

# Self-Propelled Nanofluids a Coolant Inspired from Nature with Enhanced Thermal Transport Properties

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## Abstract

Self-propelled nanofluids (SPNFs), are suspensions that contain active particles, that self-propel by converting some form of energy to mechanical work. This [theoretical](#) investigation considers the heat transfer mechanisms that may exist in an SPNF. Equations describing the effective mass diffusivity of spherical and rod-shaped particles

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were taken from the literature and used in analysis of particles of different shapes, aspect ratios, swimming velocities and suspended in water and in ethylene glycol. The analysis showed that the effective mass diffusivity of the particles was up to three orders of magnitude higher than the thermal diffusivity of the solvent. The enhancement of the mass diffusivity leads to a significant enhancement of thermal conductivity of up to an order of magnitude higher compared to that of the pure solvent. It is further discussed that in SPNFs, dispersion and clustering of particles and active turbulence mechanisms may add extra enhancement to the thermal transport. **Recent experimental investigations supporting our findings are discussed.** Combining this enhancement in thermal transport with the reported reduction of the viscosity observed for an SPNF consisting of Artificial Bacterial Flagella (ABF) particles, will help to create a highly efficient coolant. This can help to reduce energy consumption in a wide variety of the economic sectors, especially in the computing and data storage.

**Keywords:** self-propelled particles, heat transfer enhancement, nanofluid, coolant, active colloids, energy efficiency, self-propelled nanofluids, thermal conductivity enhancement

## 1 Introduction

**Humanity now faces a well-defined intertwined problem of population increase and increasing energy consumption.** Recently, an alarming finding by [1] shrinks the time available for the global human community to reduce its consumption of energy based on the fossil fuels and move to a more efficient and less energy consuming economy. Only this will avoid moving the planet earth to a state of "hothouse earth" where the average temperature will be about 4-5  $C^\circ$  above that of pre-industrial times, and sea levels will be 10-60 m higher than today. To achieve

this goal we need to develop innovative technologies to reduce energy consumption in every sector of the economy, including transportation, building technologies, computing, process engineering, and medical technologies. Developing an efficient coolant will contribute significantly to the reduction of energy consumption, especially in the areas of computing and data storage that now constitute the backbone of the social-economic activities in most developed countries. With computing, we can explore nature, develop new tools, and communicate with each other. Meanwhile, data storage, contains all our stories, memories, and intellectual achievements, and acts as a hologram of our current civilization. However, computing and data storage devices dissipate an enormous amount of heat which hampers their performance and development. **Also, the increase in energy dissipation is linked to an increase in energy consumption; for example in 2012 the energy consumed by computing devices world wide was about 300 TWh, about 1.5% of the global energy consumption [2].** However, the alarming point is that the rate of increase of computing energy consumption is faster than any other sectors of the economy [2].

When it comes to the thermal management of electronic devices, the two most important mechanisms that play a role in a successful cooling system is minimizing the thermal resistance from the substrate (chip), and increasing the convection power of the fluid to which chip is exposed. The first mechanism can be optimized by increasing the thermal conductivity of the substrate by manipulating its nanostructure [3, 4]. **However, since there is an increasing number of transistors, increasing the cooling rate by increasing the thermal conductivity of the substrate has begun to reach its limit, [5] .**

With respect to the convective cooling mechanism, there are two avenues used for thermal management. One is single-phase convection, [6–8], which is mainly a forced convection of single phase fluid such as water or air. The second avenue

is two-phase convective cooling. This can achieve lower pressure drops than single phase cooling, but the flow can become unstable due to the formation of bubbles at the first instance of boiling [9,10]. It is therefore generally preferred to stay in the single phase convective regime. However, the limitation on the performance of the single phase convective cooling is the poor transport properties of the cooling fluid such as thermal conductivity, and high viscosity.

To tackle the problem of poor thermal properties Choi and Estman [11] suggested suspending highly conductive particles into liquids. They coined the resulted colloidal suspension as a "nanofluid", and the idea attracted the attention of the international scientific community, because of its potential to replace the available heat transfer fluids. However, the idea of nanofluids suffered from a flaw, in that enhancing heat transfer by increasing the concentration of the particles is coupled with an increase in the viscosity, that increases the pumping power needed to circulate the fluid, and also hampers any convective mixing effects . This behaviour can be explained by using the Maxwell equation [12] for thermal conductivity, and the Einstein equation [13] for viscosity. Both equations are applicable for cases of dilute colloidal suspensions where the interactions between the particles are negligible. These relationships show an increase in both thermal conductivity, and viscosity with an increase in the volume fraction of the particles. International benchmarking studies have confirmed [14,15] that the thermal conductivity and viscosity of a nanofluid follow the Maxwell and Einstein equations. The conclusion from the predominant findings in the literature is that the increase in the viscosity is much higher than that of the thermal conductivity, and thus in practice nanofluids cannot meet the criteria put by [16], which states that the ratio between the enhancement of the viscosity and that of the thermal conductivity must not exceed that of 4.0 in order for a nanofluid to be useful in a large scale application.

An ideal coolant (i.e. heat transfer fluid) will have a high thermal conductivity and heat capacity, and a low viscosity. We very recently proposed a coolant inspired by nature, that possesses those characteristics [17]. We coined it as a " Self-Propelled Nanofluid". (SPNF) belong to the active suspension family, and consists of a suspension of a specific type of self-propelled particles that have similar shape and dimensions to that of the natural flagella of E-coli bacteria; they are known as artificial bacterial flagella (ABF) [18]. Those particles are self-propelled by using a weak magnetic field, which helps to control their movement in an effective way, not only for a single particle but also as a swarm of particles [18]. We choose this specific design of self-propelled particles because there is a solid evidence in the literature that suspensions that contain bacteria such as E-coli which swim as a pusher have the ability to reduce the effective viscosity of the surrounding medium to values even lower than that of the solvent alone; this evidence comes from experiments and theory [19,20].

In our work, we modelled the thermal conductivity of an SPNF to increase with volume fraction according to Maxwell's law, while the viscosity decreased with the volume fraction according to a constitutive phenomenological law for self-propelled suspensions [17]. What we found was that the significant reduction of the viscosity led to the increase of convection currents, resulting in an enhancement of the heat transfer of about three times compared to that of pure water with a meager volume fraction of the particles. However, SPNFs can contain any type artificially made self-propelled particles in principle, but there is no guarantee that all types of particles can generate the viscosity reduction mechanism observed in bacterial suspensions, because of lack of understanding of the rheological behavior of self-propelled suspensions.

