Online Bad Data Detection in Compressed Sensing based Distribution System State Estimation

James Ranjith Kumar Rajasekaran, Balasubramaniam Natarajan and Jing Jiang

***Abstract*—This letter introduces a novel approach for online bad data detection in distribution system state estimation (DSSE) by integrating compressive sensing (CS) with a modified largest normalized residual (LNR)-based detector. To the best of the authors' knowledge, this is the first work to develop a bad data detection method specifically for CS-based DSSE in unobservable distribution networks. The paper derives a closed-form solution for the compressed sensing problem, which is then used to quantify the error statistics in CS-based DSSE estimates. These statistics enable the design of a modified LNR-based detector using eigen decomposition, significantly improving anomaly detection. Extensive simulations on IEEE 37-bus and 123-bus unbalanced distribution systems demonstrate that the proposed method consistently outperforms the conventional LNR approach and neural network based technique, achieving superior detection rates with low computation effort even with a limited number of measurements. This robust approach effectively detects data anomalies from both random errors and cyber-attacks, making it highly suitable for practical DSSE applications.**

***Index Terms*—Bad Data Detection, Compressed Sensing, Distribution System, Largest Normalised Residual, Unobservability.**

I. Introduction

The integration of distributed energy resources (DER) into modern electrical distribution networks has fundamentally transformed power system operations and introduced challenges related to monitoring, control, and protection of distribution networks. As a result, distribution system state estimation (DSSE) has become essential for providing real-time operational insights into these increasingly complex systems [1]. Traditional state estimation techniques, which were originally developed for transmission systems, are not directly applicable to distribution networks due to their radial topology, lower observability, and increased uncertainty. The growing prevalence of DERs further complicates DSSE by introducing variability in generation and load, making it more susceptible to bad data, measurement noise, and potential cyber-attacks [2]. To overcome these challenges, advanced methods of state estimation and bad data detection (BDD) are necessary to ensure the accuracy and resilience of DSSE processes [3].

Several studies have compared different DSSE approaches for distribution networks, highlighting trade-offs between estimation techniques, measurement technologies, and system configurations [4], [5]. While weighted least squares (WLS) estimator has been widely used for DSSE, they are limited in their ability to detect and correct gross errors in measurements [6]. To improve the resilience of DSSE, several strategies have been proposed, such as incorporating pseudo-measurements and leveraging phasor measurement units for enhanced accuracy and robustness [2]. Other advanced techniques include interval analysis-based methods [7] and approaches for detecting leverage point attacks [8].

Machine learning (ML) approaches, such as artificial neural networks (ANN) [9] or its extensions that incorporate photovoltaic (PV) forecasting [10], have been explored as solutions for BDD in DSSE. However, these methods often require frequent retraining to adapt to evolving network dynamics [11], and training neural networks is an NP-hard problem, posing significant computational burdens [12]. Consequently, non-training-based approaches are increasingly being sought for unobservable networks [4]. Given these challenges, there is a need for non-training-based approaches for BDD in unobservable distribution networks.

Compressive sensing (CS) [13], a non-training-based approach, has emerged as a valuable tool in distribution systems with limited measurement points. CS enables accurate signal reconstruction by exploiting the inherent sparsity of the state vector. This method has been adapted for DSSE, allowing for accurate state estimation with limited measurements [14], [15]. Recent advancements have extended CS to areas such as efficient data aggregation in smart grids [16], improved state estimation through data-driven dictionaries [17], dynamic state estimation [18], harmonic state estimation [19], and Kalman filtering with intermittent observations [20], [21]. In terms of cybersecurity, studies have shown that CS-based DSSE can be vulnerable to stealthy cyber-attacks [22]. Even in scenarios with sparse or incomplete data, modified matrix completion techniques can help detect these attacks. However, matrix completion, while offering a closed-form solution as demonstrated in [23], is less accurate than CS-based DSSE in estimating system states. Robust CS based DSSE [4] is designed to generate reliable state estimates even in the presence of bad data, but they do not explicitly derive the statistical properties of residuals for formal anomaly detection.

