OPTIMAL DESIGN OF RAINBOW METAMATERIALS

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ABSTRACT

Nonperiodic metamaterials with appropriately designed resonator distributions can have superior vibration attenuation capabilities compared to periodic metamaterials. In this study, we present an optimization scheme for the resonator distribution in rainbow metamaterials that are constitutive of a Π-shaped beam with parallel plate insertions and spatially varying cantilever-mass resonators. To improve the vibration attenuation of rainbow metamaterials at specific design frequencies, two optimization strategies are proposed, aiming at minimizing the maximum and average receptance values. Objective functions are set up with the frequency response functions predicted by the displacement transfer matrix model. The masses of the two sets of resonators, clamped at different side walls of the Π-shaped beams, constitute the set of design variables. Optimization functions are solved using a genetic algorithm method. Results of case studies showed that the receptance values of the nonperiodic metamaterial is greatly reduced within the optimization frequency range, in comparison to the periodic metamaterial.

1 INTRODUCTION

Metamaterials are a new class of artificial composites engineered to have novel properties that cannot be found with natural materials. Metamaterials have attracted much attention in many research fields [1-5]. Metamaterials were originally introduced to tailor electromagnetic optical waves [1-2]. Nowadays, the concept of the metamaterial has expanded to include acoustic and elastic waves which are the focus of this work. New properties, such as negative effective mass, negative effective dynamic stiffness and negative bulk modulus, can be obtained using elastic/acoustic metamaterials. An important feature of elastic/acoustic metamaterials is the existence of bandgaps within which no waves can propagate [3-4]. Locally resonant bandgaps, relying on the resonance of internal oscillators, occur at frequencies much lower than those due to Bragg scattering.

A large number of elastic/acoustic metamaterials have been proposed with various local resonators. Most of the proposed metamaterials are periodic structures. Although these periodic metamaterials are applicable for manipulating wave propagation and providing low-frequency vibration attention, broad bandgaps are difficult to achieve. Few researchers have presented nonperiodic metamaterials with spatially varying resonators. Sun et al. [3] and Pai [4] made the first attempt at investigating metamaterials with spatially varying mass-spring-damper subsystems. They found that their metamaterials could have better vibration attenuation with properly designed nonperiodic resonators. Their design procedures were, however, mainly based on trial and error, which is unlikely to give optimal designs and possibly leaves a large design space unexplored.

In this paper, we propose a design approach for the distribution of resonators in nonperiodic metamaterials. Optimization objective functions are set up on the basis of the displacement transfer matrix model. A genetic algorithm optimization method is employed to search the optimal nonperiodic distributions of resonator masses that can generate optimal receptance values at the frequencies of interest.

2 ANALYTICAL MODELLING METHOD FOR THE RAINBOW METAMATERIAL

Figure 1 shows the schema of the proposed rainbow metamaterial. The Π-shaped beam is partitioned into substructures by periodic plate insertions. Non-symmetric cantilever-mass subsystems are clamped to the two side walls of the Π-shaped beam in each substructure. Instead of being periodic, the resonating subsystems are spatially varying along the length of the beam, hence the term "rainbow".

An analytical model built on the basis of the displacement transfer matrix model (see [5] for a more in depth description of the model) is employed to determine the receptance function R_{ec} of the rainbow metamaterial, as

$$R_{ec} = 20\log_{10}\left| w_{m,r}\big|_{x=L} \big/ F \right| \quad (1)$$

where F is the excitation force on the metamaterial beam and $w_{m,r}\big|_{x=L}$ is the displacement of the beam.

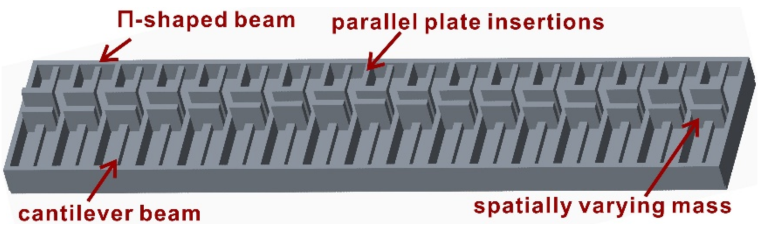


Figure 1. Illustration of the global view of the rainbow metamaterial.