Evidence has started to appear that the viscosity reduction mechanism can be

achieved not only by rod shape particles but also through a spherical shape [20]. In our previous paper [17] we did not answer the fundamental question” what are the heat transfer mechanics that can emerge if a self-propelled particle is suspended in a liquid?” Answering this inquiry is the main subject of this paper.

Before we proceed further, we offer some further considerations regarding self-propelled particles, and the possibilities that they suggest. Self-propelled particles are in general particles that they have a prescribed motion. This prescribed motion is a result of transforming some form of energy into mechanical work. One of the great examples that exists in biology is that of the protein motors that are self-propelled by consuming adenosine triphosphate (ATP) [21]. Most synthetic self-propelled particles are designed to operate within a liquid fluid, examples including colloids and nanoparticles [22] as well as micro-robots programmed to interact with their neighbours according to certain rules [23]. Self-propelled particles do not need to use a magnetic field for propulsion; a plethora of alternatives are available. For example self-propulsion can be achieved as a response to the gradient of a chemical species such as hydrogen peroxide [22], to an electrical field [24], to light [25], or to a thermal field [26]. These different propulsion mechanisms can achieve very different swimming velocities; for example self-propelled particles that are submerged in water and driven by hydrogen bubbles achieve swimming velocities of up to 3 mm/s [27], while self-propelled particles that are driven by a magnetic field can achieve swimming velocities of only about 40  $\mu\text{m/s}$  [28]. The performance of self-propelled particles will be bounded by their shape, the fluid which they swim in, and their propulsion mechanism [29].

This research presented here explores the heat transfer mechanisms that are associated with suspending active particles in fluids. In the first part of the paper consideration is given to estimating the diffusion coefficient for self-propelled particles for different particle shapes, solvents, and swimming speeds. The re-

sults of the diffusion are then used to calculate the thermal conductivity from the movement of the particles. In the remaining part of the paper, we explore the effects of mechanisms such as particle dispersion, and clustering on the heat transfer enhancement of the SPNs. Also, we will introduce the phenomena of active turbulence that is unique to active suspensions. **Finally we present experimental evidence from literature that suspending active particles will enhance the heat transfer rate compared to a pure solvent.**

## 2 Mass Diffusivity

The mass diffusivity, quantified by the diffusion coefficient, is an essential property of active and non-active colloidal suspensions that helps to elucidate their behaviour. Here, we illustrate the behaviour of the diffusion coefficient for dilute conditions (volumetric concentration  $\phi \ll 1.0$ ) of self-propelled particles for different solvents and geometrical shapes of the particles. **In this investigation, we do not consider the effect of bubble or wake formation due to the particle propulsion, or any liquid volume change around the particle, since the liquids that we consider are incompressible .**

If a micro size self-propelled particle is suspended in a viscous Newtonian solvent, it will experience two types of motion. One is translation motion due to the constant imposed swimming velocity, and the other one is tumbling due to Brownian rotational motion. The swimming velocity of the particle is defined as  $U_s$ , and it is tumbling with reorientation time  $\tau_R$ . In Ref [30] Takatori and Brady defined the mass diffusion coefficient, and the reorientation time for a single particle regardless of its shape, as:

$$D^{eff} = D_{Brownian} + D_{Swim} = \frac{K_B T}{\zeta} + \frac{U_s^2 \tau_R}{6.0} \quad (1)$$

$$\tau_R = \frac{1.0}{D_R} = \frac{\zeta_R}{K_B T} \quad (2)$$

In Eq.(1) the effective diffusivity  $D^{eff}$  of the self-propelled particle is composed of

two parts. The first part is due to the Brownian motion of the particle ( $D_{Brownian}$ ), that results from the constant bombardment of the particle by the surrounding fluid molecules. It is given by the classical Stokes-Einstein-Sutherland equation, where  $K_B$  is the Boltzmann constant,  $T$  is the temperature of the suspension, and  $\zeta$  is the transitional hydrodynamic drag resistance of the particle that depends on its geometry, and the viscosity of the medium.

The second part  $D_{Swim}$ , represents the effect of the swimming velocity of the particle on the diffusivity, referred to as the swimming diffusivity.  $D_{Swim}$  is related to the diffusivity of the random-walk particle which in general scales with  $\sim \frac{l_s^2}{\tau_R}$ ,  $l_s$  [30] being the swimmer running length that it is equal to  $U_s \tau_R$ . Here  $\tau_R$  is equal to the inverse of the Brownian rotational diffusivity  $D_R$ , and  $\zeta_R$  is the rotational hydrodynamic resistance of the particle. For further details about the mathematical derivation of the  $D_{Swim}$ , Ref [30] is an excellent resource.

For spherical shape particles, the effective swimming diffusivity is given by the following relations:

$$D_{Sph}^{eff} = \frac{K_B T}{6\pi\eta a} + \frac{U_s^2 \tau_R}{6.0} \quad (3)$$

$$\tau_{R_{Sph}} = \frac{8.0\pi\eta a^3}{K_B T} \quad (4)$$

Where  $a$  is the radius of the spherical particle. For the case of the rod-shaped ABF particles, the variation of the effective diffusivity is given by:

$$D_{rod}^{eff} = \frac{K_B T (\log [2p_a] - 0.5)}{2\pi\eta l} + \frac{U_s^2 \tau_R}{6.0} \quad (5)$$

$$\tau_{R_{rod}} = \frac{\pi\eta l^3}{3.0 K_B T (\log [2p_a] - 0.5)} \quad (6)$$

The relations for the hydrodynamic and rotational drag resistance are obtained from [31].  $l$  represents the length of the rod, and  $p_a$  its aspect ratio. Rod particles are not axi-symmetric, and thus their hydrodynamic resistance is dependent on the orientation of the particle with respect to the flow. Here, for simplicity,

we consider only the case where the particles are aligned with the flow direction. Thus the values of  $\zeta$ , and  $\zeta_R$  are for the case where the major axis of the particle is parallel to the flow direction. Also, for the current study, inertial effects are neglected, since it is assumed that fluid flow around the particles is in the Stokes regime where the Reynolds number is sufficiently small ( $Re \ll 1.0$ ) to allow this.