This letter presents a novel approach to online BDD in DSSE when faced with unobservable measurements by integrating CS with a Largest Normalized Residual (LNR)-based detector. Our approach focuses on deriving the error statistics of CS-based DSSE, enabling precise and statistically sound bad data detection. The key contributions of this paper are:

- The derivation of a closed-form solution for the compressed sensing problem, which is then applied to quantify the error statistics of CS-based DSSE estimates.
- The design of a modified LNR test-based bad data detector, employing eigen decomposition to identify data anomalies in CS-based DSSE estimates.
- Extensive testing of the proposed approach on 37-bus and 123-bus distribution test systems, demonstrating a superior performance for the modified LNR test compared to ANN based approach.

II. Proposed Approach

In a three-phase unbalanced distribution system, let $\mathcal{R}$ represent the reference bus nodes, and $\mathcal{E}$ the remaining nodes. Each phase of a bus is treated as a separate node. Voltage magnitudes $|\mathbf{v}_j|$ and real power injections $\mathbf{p}_j$ are used as measurements at nodes $j \in \mathcal{E}$. Subsets $\mathcal{A}, \mathcal{B}, \mathcal{C}$ contain nodes

J. R. K. Rajasekaran and J. Jiang are with Northumbria University, UK. B. Natarajan is with Kansas State University, USA.

for phases a, b, and c, respectively. Assuming negligible voltage angle differences due to the low X/R ratio in distribution feeders, voltage phasors are derived as $\mathbf{v}_j = |\mathbf{v}_j| \angle \boldsymbol{\delta}_j$, where $\boldsymbol{\delta}_j = 0, -2\pi/3, 2\pi/3$ for phases $\mathcal{A}, \mathcal{B}, \mathcal{C}$. The reactive power $\mathbf{q}_j$ is estimated as $\mathbf{q}_j = \mathbf{p}_j \tan\boldsymbol{\theta}$, assuming constant power factor $\cos\boldsymbol{\theta}$. The current injection $\mathbf{i}_j$ is approximated by $\mathbf{i}_j = \left(|\mathbf{v}_j|^{-1} \angle \boldsymbol{\delta}_j\right)(\mathbf{p}_j - \mathrm{i}\mathbf{q}_j)$. Let $\mathbf{v}_{\mathcal{R}}$ denote the voltage at the reference bus nodes. The relationship between current injections and nodal voltages is given by $\mathbf{i} = \mathbf{Y}_{\mathcal{R}} \cdot \mathbf{v}_{\mathcal{R}} + \mathbf{Y} \cdot \mathbf{v}$, where $\mathbf{Y}$ and $\mathbf{Y}_{\mathcal{R}}$ are the nodal admittance matrices for nodes in $\mathcal{E}$ and $\mathcal{R}$. The nodal impedance matrix is $\mathbf{Z} = \mathbf{Y}^{-1}$, allowing the relationship to be rewritten as $\mathbf{Z} \cdot \mathbf{i} = \Delta\mathbf{v}$, where $\Delta\mathbf{v} = \mathbf{Z} \cdot \mathbf{Y}_{\mathcal{R}} \cdot \mathbf{v}_{\mathcal{R}} + \mathbf{v}$.

