Chandrasekhar-Friedman-Schutz instability windows and Whitney's umbrella

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Introduction

The CFS instability is commonly accepted nowadays as one of the main triggers of gravitational radiation from single neutron stars that is the next goal for the existing (LIGO, Virgo) and planned (LISA) detectors of gravitational waves. Perturbation theory for eigenvalues in combination with numerical methods was proposed for calculation of instability windows of the CFS instability already in [1, 2]. We are extending this approach to the multiple-parameter case by adopting a new methodology to get better approximations and to put the CFS to the general context of the dissipation-induced instability theory.

Instability windows

One interesting feature of the CFS mechanism is to be able to relate the spin of a rotating neutron star directly to its temperature following a scattering law. The computation of such instability windows is of high interest for the astrophysical community and the next possible detection of gravitational waves from a single source.

While lots of authors have proposed different versions of stability domains including additional dissipative effects or coupling between modes, our approach is classical and rely on the fundamental $m = 2$ mode of the Maclaurin spheroids presented by Chandrasekhar [1].

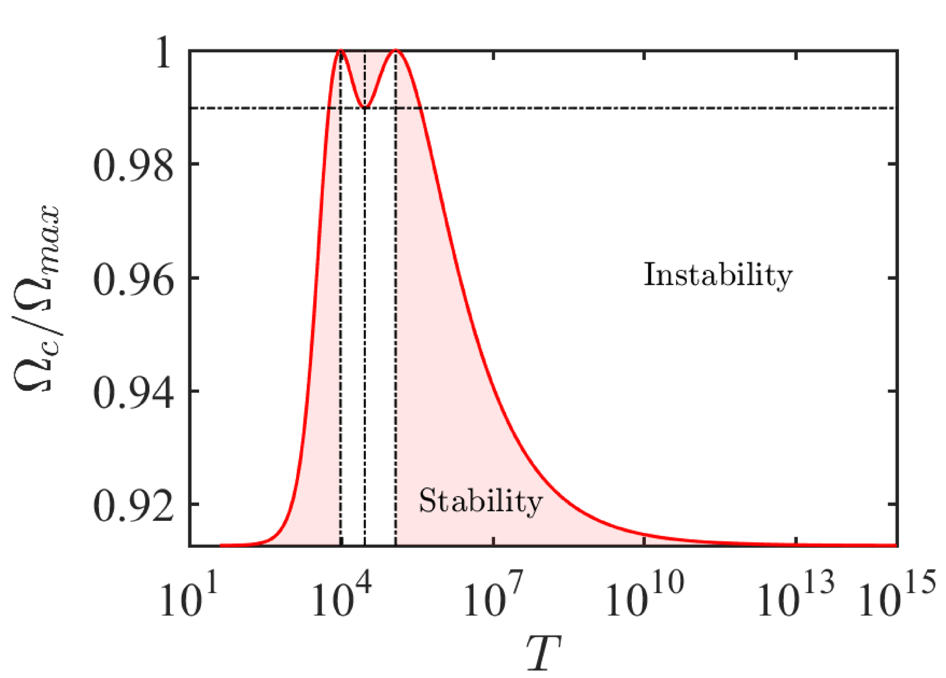

Figure 1: Instability windows of a Maclaurin spheroid with $1.5M_\odot$ and a radius of 17.171km, subject to viscous dissipation and GW emission.

Dynamical and secular instabilities

In the undamped system, the characteristic polynomial $p(\lambda) = \det \mathcal{L}(\lambda)$ can be solved analytically as

$$\lambda_0^\pm = i\Omega \pm i\sqrt{4b-\Omega^2}, \qquad (1)$$

where Ω and b are two functions of the eccentricity of the ellipsoid [1].

We can determine the point of neutral stability (the same point where the Maclaurin sequence bifurcates to the Jacobi family as Meyer and Liouville shown) from (1). At this point $\lambda = 0$ and the critical eccentricity is equal to $e_L = 0.812670...$, which is a solution of the equation $4b(e) = 2\Omega^2(e)$.