We consider two different solvents, water and ethylene glycol, each of which is widely used for industrial applications, and we plot the effect of Eqs(3) to (6). ). For the case of the spherical self-propelled particles immersed in water, the variation of the  $D_{Sph}^{eff}$  is shown in Figure 1 for different particle sizes and swimming velocities. Two regimes are clear. In the first, the effective diffusivity decreases as the particle size increases. This regime applies when the particle size is small such that Brownian motion plays a major role, and the diffusivity of the particle is controlled by its Brownian component. In the Brownian regime, the particles reorient in a short time, as reflected by the cubic dependence of the reorientation time on the particle size. This means that the Brownian component of the diffusivity dominates over that of swimming diffusivity. This behaviour is similar to the behaviour of non-active particles at the colloidal size, which rely only on Brownian motion for their transport.

In the second non-Brownian regime which starts when a minimum effective diffusion coefficient is reached, the diffusivity starts to increase as the particle size increases. In this regime, the particle needs longer to reorient, which gives rise to the dominance of the swimming diffusivity component of the effective diffusivity. There is a substantial enhancement of the effective diffusivity in the non-Brownian regime, and this enhancement increases with increasing swimming velocity. For example, at the lowest swimming velocity considered ( $U_s = 10\mu\text{m/s}$ ), and particle radius of  $10^{-6}\text{m}$  the diffusivity is enhanced about ten times compared with that of a particle with radius  $10^{-9}\text{m}$ . Meanwhile, for the case of  $U_s = 200\mu\text{m/s}$ , the

effective diffusivity can increase up to 1000 times if the same previous particle sizes are compared. Another interesting aspect shown in Figure 1 is that at a certain particle size and swimming velocity the effective mass diffusivity of the particle is higher than the thermal diffusivity of the water ( $\alpha = 1.5 \times 10^{-7} \frac{m^2}{s}$ ). Having a mass transport coefficient higher than the thermal transport coefficient will enhance the thermal transport [32, 33], as will be shown in the following section, because the two processes are coupled.

By suspending self-propelled spheres into ethylene glycol, a solvent that has a viscosity around an order of magnitude higher than that of water, their effective diffusivity is enhanced significantly, as shown in Figure 2. Increasing the viscosity of the solvent it extends the non-Brownian regime where the mass diffusivity is enhanced significantly to lower particle sizes compared to the case of water. Also, for all swimming velocities except that of  $10 \frac{\mu m}{s}$  the mass diffusivity of the particles exceeds the thermal diffusivity of the solvent at particle sizes approaching  $\mu m$ . For example, at  $U_s = 200 \frac{\mu m}{s}$ , and a particle diameter of  $1 \mu m$  the mass diffusivity is about two orders of magnitude higher than the thermal diffusivity, which will significantly enhance the thermal transport process.

The substantial enhancement of the effective mass diffusivity of the self-propelled particles swimming in ethylene glycol, compared with that of water is because of the increase in the viscosity of the medium. As Eq.(4) shows the reorientation time will increase as the viscosity of the medium increases. The reorientation time is the time needed for the particle to change its orientation or direction. For higher values of orientation time, the particle will experience longer translational strokes, and thus its mass diffusivity will increase.

Rod-shaped ABF were considered because of their viscosity reduction characteristics [19, 20, 34], again using water and ethylene glycol as working solvents, using

two different lengths for the rod particles ( $l = 1.0 \times 10^{-7}$  and  $l = 5.0 \times 10^{-6}m$ ) and various aspect ratios. Results are according to Eq.(5) and Eq.(6). Note that in the analysis, the length of the particle was kept fixed, and the diameter changed to give changing aspect ratios.

For the case of rod particles ( $l = 1.0 \times 10^{-7}$ ) suspended in water, the variation of the effective diffusivity coefficient for the different aspect ratio of the particles is shown in Figure 3. The overall behaviour is similar to that of the spherical particles, in that the diffusion coefficient increases with increase of the swimming velocity. However, the variation of the diffusion coefficient with aspect ratio shows two distinct behaviours. The first is that the diffusion coefficient decreases with increase of the aspect ratio, until a certain point, after which it reaches an asymptotic value. The second behaviour is observed for the lower swimming velocity used, of only ( $U_s = 10\mu m/s$ ). Here, the diffusion coefficient increases with aspect ratio until it reaches an asymptotic value.

We observe the same Brownian and non-Brownian regimes as for spherical particles; particles of lower aspect ratio are in the non-Brownian regime since their larger diameter gives rise to a longer reorientation time. All the values of the effective mass diffusivity coefficient are lower than the thermal diffusivity of the water, by about three orders of magnitude. Also, the values of effective diffusivity for the case of rod particles ( $l = 1.0 \times 10^{-7}m$ ) are lower than those of spheres with the same radius. This shows that for low viscosity fluids, choosing a particle less than  $1 \mu m$  in length will not provide a significant mass transport enhancement. If the particle length increases to  $5 \times 10^{-6}\mu m$  a substantial enhancement in the diffusion coefficient is achieved, as shown in Figure 4. The diffusion coefficient decreases with increasing aspect ratio until it reaches an asymptotic value, and it increases as the swimming velocity increases. Except for the case of  $U_s = 10\mu m/s$ , all the other cases show that for a specific range of aspect ratios ( $1 \leq p_a \leq 10$ ), the values of the effective mass diffusivity are higher than that of the thermal diffusivity of

the solvent. This range of aspect ratios, in which the diffusivity is higher than the thermal diffusivity, increases as the swimming velocity increases. With the case of  $U_s = 200\mu m/s$ , the values of the effective diffusivity are higher for the whole range of  $p_a$  considered in the current investigation, with the diffusivity being around two orders of magnitude higher than the thermal diffusivity of pure water.

For the case of suspensions with ethylene glycol as the suspending medium, and rod-shaped ABF particles with  $l = 1 \times 10^{-7}m$ , the variation of the effective diffusivity with aspect ratio and swimming velocity is shown in Figure 5. The trend is similar to that of the water, with the effective diffusivity decreasing with the aspect ratio. However, the values of diffusivity are higher than those of water suspensions. Increasing the size of the particles enhances the effective mass diffusivity significantly compared with all other cases considered in the current investigation, as shown in Figure 6. The enhancement is so significant that even for the lowest swimming velocity ( $U_s = 10\mu m/s$ ) the effective mass diffusivity is higher than the thermal diffusivity for a narrow aspect ratio range. For the other swimming velocities considered, the effective diffusivity is always higher than that of the thermal diffusivity of water for the whole range of the aspect ratios, with the effective diffusivity being three orders of magnitude higher than thermal diffusivity for the case of the highest swimming velocity considered. This shows that suspensions of rod particles with  $\mu m$  length, suspended in high viscosity fluids, can achieve very high mass diffusivities, even for medium swimming speeds. This feature will have significant benefits for heat transfer applications.