A. Closed Form Solution of Compressed Sensing Approach

Let $\Delta\mathtt{v} = [\Re(\Delta\mathbf{v})' \ \Im(\Delta\mathbf{v})']'$ and $\mathtt{i} = [\Re(\mathbf{i})' \ \Im(\mathbf{i})']'$ represent real domain vectors. As shown in [14], $\Delta\mathtt{v}$ and $\mathtt{i}$ are compressible in given sparsifying bases $\boldsymbol{\Psi}_v$ and $\boldsymbol{\Psi}_i$, where $\Delta\mathtt{v} = \boldsymbol{\Psi}_v \mathbf{s}_v$ and $\mathtt{i} = \boldsymbol{\Psi}_i \mathbf{s}_i$, with $\mathbf{s}_v$ and $\mathbf{s}_i$ being K_v-sparse and K_i-sparse. Given the set of available measurements $\mathcal{M}$, voltage and current measurement vectors are $\mathbf{y}_v = \mathbf{C}\Delta\mathtt{v}$ and $\mathbf{y}_i = \mathbf{C}\mathtt{i}$, with $\mathbf{C}$ as the projection matrix. The sparse estimates $\hat{\mathbf{s}}_v$ and $\hat{\mathbf{s}}_i$ are obtained by solving the following composite minimization problem:

$$(\hat{\mathbf{s}}_v, \hat{\mathbf{s}}_i) = \min_{\mathbf{s}_v, \mathbf{s}_i} \|\mathbf{s}_v\|_1 + \|\mathbf{s}_i\|_1 + \|\boldsymbol{\Psi}_v \mathbf{s}_v - \mathtt{Z}\boldsymbol{\Psi}_i \mathbf{s}_i\|_2 \quad \text{s.t.} \quad \mathbf{y}_v = \boldsymbol{\Theta}_v \mathbf{s}_v, \quad \mathbf{y}_i = \boldsymbol{\Theta}_i \mathbf{s}_i \tag{1}$$

where, $\boldsymbol{\Theta}_v = \mathbf{C}\boldsymbol{\Psi}_v$, $\boldsymbol{\Theta}_i = \mathbf{C}\boldsymbol{\Psi}_i$ and

$$\mathtt{Z} = \begin{bmatrix} \Re\{\mathbf{Z}\} & -\Im\{\mathbf{Z}\} \\ \Im\{\mathbf{Z}\} & \Re\{\mathbf{Z}\} \end{bmatrix}. \tag{2}$$

It is evident that (1) extends the typical CS problem formulation for DSSE [14], where the ℓ_2-norm component minimizes estimation error using the system model. From the estimated values of $\hat{\mathbf{s}}_v$ and $\hat{\mathbf{s}}_i$, the signal can be reconstructed as, $\Delta\hat{\mathtt{v}} = \boldsymbol{\Psi}_v \hat{\mathbf{s}}_v$ and $\hat{\mathtt{i}} = \boldsymbol{\Psi}_i \hat{\mathbf{s}}_i$. To derive a closed-form solution of (1), the formulation is simplified as:

$$\hat{\mathbf{s}} = \min_{\mathbf{s}} \|\mathbf{s}\|_1 + \|\mathbf{T}\mathbf{s}\|_2 \quad \text{s.t.}\, \mathbf{y} = \boldsymbol{\Theta}\mathbf{s} \tag{3}$$

where, $\mathbf{s} = [\mathbf{s}_v' \ \mathbf{s}_i']'$, $\mathbf{y} = [\mathbf{y}_v' \ \mathbf{y}_i']'$, $\mathbf{T} = [\boldsymbol{\Psi}_v \ \ -\mathtt{Z}\boldsymbol{\Psi}_i]$ and

$$\boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{\Theta}_v & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Theta}_i \end{bmatrix}. \tag{4}$$

Equation (3) can be reformulated using Lagrange multipliers as, $\left(\hat{\mathbf{s}}, \hat{\boldsymbol{\lambda}}\right) = \min_{\mathbf{s}, \boldsymbol{\lambda}} \|\mathbf{s}\|_1 + \|\mathbf{T}\mathbf{s}\|_2 + \boldsymbol{\lambda}(\boldsymbol{\Theta}\mathbf{s} - \mathbf{y})$. The analytical solution for the optimal value can be found by applying the first-order condition on this above reformulation, which is:

$$\nabla_{\boldsymbol{\lambda}} f \Rightarrow \boldsymbol{\Theta}\hat{\mathbf{s}} - \mathbf{y} = \mathbf{0} \tag{5}$$

$$\nabla_{\mathbf{s}} f \Rightarrow \boldsymbol{\Theta}'\hat{\boldsymbol{\lambda}} + \operatorname{sign}(\hat{\mathbf{s}}) + 2\mathbf{T}'\mathbf{T}\hat{\mathbf{s}} = \mathbf{0} \tag{6}$$

