to repurpose existing natural gas networks. Future work may also encompass aspects of energy import in the form of electricity and natural gas, along with the emergence of the international hydrogen market.

### CRedit authorship contribution statement

**Akhil Joseph:** Writing – review & editing, Writing – original draft, Methodology, Data curation, Conceptualization. **Adib Allahham:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization. **Sara Louise Walker:** Methodology, Funding acquisition, Conceptualization.

### Declaration of competing interest

I confirm that all the authors of this paper declare no conflict of interest.

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### Data availability

Data will be made available on request.

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