### 3 Heat Transfer Enhancement Mechanisms

This section discusses possible heat transfer mechanisms related to SPNFs. So far as the authors are aware, there is no reported investigation into the heat transfer mechanisms that are associated with the active suspensions. Therefore we revisit

the heat transfer mechanisms that are established for the nanofluids, and we consider the active turbulence mechanism that is related to active colloidal suspensions and may lead the enhancement of heat transfer.

### 3.1 Particle motion (diffusion)

The motion of particles plays an essential role in the heat transfer of suspensions. For the case of traditional nanofluids, Brownian motion was previously thought to play a vital role in enhancing thermal conductivity [35–37]. However, this claim has been disproved, since the time required for the heat to transfer a distance equal to the particle diameter is orders of magnitude smaller than the time required for the particle to travel the same distance [38, 39]. However, as shown in the preceding section for the case of the SPNFs, the diffusivity of the particles could be orders of magnitude higher than the thermal diffusivity for certain cases.

In order to illustrate the effect of effective mass diffusivity on the thermal conductivity of the active suspensions, the simple approach recommended by [38] is followed, assuming that the kinetic theory of gases is still valid for the case of suspensions. Thermal conductivity of the solvent is then defined as:

$$\kappa_T = c_p \alpha \quad (7)$$

Where the  $\kappa_T$ , is the thermal conductivity of the fluid,  $c_p$  is the heat capacity of the fluid, and  $\alpha$  is the thermal diffusivity of the fluid.

The thermal conductivity due to diffusivity of self-propelled particles is given following by the equation:

$$\kappa_D = c_p D^{eff} \quad (8)$$

To calculate the relative thermal conductivity of the mixture.  $K_r = \frac{\kappa_{eff}}{\kappa_T}$ , where  $\kappa_{eff}$  is the effective thermal conductivity, a simple mixture model is employed:

$$K_r = (1 - \phi) + \phi \frac{D^{eff}}{\alpha} \quad (9)$$

Here  $\phi$  is the volume fraction of the particles. For the case of the spherical geometry, results for the ethylene glycol solvent for  $\phi = 1\%$  are shown in Figure 7. The results show that for small particle sizes, there is no variation in the value of the effective thermal conductivity, and the thermal conductivity of the SPNF is that of the solvent. As the size of the particles approaches  $1.0 \mu m$  we start to observe an increase in the value of the relative effective thermal conductivity of the SPNF. This reaches 40% for the case of  $U_s = 200 \mu m/s$ , and this increase is substantial for the volume fraction of the particles used.

For the case of the rod-shaped ABF-based SPNF, the variation of  $K_r$  with respect to aspect ratio is shown in Figure 8 for the case of water solvent, and  $\phi = 1\%$ . For the lower aspect ratios, the increase in the thermal conductivity is almost 40% for the case of  $U_s = 200 \mu m/s$  and as  $p_a$  increases the  $K_r$  attains an asymptotic value greater than 1.0. For the lower swimming velocity cases, the increase in thermal conductivity is restricted to a narrow region of low aspect ratios.

For the case of ethylene glycol SPNF with rod ABF particles (Figure 9), a significant enhancement of mixture relative thermal conductivity is observed for nearly all the cases considered, and the thermal conductivity of the suspension is also higher than that of the solvent for a wider range of aspect ratios. Specifically, for the case of  $U_s = 200 \mu m/s$  a maximum enhancement of thermal conductivity of about 12 times is obtained, and enhancement of over two times is sustained for the rest of the aspect ratios.

This analysis demonstrates in an elementary way that there is a clear link between the diffusivity of self-propelled particles and the overall suspension thermal conductivity. It clearly shows that suspending active particles into liquid can lead to an enhancement of the effective thermal conductivity of the medium. **This observation is confirmed by molecular simulations performed by Peng et al. [40].**

Also, if the particles are of the ABF type, the resulting suspension will show enhancement in thermal transport combined with a reduction in viscosity [17], thus manifesting the characteristics of a perfect coolant.

These results require experimental investigation, but in experiments, proving the enhancement of thermal transport in an active suspensions will be substantially easier than the proving the viscosity reduction mechanism.

### 3.2 Particle dispersion and clustering

A complimentary heat transfer mechanism for suspensions is the dispersion of particles that help to transfer heat during particle movement. Buongiorno [33] proposed an energy equation that describes the energy transport for classical nanofluids, and a starting assumption is that a similar equation is valid for the case of SPNFs. If we exclude the thermophoresis term, the non-dimensional form of the energy equation (Eq.(28) in [33]) is as follows:

$$\frac{\partial \theta}{\partial \xi} + \mathbf{V} \cdot \nabla \theta = \frac{1}{PrRe} \left[ \nabla^2 \theta + \frac{\nabla \Phi \nabla \theta}{Le} \right] \quad (10)$$

Here  $\theta$ ,  $\xi$ ,  $\mathbf{V}$ ,  $Pr$ ,  $Re$ ,  $\Phi$ , and  $Le$ , are the non-dimensional temperature, non-dimensional spatial coordinate, non-dimensional velocity, Prandtl number, Reynolds number, non-dimensional volume fraction, and Lewis number respectively. The dispersion term in Eq.(10), is the second term on the right-hand side, and the parameter that signifies the dispersion  $Le$ .  $Le$  is defined as the ratio between thermal diffusivity and particle diffusivity:

$$Le = \frac{\alpha}{D^{eff}_{\phi}} \quad (11)$$

For classical nanofluids,  $Le$  is always much higher than 1, which limits the importance of the dispersion term. However, SPNF  $Le$  values are much lower than unity because the particle diffusivity exceeds that of the thermal by orders of magnitude. This triggers the activation of the particle dispersion mechanism, which together

with the enhancement of the thermal conductivity will significantly enhance the thermal transport in an SPNF.

However, there remains a need for the derivation of an energy equation specifically for the case of SPNFs (active fluids) that may account for extra heat transfer mechanisms that have not been explored by the scientific community up to the current time.

The main reason behind the significant increase in the thermal conductivity of nanofluids was found by Koblinski et al. to be clustering [39]. Clustering is the process of particles coming into a close proximity, due to attractive inter-particle forces. Clustering shortens the distance between the particles, making the heat paths shorter, and thus enhancing heat transfer rates. However, clustering is not beneficial for the flow properties of the suspension since it increases the viscosity of the suspension, and it also increases the rate of sedimentation of particles out of the mixture. Active matter (such as active suspensions) are non-equilibrium systems, that have a plethora of clustering morphologies such as flocs, schools, swarms, and active crystals as reported by [22, 41, 42]. For active suspensions, clustering could happen even for the case of dilute suspensions, and in the absence of any attractive forces [42]. For example when two particles collide, they block the progress of each other, due to their continuous motion in the same direction. Such a cluster would decompose in a time scale comparable to that of the reorientation time scale.