Since $\operatorname{sign}(\hat{\mathbf{s}})$ is non-linear and discontinuous, it is rewritten to facilitate its incorporation into a matrix representation:

$$\operatorname{sign}(\hat{\mathbf{s}}_j) = \frac{1}{|\hat{\mathbf{s}}_j|}\hat{\mathbf{s}}_j \tag{7}$$

The objective is to derive a closed-form solution for the CS problem and to achieve this, the solution $\hat{\mathbf{s}}$ from (1) is substituted to linearize the expression in (7). The resulting function can be represented as a diagonal matrix $\boldsymbol{\Delta} \in \mathbb{R}^{2\mathcal{E} \times 2\mathcal{E}}$, with its diagonal elements expressed as:

$$\boldsymbol{\Delta}_{jk} = \begin{cases} \dfrac{1}{|\hat{\mathbf{s}}_j|} & , \text{if } j = k \\ 0 & , \text{if } j \neq k \end{cases} \tag{8}$$

With this definition, the first-order conditions provided in (5)–(6) can be expressed in matrix form as:

$$\begin{bmatrix} \boldsymbol{\Theta} & \mathbf{0} \\ \boldsymbol{\Delta} + 2\mathbf{T}'\mathbf{T} & \boldsymbol{\Theta}' \end{bmatrix} \begin{bmatrix} \hat{\mathbf{s}} \\ \hat{\boldsymbol{\lambda}} \end{bmatrix} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0} \end{bmatrix} \tag{9}$$

Using the closed-form relationship between $\hat{\mathbf{s}}$ and $\mathbf{y}$ provided in (9) for the problem in (1), the noise statistics of the estimated value $\hat{\mathbf{s}}$ can be derived. These noise statistics are then utilized to design a LNR based bad data detector to identify any data anomalies. To facilitate the design of such a detector, the expression in (9) can be further simplified using the following definition:

$$\mathbf{D} \Rightarrow \begin{bmatrix} \mathbf{D}_{\mathbf{s},\mathbf{y}} & \mathbf{D}_{\mathbf{s},\mathbf{0}} \\ \mathbf{D}_{\boldsymbol{\lambda},\mathbf{y}} & \mathbf{D}_{\boldsymbol{\lambda},\mathbf{0}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Theta} & \mathbf{0} \\ \boldsymbol{\Delta} + 2\mathbf{T}'\mathbf{T} & \boldsymbol{\Theta}' \end{bmatrix}^{-1} \tag{10}$$

Using this block representation of $\mathbf{D}$, the reduced-order closed-form solution for the CS formulation in (1) can be expressed as $\mathbf{s} = \mathbf{D}_{\mathbf{s},\mathbf{y}}\mathbf{y}$. By defining $\tilde{\mathbf{D}} = \boldsymbol{\Psi}\mathbf{D}_{\mathbf{s},\mathbf{y}}$, the closed-form expression of the CS problem is used to establish the relationship between the measurements and their estimates as:

$$\left[\Delta\hat{\mathtt{v}}' \ \ \hat{\mathtt{i}}'\right]' = \tilde{\mathbf{D}}\left[\mathbf{y}_v' \ \ \mathbf{y}_i'\right]' \tag{11}$$

