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The effect of the cone rotation in the supersonic gas flow on the velocity propagation of perturbations in the boundary layer.

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Abstract:	The paper aimed at studying the peculiarities of boundary layer on a rotating body. Rotation of axisymmetric bodies is often used for their stabilization at a high velocity motion in the atmosphere. Presented are calculation of boundary layer flow on a rotating cone at zero angle of attack. Longitudinal and full enthalpy profiles in the laminar boundary layer obtained as a result of calculations were used to determine upstream disturbances propagation velocity. New integral relation was determined to find out disturbances propagation velocity for the regime of weak hypersonic viscous-inviscid interaction. Effects of surface rotation velocity along with the temperature factor influence were investigated.
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Sincerely Yours,
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Igor Lipatov

- Effects of upstream disturbances propagation in the boundary layer are investigated.
- Disturbances propagation was analyzed for the weak interaction regime.
- New results have been obtained for 3D flow near the rotating cone .
- Quantitative data were obtained for velocity of disturbances propagation.

The effect of the cone rotation in the supersonic gas flow on the velocity propagation of perturbations in the boundary layer.

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Abstract

The paper aimed at studying the peculiarities of boundary layer on a rotating body. Rotation of axisymmetric bodies is often used for their stabilization at a high velocity motion in the atmosphere. Presented are calculation of boundary layer flow on a rotating cone at zero angle of attack. Longitudinal and full enthalpy profiles in the laminar boundary layer obtained as a result of calculations were used to determine upstream disturbances propagation velocity. New integral relation was determined to find out disturbances propagation velocity for the regime of weak hypersonic viscous-inviscid interaction. Effects of surface rotation velocity along with the temperature factor influence were investigated.

Nomenclature

M – Mach number

τ the dimensionless thickness of the boundary layer

l characteristic length

γ specific heat ratio

a a velocity of upstream disturbances propagation

μ_0 the value of the dynamic coefficient of viscosity, determined at the temperature of inhibition

z transformed longitudinal coordinate

u longitudinal velocity

v normal velocity

w transversal velocity

H total enthalpy

ρ density

x longitudinal coordinate

y normal coordinate

f transformed flow function

ξ transformed longitudinal coordinate

η transformed normal coordinate

ω transformed angular velocity

Keywords: boundary layer, propagation of disturbances, supersonic, weak viscous-inviscid interaction, flow near a cone.

1. Introduction

Aerodynamics of bodies in high-speed flight in the atmosphere was always on the top of research needs of aerospace science and technology [1, 2, 3]. Effective estimates of aero-dynamical loads are necessary for precise evaluation of deorbiting time for satellites [4]. These are aspects ensuring safety of Space missions [5, 6]. Rotation of streamlined bodies is often used to ensure flight stability at high velocities. Rotation of a streamlined body also affects the stability of the flow and the position of the laminar-turbulent transition. A lot of work is devoted to the study of boundary layers on rotating bodies [7], including on a cone, since such studies can be used

directly, both in ballistics and in aerospace industries. In particular, it is interesting to investigate what happens to the propagation of perturbations in the boundary layer on a cone if it is rotating. Undoubtedly, rotation affects the velocity and enthalpy profiles in the boundary layer, thus affecting the propagation characteristics of the perturbations. In this paper, we investigate the characteristics of the boundary layer on the surface of a rotating cone and determine the propagation velocities of the perturbations in the boundary layers.

2. Statement of the problem.

We consider the flow past a cone by a supersonic flow at zero angle of attack, assuming that the Mach number of the oncoming flow is large and that the regime of weak hypersonic interaction is realized

$$M_{\infty}\tau \ll 1$$

where τ is the dimensionless thickness of the boundary layer.

For Cartesian coordinates measured in the direction of the incident flow, along the normal to the surface and in the transversal direction, the time, the corresponding components of the velocity vector, density, pressure, total enthalpy, dynamic viscosity coefficient, the following notation is adopted: $lx, ly, lt / u_{\infty}, u_{\infty}u, u_{\infty}v, u_{\infty}w, \rho_0\rho, \rho_{\infty}u_{\infty}^2 p, H_{\infty}g, \mu_0\mu$. The parameter l is some characteristic length (for example, the length of the generatrix of the surface in the longitudinal direction); $\tau = (\rho_0 u_{\infty} l / \mu_0)^{-1/2}$, where the subscript " ∞ " denotes dimensional values in the oncoming stream; - μ_0 the value of the dynamic coefficient of viscosity, determined at the temperature of inhibition. It is assumed that the gas is thermodynamically perfect and is characterized by a constant value of the specific heat ratio γ . Although in hypersonic flows the effects of real gas are significant, in this paper they are not considered, since their accounting in principle does not change the relationships obtained below. The Reynolds number is large, but does not exceed the critical value at which a laminar-turbulent transition occurs. It is known that for supersonic and hypersonic flows the Reynolds number of the transition is large enough. It is also assumed that the dynamic viscosity depends linearly on temperature, so that $\rho\mu = const$. The angle of the cone solution is such that an associated shock wave is realized, the angle of attack is zero. The coordinate system is shown in Fig.1