The critical eccentricity e_0 at the point of dynamical instability (established by Riemann) is a root of the equation $4b(e) = \Omega^2(e)$, yielding $e_0 = 0.952886...$

Movements of eigenvalues

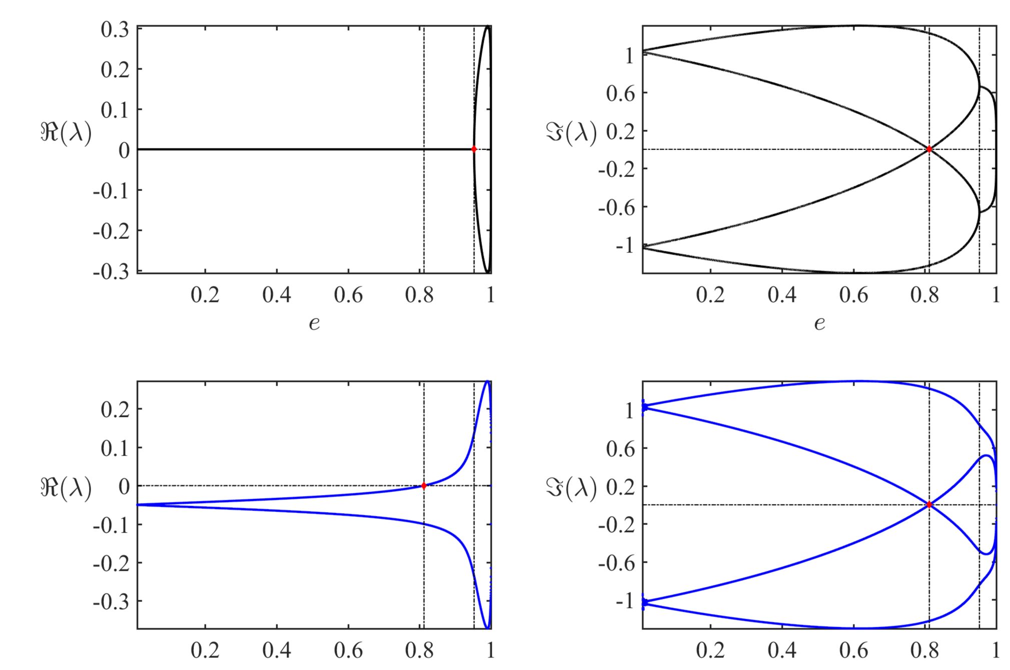

Figure 2: Spectrum of the ideal and viscous Maclaurin spheroids related to the eccentricity. Red dots represent the onset of either dynamical or secular instability.

Important Result

The neutral stability surface at the Riemann eccentricity leads to a topological singularity known as the Whitney's umbrella. The Liénard-Chipart criterion does not require to be set in the viscinity of dissipation and extend therefore the previous results [2] to the whole range of damping coefficients.

Stability analysis

The linear stability of the Maclaurin spheroids is governed by a system of two linear ordinary differential equations with constant coefficients [1], which yields the following matrix polynomial

$$\mathcal{L}(\lambda) = \lambda^2\mathcal{M} + \lambda(\mathcal{G}+\mathcal{D}) + \mathcal{K} + \mathcal{N}(\lambda) = 0, \qquad (2)$$

where $\mathcal{M}, \mathcal{K}$ are symmetric mass and stiffness matrices, $\mathcal{G}$ is a skew-symmetric gyroscopic matrix and $\mathcal{D}$, $\mathcal{N}$ represent nonconservative effects due to viscous dissipation and gravitational radiation reaction.

Let consider the Maclaurin series of a simple root of the characteristic polynomial $p(\lambda) = \det \mathcal{L}(\lambda)$ in the neighborhood of one of the four roots $\pm\lambda_0^\pm$ computed in (1) as $\lambda \approx \lambda_0 + \Delta\lambda$

$$\Delta\lambda_0^\pm = \left.\frac{\partial\lambda^\pm}{\partial\mu}\right|_{\mu,\delta=0}\mu + \left.\frac{\partial\lambda^\pm}{\partial\delta}\right|_{\mu,\delta=0}\delta + o(\mu,\delta).$$

The latter leads to the following expression [2]

$$\Delta\lambda_0^\pm = \frac{2i\delta}{5(\Omega+i\lambda_0^\pm)}\left[(\lambda_0^\mp)^5 - \chi\Omega_0^4\lambda_0^\pm\right] + o(\mu,\delta),$$

where

$$\chi = \frac{25}{2\Omega_0^4}\frac{\mu}{\delta},$$

and Ω_0 is the Maclaurin angular velocity evaluated at the Riemann eccentricity.