B. Derivation of Estimation Error Statistics

To quantify the presence of noise, let $\boldsymbol{\eta}$ represent a gaussian random variable with zero mean and covariance matrix $\boldsymbol{\Sigma}$. Considering that $\bar{\mathbf{y}}_v$ and $\bar{\mathbf{y}}_i$ are the true values of the voltage and current measurements, which can be expressed as:

$$\begin{bmatrix} \mathbf{y}_v' & \mathbf{y}_i' \end{bmatrix}' = \begin{bmatrix} \bar{\mathbf{y}}_v' & \bar{\mathbf{y}}_i' \end{bmatrix}' + \boldsymbol{\eta}, \quad \boldsymbol{\eta} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}) \tag{12}$$

With the closed form expression given in (11), the estimation error can be quantified as:

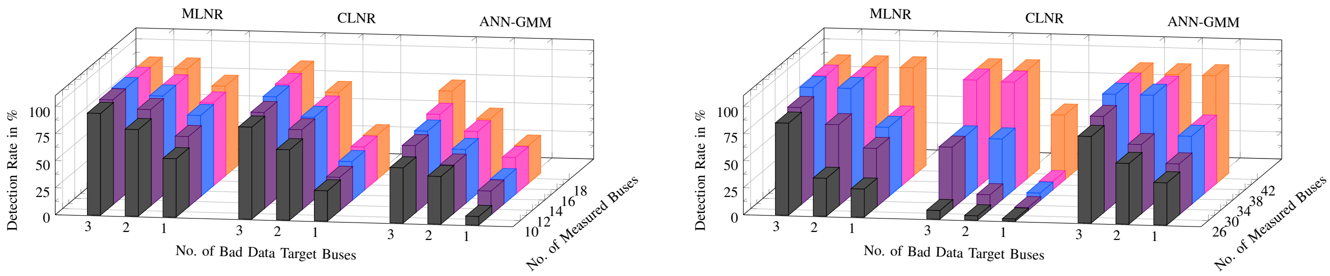
$$\begin{bmatrix} \Delta\hat{\mathtt{v}} \\ \hat{\mathtt{i}} \end{bmatrix} = \begin{bmatrix} \Delta\bar{\mathtt{v}} \\ \bar{\mathtt{i}} \end{bmatrix} + \boldsymbol{\mu}, \quad \boldsymbol{\mu} \sim \mathcal{N}\left(\mathbf{0}, \tilde{\mathbf{D}} \cdot \boldsymbol{\Sigma} \cdot \tilde{\mathbf{D}}'\right) \tag{13}$$

where $\Delta\bar{\mathtt{v}}$ and $\bar{\mathtt{i}}$ represent the true state values of the voltage deviations and current injections, respectively. Since $\Delta\bar{\mathtt{v}}$ and $\bar{\mathtt{i}}$ are not directly available, the system model is integrated into the LNR test designed to identify anomalous measurements. The relationship $\Delta\bar{\mathtt{v}} = \mathtt{Z} \cdot \bar{\mathtt{i}}$ can then be rewritten as $\mathtt{T}\left[\Delta\bar{\mathtt{v}}' \ \ \bar{\mathtt{i}}'\right]' = \mathbf{0}$ where $\mathtt{T} = [\mathbf{I} \ \ -\mathtt{Z}]$ and $\mathbf{I}$ is an identity matrix.

Through the linear transformation on the CS estimates with $\mathtt{T}$, the mean value of the result becomes zero. This resulting residual, $\mathtt{r} = \mathtt{T}\left[\Delta\hat{\mathtt{v}}' \ \ \hat{\mathtt{i}}'\right]' = \mathtt{T} \cdot \tilde{\mathbf{D}} \cdot [\mathbf{y}_v' \ \ \mathbf{y}_i']'$ can be used as the test statistic for the LNR test. With zero mean, the distribution of $\mathtt{r}$ is given as $\mathtt{r} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega})$ where, $\boldsymbol{\Omega} = \mathtt{T} \cdot \tilde{\mathbf{D}} \cdot \boldsymbol{\Sigma} \cdot \tilde{\mathbf{D}}' \cdot \mathtt{T}'$.