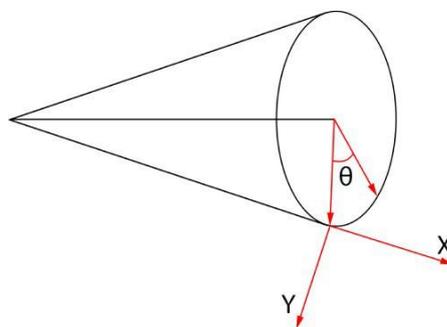


Fig. 1. The coordinate system used in the calculation

The system of boundary-layer equations on a rotating cone has the form

$$\frac{\partial \rho u r_w}{\partial x} + \frac{\partial \rho v r_w}{\partial y} = 0$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} - \rho \frac{r_w'}{r_w} w^2 = \frac{\partial}{\partial y} \mu \frac{\partial u}{\partial y}$$

$$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho \frac{r_w'}{r_w} u w = \frac{\partial}{\partial y} \mu \frac{\partial w}{\partial y}$$

$$\rho u \frac{\partial g}{\partial x} + \rho v \frac{\partial g}{\partial y} = \frac{\partial}{\partial y} \frac{\mu}{\text{Pr}} \frac{\partial g}{\partial y}$$

$$\text{Pr} = 1$$

with boundary conditions:

$$u \rightarrow 1; w \rightarrow 0; H \rightarrow 1, \quad y \rightarrow \infty$$

$$u = v = 0; h = T_w / T_e; w = w(x), \quad y = 0$$

$$u = 1; v = w = 0; H = 1, \quad x = 0$$

This system of equations under rotational conditions includes the effect of centrifugal forces and Coriolis forces

To integrate this system, we pass to the Dorodnicyn-Lees variables

$$(x, y) \rightarrow (z, \eta) \quad [8]$$

$$\xi(x) = \int_0^x \rho_w \mu_w u_\infty r_w^2 dx$$

$$\eta(x, y) = \frac{u_\infty r_w}{\sqrt{2\xi}} \int_0^y \rho dy$$

$$z = \xi^3, \quad u = \frac{\partial f}{\partial \eta}$$

, the system of equations in the new variables takes the form

$$(Nf'')' + ff'' + 2/3w^2 = 2/3z(ff' - \dot{f}f'')$$

$$(Nw')' + fw' - 2/3f'w = 2/3z(f'w - \dot{f}w')$$

$$\left(\frac{N}{\text{Pr}}g\right)' + fg' = 2/3z(fg - \dot{f}g')$$

$$N = \frac{\rho\mu}{\rho_w\mu_w}, \quad ()' = \frac{\partial}{\partial \eta}, \quad \dot{()} = \frac{\partial}{\partial z}$$

With boundary conditions

$$\xi = 0; f = \eta, w = 0, g = 1$$

$$\eta = 0; f = f' = 0, w = \omega_1 z, g = g_w, \omega_1 = \omega u_\infty^{2/3} \left(\frac{3 \sin \varphi}{\rho_w \mu_w} \right)^{1/3}$$

$$g_w = \frac{T_w}{T_0} + w_w^2$$

$$\eta \rightarrow \infty; f' = 1, w = 0, g = 1$$

3. Propagation of perturbations in boundary layers with weak interaction

Earlier, an analysis of the propagation processes of perturbations was carried out for supersonic flows under strong interaction conditions. Although the original work was performed for channel flows (the Pearson integral), the question of the applicability of the integral for other flows or for other modes of interaction, for example, for the regime of weak hypersonic interaction, remained. We leave the assumption of a large supersonic velocity of the external flow, which is necessary to obtain a fixed thickness of the boundary layer due to the discontinuous density distribution near the outer boundary of the boundary layer.

We confine ourselves here to the consideration of the two-dimensional case, assuming that propagation to the three-dimensional case can be carried out in the same way as in the works performed earlier for the strong-interaction regime.