3 OPTIMIZATION STRATEGY

In order to maximize the vibration attenuation of rainbow metamaterials in a prescribed frequency range, two optimization strategies are proposed that use two objective functions individually. One objective function is set based on the maximum receptance value within the prescribed frequency range, given by

$$\min \quad \max\left(R_{ec}\left(\mathbf{M}_1,\mathbf{M}_2,\mathbf{\Phi}\right)\right) \tag{2}$$

where $\mathbf{M}_1=\left(m_{11},m_{12},...,m_{1t}\right)$ and $\mathbf{M}_2=\left(m_{21},m_{22},...,m_{2t}\right)$ represent the mass of the resonators at different sides of the complex beam respectively, t is the total number of segments in the metamaterial beam and $\mathbf{\Phi}=\left(f_1 \sim f_2\right)$ is the prescribed frequency regime. The receptance within the prescribed frequency range is low when the maximum value remains minimal; the vibration attenuation is thus optimized. The mass of each resonator cannot be less than zero, constraints of the design variables are hence given by

$$s.t. \quad m_{ij} \geq 0.\ i=1,2.\ \ j=1,2,...,t \tag{3}$$

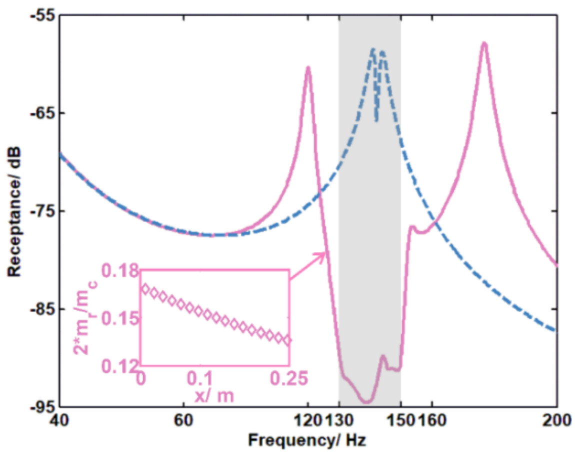
The mean value is another indicator of the receptance value quality within a prescribed frequency range, therefore, the other objective function is set based on the mean receptance value within the prescribed frequency range, given by

$$\min \quad \frac{\int_{\mathbf{\Phi}} R_{ec}\left(\mathbf{M}_1,\mathbf{M}_2,\mathbf{\Phi}\right)df}{f_2-f_1}. \tag{4}$$

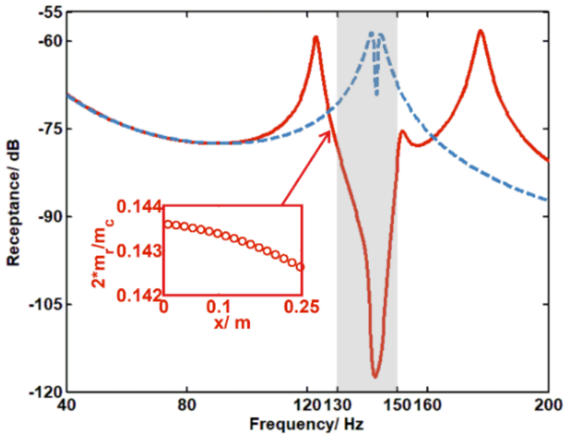
The constraints on the design variables defined by Eq. (4) are similarly applicable to the maximum value-based optimization objective function. A genetic algorithm method is employed to solve the objective functions in Eqs. (2) to (4).

4 OPTIMIZATION EXAMPLES

For the purpose of reducing vibration at low frequencies, a prescribed frequency range 130 Hz to 150 Hz, which is around the first resonance frequency of the backbone beam, is used for the optimization examples. Receptance values of the rainbow metamaterials with optimal resonator mass distributions obtained by the two optimization strategies are compared with those of complex beams of the same mass but without resonators, as shown in Figs. 2(a) and (b) . As can be seen, both of the two optimal rainbow metamaterials show bandgaps within the prescribed frequency range, hence, the receptance values are significantly reduced. The optimal metamaterial in Fig 2(a) has a maximal receptance approximately 38 dB less than that of the structure without resonators. The mean receptance difference between the optimal structure and the no-resonator beam, shown in Fig 2(b), is approximately 33 dB. Both maximum and mean displacements within 130 Hz to 150 Hz can be reduced by a factor of more than 70 with the optimization process.



(a)



(b)

Figure 2. Receptance value comparison between optimal rainbow metamaterial beams (solid line) by maximal receptance value based objective function (a) and by average value based objective function (b) and no-resonator Π-shaped beam (dashed line) of the same mass. The ratios of resonator masses to that of the backbone structure of the optimal beams are plotted in the subfigure.

In addition, as shown in Figs. 2(a) and (b), the optimal structure, derived using a maximum value based objective function, has a broader bandgap of higher receptance value within the prescribed frequency range, which is opposite to the optimal structure derived by the mean value based objective function. The optimization strategy can be chosen according to the requirements of different applications.

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