C. Modified LNR (MLNR) Approach

The residual covariance matrix $\boldsymbol{\Omega}$ has a rank of $|\mathcal{M}|$, since its rows and columns are linearly dependent on those corresponding to the set $\mathcal{M}$. Let, $\tilde{\boldsymbol{\Omega}} = \{\boldsymbol{\Omega}_{jk} \mid j \in \mathcal{M}, k \in \mathcal{M}\}$ be the positive definite sub-matrix of $\boldsymbol{\Omega}$ that includes only the rows and columns associated with $\mathcal{M}$. Similarly let, $\tilde{\mathtt{r}} = \{\mathtt{r}_j \mid j \in \mathcal{M}\}$ be the sub-vector of $\mathtt{r}$ corresponding to $\mathcal{M}$. The error statistics of the CS estimates is utilised to normalise the residuals which is further used to test the presence the bad data in the measurements. A conventional LNR approach [3] normalizes the residual $\tilde{\mathtt{r}}$ using only the diagonal elements of the covariance matrix $\tilde{\boldsymbol{\Omega}}$. This conventional approach, however, overlooks the off-diagonal elements of $\tilde{\boldsymbol{\Omega}}$, which can significantly affect the accuracy of bad data detection. To address this limitation, a modified LNR approach is proposed, incorporating eigen-decomposition to diagonalize the sub-matrix $\tilde{\boldsymbol{\Omega}}$. This technique normalizes the distribution of $\tilde{\mathtt{r}} \sim \mathcal{N}(\mathbf{0}, \tilde{\boldsymbol{\Omega}})$ more effectively. Let $\mathbf{Q}$ denote the matrix



(a) 37-bus system (b) 123-bus system

Fig. 1: Detection Rate with CLNR and MLNR approaches.

TABLE I: Comparison of computation time

(a) 37-bus system

No. of Measured Buses	10	12	14	16	18
ANN Training Time (s)	3.4581	3.3306	3.6042	3.796	3.3655
GMM Estimation Time (s)	0.1797	0.1564	0.1165	0.1127	0.0973
MLNR Detection Time (s)	0.7263	0.764	0.715	0.8168	0.7393

(b) 123-bus system

No. of Measured Buses	26	30	34	38	42
ANN Training Time (s)	11.588	11.097	10.642	20.035	20.763
GMM Estimation Time (s)	0.1131	0.384	0.5722	0.6282	0.2615
MLNR Detection Time (s)	4.4636	4.631	4.3483	5.8205	5.3156

of eigenvectors and $\mathbf{\Lambda}$ be the diagonal matrix of eigenvalues of $\tilde{\mathbf{\Omega}}$. Consequently, $\tilde{\mathbf{\Omega}}$ can be expressed as $\mathbf{\Lambda} = \mathbf{Q}' \cdot \tilde{\mathbf{\Omega}} \cdot \mathbf{Q}$. Finally, the normalized residuals can be given as:

$$\mathtt{r}^N = \sqrt{\mathbf{\Lambda}^{-1}} \cdot \mathbf{Q}' \cdot \tilde{\mathtt{r}}, \quad \mathtt{r}^N \sim \mathcal{N}(\mathbf{0}, \mathbf{1}) \tag{14}$$

To account for 99.99 % of the normally distributed noise component in the measurements, a threshold of 4 is used. With this consideration, the test condition for detecting the presence of bad data is $\max \left\{ \left| \mathtt{r}_j^N \right|, j \in [1, |\mathcal{M}|] \right\} > 4$.

III. Simulation Results

To evaluate the performance of the proposed CS based MLNR approach for bad data detection in DSSE, simulations were conducted on the IEEE 37-bus and 123-bus unbalanced distribution systems [24]. Load variability was set at 30% from the base condition to account for real-world fluctuations in load values. The measurements used consists of nodal voltage magnitudes and real power injections, communicated to the control station. The number of measured buses was varied, starting from 10 in 37-bus system and 26 in 123-bus system, with bus locations selected randomly. This low number of measurements rendered the systems unobservable under conventional WLS-based state estimation, reflecting real-world conditions. To simulate typical measurement imperfections, a 1% error tolerance was applied [25].