Clustering in active suspensions depends on swimming velocity and the concentration of the particles, and the clusters tend to be in a metastable condition, because if the particle swimming velocity changes, the clusters cease to exist.

These metastable cluster [43] structures could be useful for SPNFs, since if the particles could sense high temperature gradients, they could adjust their velocity,

and form a cluster at the hot location, which would increase the local heat transfer rate, and thus reduce the local temperature. After the reduction of the temperature, the cluster could be decomposed by just changing the swimming velocity of the particles to a certain level. This way, the formation of long term clusters that can significantly reduce the fluidity of the suspension could be avoided. **Freezing of SPNFs will also be an exciting research path, especially comparing their behaviour with the freezing of colloidal suspensions [44–47] which may lead to new applications in energy storage and material processing.**

### 3.3 Active turbulence

Active turbulence may play a role in enhancing heat transfer in SPNFs. Active turbulence is observed to emerge in dense active suspensions, such those of bacteria, as a result of their collective motion. Active turbulence is characterised by the continuous formation and decay of jets, vortices, and whirls [42, 48–51]. For non-active fluids, we know that turbulence emerges from the inertia of the fluid (i.e.,  $Re \gg 1.0$ ). However, the reported observation of active turbulence in bacterial suspensions is within the creeping flow regime (i.e.  $Re \ll 1.0$ ). The relation between active turbulence and turbulence in passive fluids remains largely unknown, and various mechanisms behind it are still under investigations, such as the role of long and short range hydrodynamic interaction between the particles and the fluid, or the way the particles swim [52]. Very recently, a model equation based on a continuum approach was proposed to simulate the hydrodynamic behaviour of the active fluids at a macro-scale [52–55]. This equation applies to the case of a very dense suspension, where the density of particles is nearly constant.

The equation has the form of Toner-Tu equation with Swift-Hohenberg-type-fourth order term, and it can be reduced to the classical Navier-Stokes when the activity of the particles goes to zero. The model equation was tested successfully against experiments [52]. Using this model Slomka and Dunkel [55] found that the

nature of active turbulence is to break the symmetry and thus to increase the mixing of a fluid significantly. This mechanism could be beneficial to the enhancement of the heat transfer. We know that the active turbulence mechanism is observed only for high concentrations, and may not be suitable for the SPNFs. However, the study of this phenomenon is still in its infancy, and may it can exist at low concentrations as well. Otherwise, this mechanism will play a significant role in increasing the heat transfer rate for the case of SPNFs used at high concentrations.

## 4 Experimental Evidence

Up until now, there has been no experimental investigation that proves that self-propelled particles (active particles) can enhance the heat transfer process. However, very recently, two experimental investigations by Wang et al. [56, 57] show that using active particles can enhance heat transfer rate significantly. Their experiments were carried out with Rayleigh-Bernad convection up to the turbulent regime at extremely high Rayleigh numbers ranging from 2.6 to  $4.8 \times 10^{10}$ . The working fluid was water, and they added a low volume fraction ( $\phi = 0.5, 1.0,$  and 4%) of low boiling temperature heavy immiscible liquid (HFE-7000).

As the temperature of the bottom plate increased and reached the boiling temperature of HFE-2000, HFE -2000 bubbles began to nucleate at the bottom plate, and started to rise due to buoyancy. These newly formed bubbles were termed biphasic species. The rising velocity of the biphasic species reached up to 40 cm/s. The biphasic species acts as a heat carrier, moving from the bottom to the top plate, and then falling. This movement created plumes similar to that of turbulence, which helped to enhance the heat transfer significantly.

Wang et al. reported that the heat transfer enhancement is about 800% compared to the pure water case for the case of  $\phi = 4 \%$ . They believed that particle

dispersion and active turbulence mechanisms similar to those described in sections 3.2, and 3.3 above were responsible for the heat transfer enhancement. The experiments of Wang et al. [56,57] show without any doubt that suspending active particles into fluids will enhance the heat transfer significantly.

## 5 Conclusions

A theoretical investigation has been carried out into the significance of active suspensions of self-propelled particles, or self-propelled nanofluids (SPNFs) in thermal transport applications. This brief investigation shows without a doubt that adding active particles can result in a significant enhancement of the thermal conductivity of a suspension, with consequent benefits in a wide range of industrial applications using heat transfer, for example cooling systems for computation and data storage. The following specific conclusions are made:

- The shape of the particles, plus the swimming velocity of the particles plays an important role in the enhancement of the thermal conductivity of an SPNF. Increasing the swimming velocity will increase the enhancement of thermal conductivity. This observation is confirmed by recent molecular dynamics simulations.
- Suspensions that consist of a rod-shaped artificial bacterial flagella (ABF) particles will enhance the thermal conductivity more compared than an equivalent suspension of spherical self-propelled particles.
- Adding self-propelled particles to the fluid will bring an array of different thermal transport mechanisms into play. Particle dispersion, clustering and active turbulence mechanisms may play a significant role in the further enhancement of thermal transport in SPNFs.
- **Experimental evidence from the literature clearly shows that suspending active particles into a liquid significantly enhances the heat transfer rate**

compared with pure solvent.

## References

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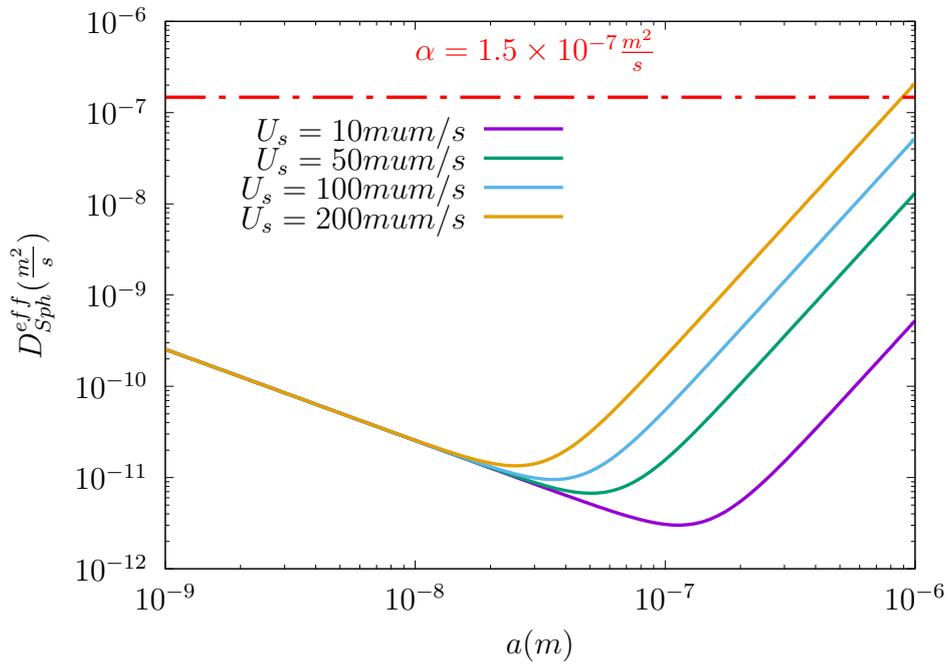