Setting the increment to be identically zero and some algebraic manipulations later, we find the following quadratic expression for the variable χ

$$\chi^2 + \chi\frac{16b(3\Omega^4 - 12b\Omega^2 - 4b^2)}{\Omega_0^4(\Omega^2-2b)} + \frac{16(\Omega^2-2b)^4}{\Omega_0^8} = 0.$$

Applying the well-known Liénard-Chipart criterion on the 10^{th}-order characteristic polynomial, it leads to the following stability criterion for the system

$$Q = -\frac{\Omega^2-2b}{512}\left\{\left(\frac{25\mu}{2}\right)^2 + 16(\Omega^2-2b)^4\delta^2 + \frac{16b(3\Omega^4-12\Omega^2 b-4b^2)}{\Omega^2-2b}\frac{25\mu\delta}{2}\right\} > 0. \qquad (3)$$

The equation in brackets in the latter expression reduces exactly to the result of the perturbation theory.

Stability domain

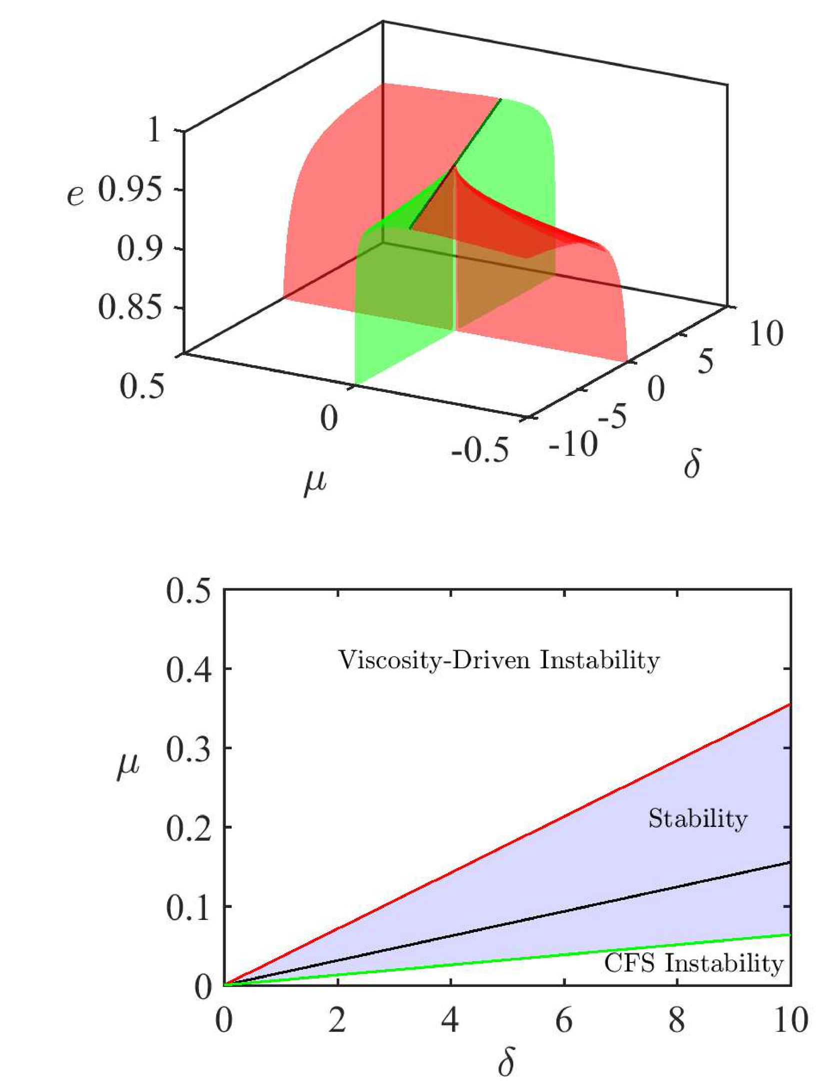

Figure 3: Neutral stability surface (Whitney's umbrella) and stability domain of the CFS mechanism from Liénard-Chipart criterion (3). The system is dominated by viscosity (respectively, GW emission) in the red (respectively, green) domain. The black line correspond to $\chi = 1$.

References

[1] Chandrasekhar, S. Yale Univ. Press, 1969; *Phys. Rev. Lett.*, **24**(11), 611 (1970); *Astrophys. J.*, **161**, 561 (1970).

[2] Lindblom, L. & Detweiler, S. L. *Astrophys. J.*, **211**, 565 (1977).

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