A. Detection Performance of the Proposed Approach

The method described in [16], using principal component analysis, was employed to identify the sparsifying bases for both test systems. Bad data was injected at randomly selected target buses, varying between 1 and 3 buses, by flipping the sign of the measured values. This method simulates common human errors that occur during network servicing. The proposed MLNR approach and conventional LNR (CLNR) method were tested against the ANN-based approach for bad data detection developed in [9], [10]. For training the ANN, in both test systems, 17,520 time steps of load flow converged solutions with random loads were obtained through Monte Carlo simulations. As described in [9], [10], the error statistics of the estimates were obtained from a Gaussian mixture model (GMM), which was used for LNR-based bad data detection. The simulations were executed in MATLAB, with 1,000 Monte Carlo simulations performed for both systems under scenarios with and without bad data injection. No false alarms were observed in any scenario, so those results were excluded for brevity. The detection rates for the composite ANN-GMM approach [9], [10] along with the CLNR & MLNR approaches that employ the proposed closed-form solution of the CS problem are shown in Fig. 1 (a) & (b) for the 37-bus and 123-bus systems, with varying numbers of measured and target buses (1 to 3) under bad data injection. The CLNR method showed lower detection rates, especially with fewer measurements or small injection magnitudes of bad data, with this effect being more pronounced in the 123-bus system as compared to 37-bus system. The detection rates of the ANN-GMM approach were relatively lower compared to the MLNR approach for the 37-bus system, whereas both MLNR and ANN-GMM approaches exhibited similar detection rates for the 123-bus system. The results demonstrate that the proposed MLNR approach can detect significant amounts of bad data injection with over 90% detection rates, even when the number of measurements is only 25% of the total number of buses.

B. Computational Complexity and Execution Time Analysis

The computational complexity of the proposed CS-based estimation method is $\mathcal{O}(m^2 n^3)$ with a convex approach and $\mathcal{O}(nm^2)$ with the bayesian approach [26]. In contrast, training ANNs with ReLU activation is NP-hard [12], with a complexity of $\text{poly}(n^{2^t})$, where $\text{poly}(x) = 2^{\mathcal{O}(\log(x))}$ [27]. Tables I (a) and (b) provide execution time comparisons between the proposed MLNR approach and the composite ANN-GMM approach [9], [10]. It is observed that, the proposed approach takes, on average, one-third of the time required to train ANN. The computational burden of ANN-based approaches will be significant where periodic retraining with the latest measurements is required in order to adapt to changes in network topology and system dynamics [11]. In contrast, the proposed approach leverages system models, avoiding the need for retraining and demonstrating significantly lower computational effort, making it well-suited for real-time applications.

IV. Conclusion

In this letter, we introduced a method for detecting bad data in DSSE by combining CS with a MLNR-based detector. The closed-form solution for the CS problem enables accurate error quantification in CS-based DSSE estimates. Using these error statistics and eigen decomposition, the MLNR detector effectively detects bad data, especially in systems that are unobservable. Simulations on IEEE test systems demonstrate the superior performance of this approach in terms of detection rate and computational complexity compared to CLNR and ANN-GMM approaches. Specifically, the MLNR approach consistently achieves detection rates exceeding 90%, even under low observability conditions, while maintaining significantly lower computational complexity than ANN-based techniques. The proposed method is particularly applicable in distribution systems with limited measurement points, where obtaining training dataset is a challenge. It is especially relevant in scenarios involving high penetration of DERs, such as PV systems and electric vehicles, which introduce variability and uncertainty into load forecasts. Future work may include hybridizing model-based and training-based approaches, as well as extending this method to other distribution management functions and real-time optimization tasks.

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