Figure 1: The variation of the effective mass diffusion coefficient  $D_{Sph}^{eff}$  of self-propelled spherical particles suspended in water for different swimming velocities.  $\alpha$  is the thermal diffusivity of water

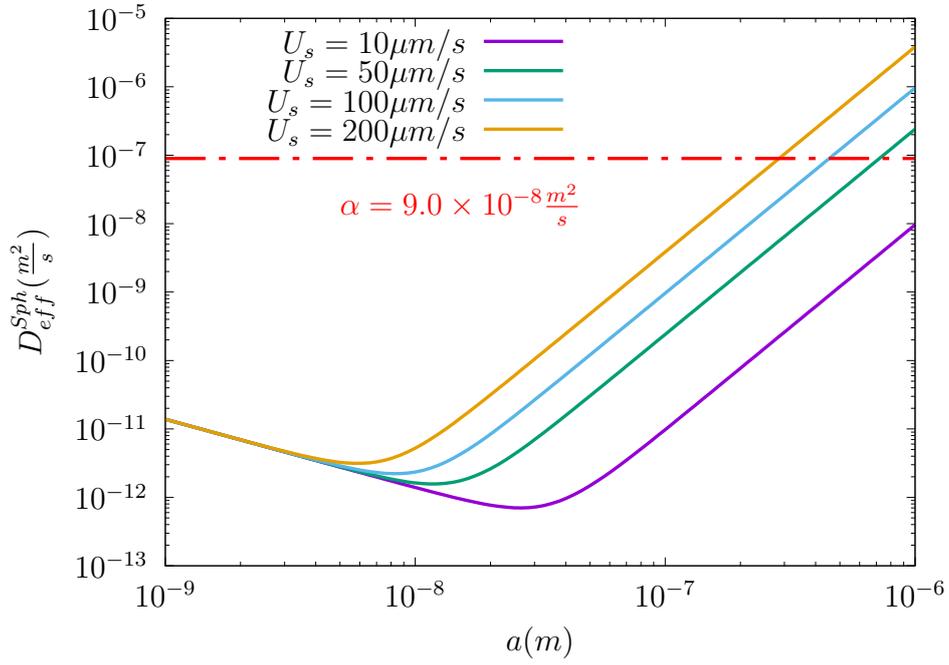


Figure 2: The variation of the effective mass diffusion coefficient  $D_{Sph}^{eff}$  of self-propelled spherical particles suspended in ethylene glycol for different swimming velocities.  $\alpha$  is the thermal diffusivity of ethylene glycol.

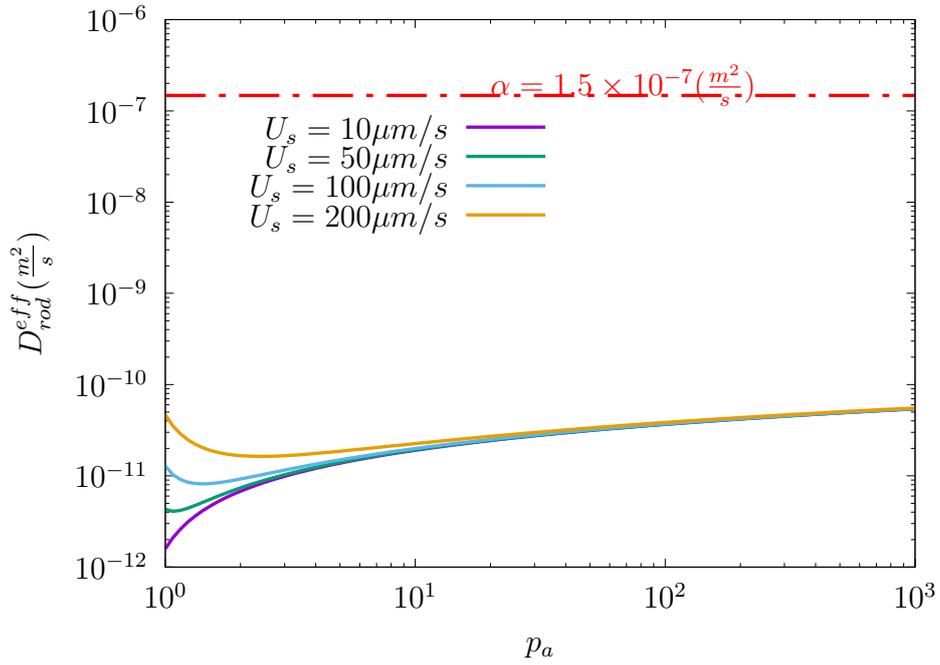


Figure 3: The variation of the effective diffusion coefficient  $D_{rod}^{eff}$  of self-propelled rod particles ( $l = 1.0 \times 10^{-7} m$ ) suspended in water for different swimming velocities and different aspect ratios.  $\alpha$  is the thermal diffusivity of water.