In the conditions of a rapid transition in the boundary layer, a number of terms can be neglected. Then the system of equations needed to describe the transition has the form

$$f \dot{f}' - \dot{f} f'' = -\frac{(\gamma-1)}{2\gamma p} \frac{dp}{d\xi} (g - f'^2) \quad (1)$$

$$f \dot{g} - \dot{f} g' = 0 \quad (2)$$

$$\delta = \frac{c}{p_0} \int_0^\infty (g - f'^2) d\eta \quad (3)$$

The form of the constant in the expression for the thickness of displacement does not matter, since further analysis is connected with finding the derivative of the thickness of displacement by pressure and equating this derivative with zero. This corresponds to finding the state in which the average Mach number is equal to one.

$$\frac{d\delta}{dp} = -\frac{c}{p^2} \int_0^\infty (g - f'^2) + \frac{c}{p_0} \int_0^\infty \left(\frac{\partial g}{\partial p} - 2f' \frac{\partial f'}{\partial p} \right) d\eta = 0 \quad (4)$$

Let us find the derivatives that appear in the above formula. For this, equation (1) can be rewritten in the form

$$f'^2 \frac{\partial}{\partial \eta} \left(\frac{\dot{f}}{f'} \right) = -\frac{(\gamma-1)}{2\gamma p} \frac{1}{p} \frac{dp}{d\xi} \quad (5)$$

then

$$\dot{f} = -(f' + a) \frac{(\gamma - 1)}{2\gamma p} \frac{dp}{d\xi} \int_0^\eta \frac{(g - f'^2)}{(f' + a)^2} d\eta \quad (6)$$

Passing to differentiation with respect to pressure, one can obtain

$$\frac{\partial f}{\partial p} = -(f' + a) \frac{(\gamma - 1)}{2\gamma p} \int_0^\eta \frac{(g - f'^2)}{(f' + a)^2} d\eta \quad (7)$$

Using equation (2), one can obtain

$$\frac{\partial g}{\partial p} = -g' \frac{(\gamma - 1)}{2\gamma p} \int_0^\eta \frac{(g - f'^2)}{(f' + a)^2} d\eta \quad (8)$$

We transform expression (7)

$$\frac{\partial f'}{\partial p} = -f'' \frac{(\gamma - 1)}{2\gamma p} \int_0^\eta \frac{(g - f'^2)}{(f' + a)^2} d\eta - (f' + a) \frac{(\gamma - 1)}{2\gamma p} \frac{(g - f'^2)}{(f' + a)^2} \quad (9)$$

Now we can calculate the integrals in (4)

$$\int_0^\infty \frac{\partial g}{\partial p} d\eta = -\frac{(\gamma - 1)}{2\gamma p} g \int_0^\infty \frac{(g - f'^2)}{(f' + a)^2} d\eta + \frac{(\gamma - 1)}{2\gamma p} \int_0^\infty \frac{g(g - f'^2)}{(f' + a)^2} d\eta \quad (10)$$

$$\int_0^\infty f' \frac{\partial f'}{\partial p} d\eta = -\frac{(\gamma - 1)}{2\gamma p} \int_0^\infty f''(f' + a) d\eta \int_0^\infty \frac{(g - f'^2)}{(f' + a)^2} d\eta - \frac{(\gamma - 1)}{2\gamma p} \int_0^\infty (g - f'^2) d\eta \quad (11)$$

After a number of transformations of (11), we obtain

$$\int_0^\infty f' \frac{\partial f'}{\partial p} d\eta = -\frac{(\gamma - 1)}{2\gamma p} \frac{(f' + a)^2}{2} \int_0^\infty \frac{(g - f'^2)}{(f' + a)^2} d\eta - \frac{(\gamma - 1)}{4\gamma p} \int_0^\infty (g - f'^2) d\eta \quad (12)$$

Combining expressions (10) and (11), one can obtain

$$\int_0^\infty \frac{\partial g}{\partial p} - 2f' \frac{\partial f'}{\partial p} d\eta = \frac{(\gamma - 1)}{2\gamma p} \int_0^\infty \frac{g(g - f'^2)}{(f' + a)^2} d\eta + \frac{(\gamma - 1)}{2\gamma p} \int_0^\infty (g - f'^2) d\eta \quad (13)$$

Substituting (13) into (4) we finally obtain

$$\frac{d\delta}{dp} = c \frac{(\gamma - 1)}{2\gamma p} \int_0^\infty \frac{(g - f'^2)^2}{(f' + a)^2} d\eta - c \frac{1}{\gamma} \int_0^\infty (g - f'^2) d\eta$$

Where does the expression come from. which determines the rate of propagation of disturbances

$$\frac{(\gamma - 1)}{2} \int_0^\infty \frac{(g - f'^2)^2}{(f' + a)^2} d\eta - \int_0^\infty (g - f'^2) d\eta = 0$$

The propagation of perturbations in boundary layers in the viscous-inviscid interaction theory is considered in [9-11] and an expression is obtained for finding the propagation velocity of the perturbations, here, u , w the longitudinal and transverse velocity profiles, g the total enthalpy profile, the θ direction of propagation of the perturbations:

$$\frac{(\gamma-1)}{2} \int_0^{\infty} \frac{(H-u^2-w^2)^2}{(u \cos \theta + w \sin \theta - a)^2} dy - \int_0^{\infty} (H-u^2-w^2) dy = 0$$

4. Results of calculations

The system of equations is integrated numerically and for the obtained profiles of velocity and enthalpy components u, w, g , using the above integral, one can also find the values of the velocity of propagation of perturbations in the boundary layer. The results are obtained for the magnitude of the propagation angle, $\theta = \pi$ i. upstream.