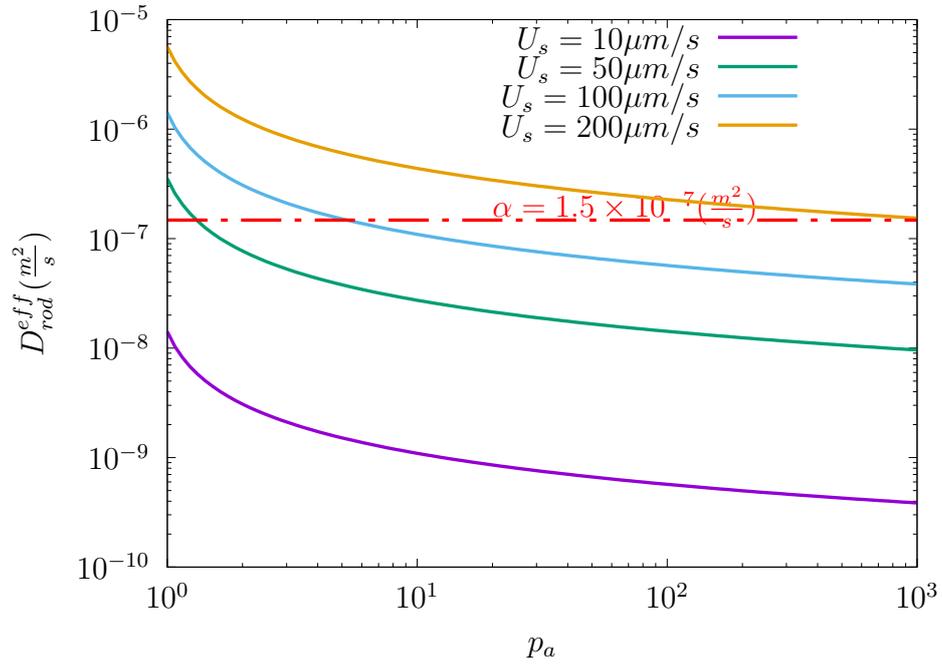


Figure 4: The variation of the effective diffusion coefficient  $D_{rod}^{eff}$  of self-propelled rod particles ( $l = 5.0 \times 10^{-6}m$ ) suspended in water for different swimming velocities and different aspect ratios.  $\alpha$  is the thermal diffusivity of water.

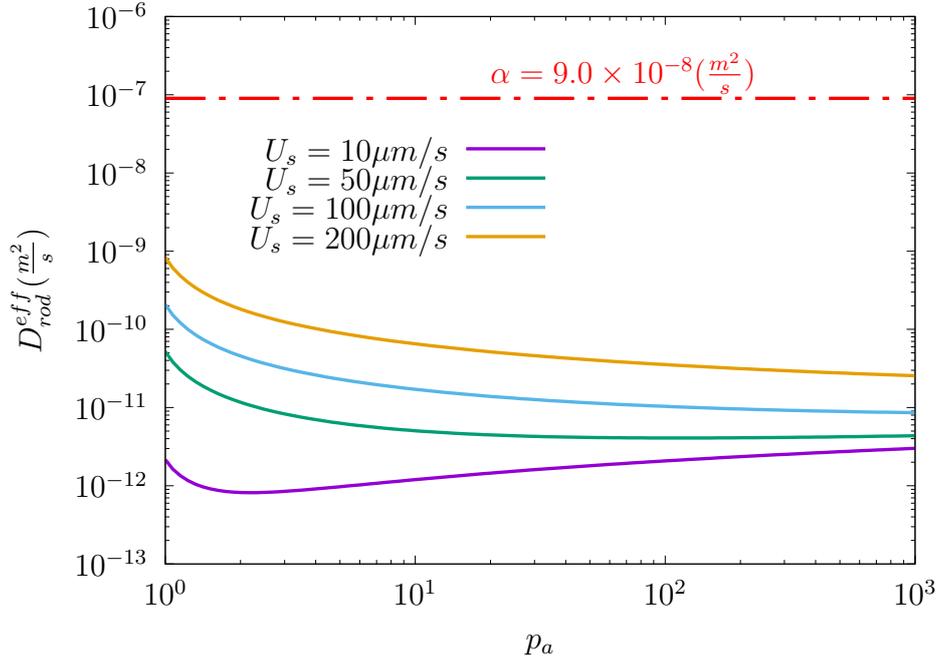


Figure 5: The variation of the effective diffusion coefficient  $D_{rod}^{eff}$  of self-propelled rod particles ( $l = 1.0 \times 10^{-7} m$ ) suspended in ethylene glycol for different swimming velocities and different aspect ratios.  $\alpha$  is the thermal diffusivity of ethylene glycol.

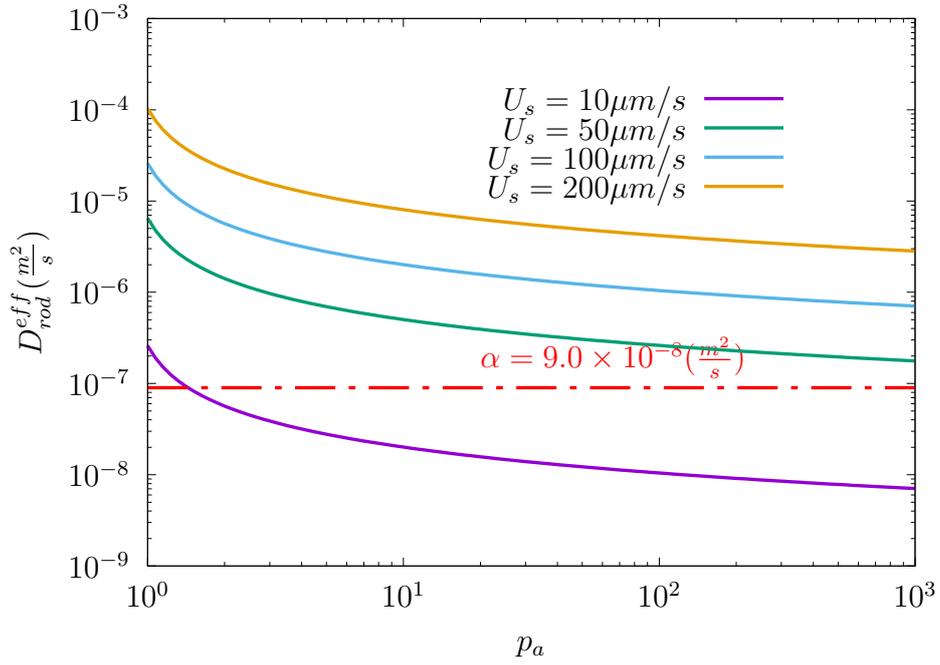


Figure 6: The variation of the effective diffusion coefficient  $D_{rod}^{eff}$  of self-propelled rod particles ( $l = 5.0 \times 10^{-6} \text{m}$ ) suspended in ethylene glycol for different swimming velocities and different aspect ratios.  $\alpha$  is the thermal diffusivity of ethylene glycol.