Data were obtained for a number of values of the temperature factor g_w , and also for a number of values of the velocity of rotation of the cone.

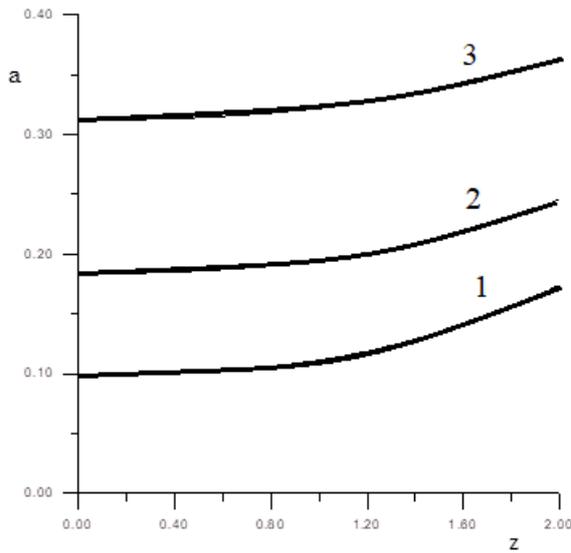


Fig. 2 The dependence of the upstream perturbations propagation velocity a on the longitudinal coordinate z for three values of the temperature factor $g_w = T_w / T_0$ ($g_w = 0.5, 2, g_w = 1, g_w = 2$)

It can be noted that as the longitudinal coordinate increases, the velocity of perturbation propagation increases. It can also be noted that the temperature factor also leads to the same effect. This effect is explained by the increase in temperature in the near-wall region and the relative increase in the region of subsonic flow in the boundary layer. We note that with a relative decrease in the subsonic flow region, the propagation velocity of the perturbations tends to zero if the temperature factor tends to zero. Results presented on the fig.3 show that with the

growth of the longitudinal coordinate, the transverse velocity associated with the rotation of the cone increases, which also leads to a relative increase in the subsonic flow region.

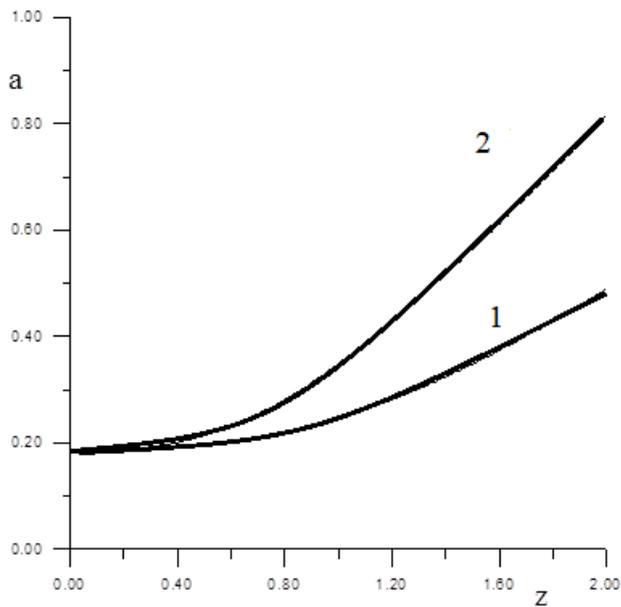


Fig. 3. Dependence of the disturbances propagation velocity on the dimensionless angular velocity of rotation of the cone $\omega_1 = \omega u_\infty^{2/3} (\frac{3 \sin \varphi}{\rho_w \mu_w})^{1/3}$ (1- $\omega_1 = 1$, 2- $\omega_1 = 2$)

It can be noted that with an increase in the speed of rotation of the cone, the rate of upward flow of perturbations increases. As noted above, this effect can be explained by a relative increase in the subsonic flow region.

5. Conclusions

The effects of propagation of disturbances in boundary layers near the surface of a rotating cone are investigated. It is shown that the relations obtained earlier for determining the propagation velocity of perturbations for the regime of strong hypersonic interaction are also valid for describing the regime of weak hypersonic interaction. The dependences of the propagation velocity of the perturbations on the longitudinal coordinate are determined for different values of the temperature factor and for various velocities of rotation of the cone. The obtained data can be used in determining the aerodynamic characteristics of a rotating cone.

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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