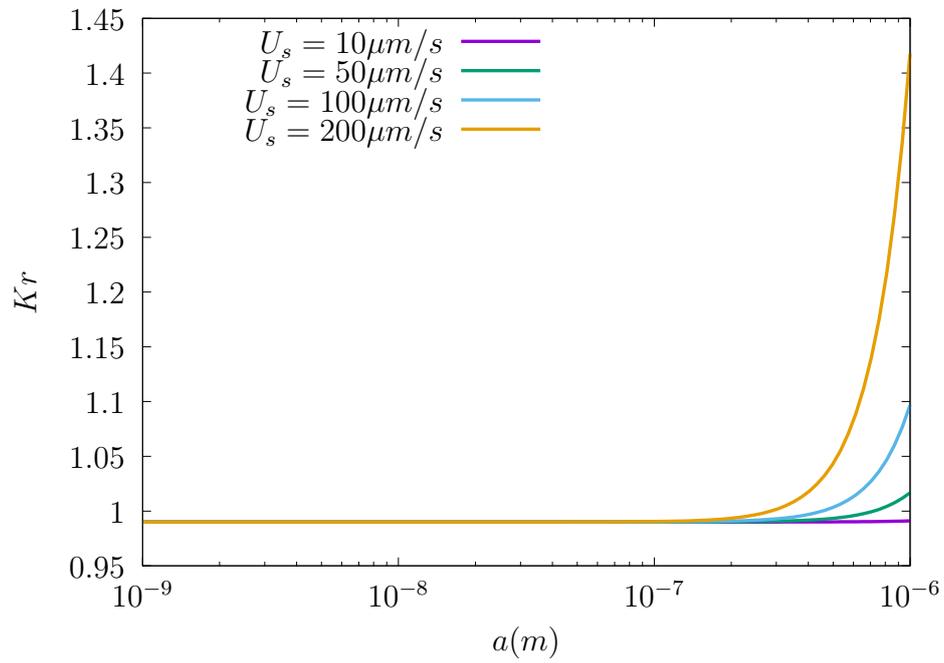


Figure 7: The variation of mixture relative thermal conductivity  $Kr$  for a suspension of spherical particles with different radii, and propelled with different velocities, in an ethylene glycol solvent with volume fraction  $\phi = 1\%$

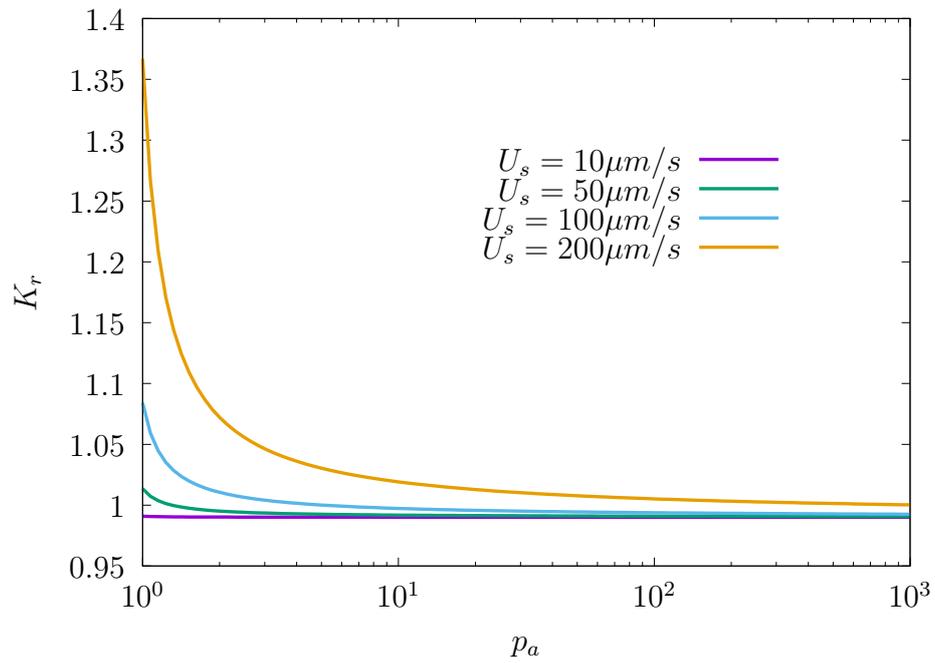


Figure 8: The variation of mixture relative thermal conductivity  $K_r$  for a suspension of rod-shaped ABF particles ( $l = 5.0 \times 10^{-6}m$ ) in water, for different aspects ratios, and propelled by different velocities, with volume fraction  $\phi = 1\%$

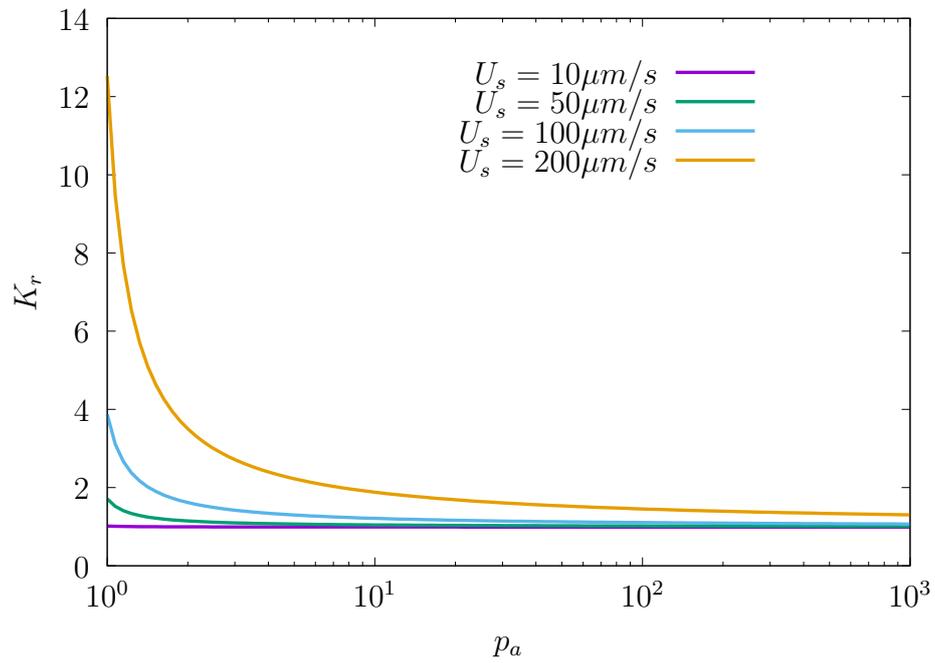


Figure 9: The variation of  $Kr$  for a suspension of rod-shaped ABF particles ( $l = 5.0 \times 10^{-6} m$ ) in ethylene glycol, for different aspects ratios, and propelled by different velocities, with volume fraction  $\phi = 1\%$