Age of Information for Actuation Update in Real-Time Wireless Control Systems

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***Abstract*—In this paper, we introduce a generalized definition of *age of information* (AoI) for actuation update in real-time wireless control systems. In such a system, a general queueing model, i.e., $M/M/1/1$ queueing model, is used to describe the actuation update, in which the sampling packets arrive at the remote controller following the Poisson process, the process from the controller to the actuator follows the exponential distribution, and the actuation intends to update at the actuator at the predictive time. Then, the initial time of the AoI for the new actuation update is the predictive time for the latest update, which is significantly different from the traditional calculation in status update. By the relationship between communication time from the controller to the actuator and predictive time, the AoI calculation falls into two cases, where the conventional AoI in status update is a specific case in this paper. Simulation results show the performance of our method.**

I. Introduction

Timeliness is critical to guarantee good control performance in real-time wireless feedback control systems [1]–[4]. In such a system, the update of the control process includes both *status update* and *actuation update*. Specifically, the controller receives the latest samples of plant state from the sensor, which is the *status update*. Then, the actuator receives the control command generated by the controller based on the status update, and updates the plant state, which is the *actuation update*.

Age of information (AoI) is first proposed in 2011 and becomes a metric to measure the timeliness of status update [10], where AoI is denoted as $\Delta(t)$ and defined as the amount of time elapsed since the moment that the freshest delivered update was generated [5], i.e., $\Delta(t) = t - S(t)$. Here, t represents the observation time and $S(t)$ represents the sampling time of the latest received status update at the controller. Based on the definition, the research on AoI in status updates starts from direct source-to-destination communication link [5][6], where a sensor measures a random process that represents a physical variable and sends samples to a controller via a communication network. For instance, the authors in [5] calculated average AoI and peak AoI in a single source-to-destination link with different queueing models. Then, more research has been done on multiple source-to-destination communication links in different scenarios [7]–[9] and [11]–[16].

From the above discussion, the research on AoI for status update is maturely studied. However, how to describe AoI for actuation update is still open, where the actuation update is very important for the control performance. In practical control systems, each actuation update usually represents the actuation that is designed to be done at a certain time in the future [17][18]. This makes the conventional AoI invalid in actuation update, where it does not make sense to follow the conventional definition to calculate the AoI in actuation update.

In this paper, we focus on AoI for actuation update. The most relevant works to this paper are AoI in control systems [19]–[21]. For instance, the authors in [19] analyzed the trade-off between the AoI and control performance for a single plant-to-controller link in real-time control systems. However, all the above works are based on conventional AoI that was developed for status update, instead of actuation update.

In this paper, we investigate AoI for actuation update in real-time wireless feedback control systems. In particular, we propose a generalized definition of AoI for actuation update. In addition, a general queueing model, i.e., $M/M/1/1$ queueing model, is used to describe the AoI. Then, we find that the initial time of the AoI for each new actuation update is the predictive time for the latest actuation update. Furthermore, by the relationship between communication time from the controller to the actuator and predictive time, the AoI calculation falls into two cases, where the traditional calculation in status update is a specific case in this paper. Based on the queueing model and initial time, we can calculate the AoI for actuation update, which can be served as a critical metric in overall system design.

The rest of this paper is organized as follows. In Section II, the system model and problem statement are presented. In Section III, a new AoI definition is introduced for actuation update in predictive wireless control systems, and the average AoI in different cases is analyzed, where we obtain the closed-form expressions for different cases. In Section IV, simulation results are provided to show the performance. Finally, Section VI concludes the paper.

II. System Model and Problem Statement

As shown in Fig. 1, we consider AoI for actuation update in a typical predictive wireless feedback control system [23] [24]. In such a system, each control process is as follows. First, the sensor takes samples of the plant at time $t_k - \alpha$,

This work was supported in part by the National Natural Science Foundation of China under Grant 61631004, and in part by the funding for the Scotland 5G Centre.

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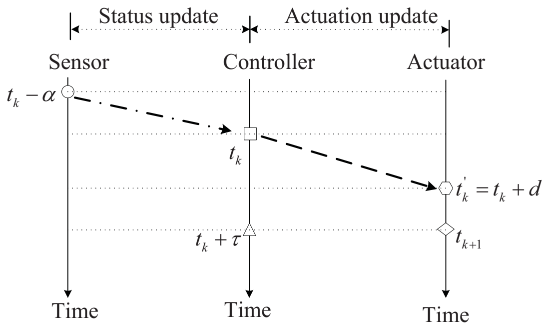


Fig. 1: Actuation update in a typical predictive wireless feedback control system.

and sends the state sample $X(t_k-\alpha)$ to the controller. Then, the remote controller receives the sample packet at time t_k, which is represented as $X(t_k)$. The above update from the sensor to the controller is the status update. After that, the controller would predict the control command $u(t_k+\tau)$ for the time $t_k^A = t_k+\tau$ and send it to the actuator. Finally, the actuator receives the control command at time $t_k' = t_k+d$ and performs it at time t_{k+1} to update the plant state to $X(t_{k+1})$. The above update from the controller to the actuator is the actuation update.

If the predictive time $t_k^A = t_k+\tau$ is less than the arrival time $t_k' = t_k+d$ at the actuator, i.e., $\tau < d$, the available time of the command at the actuator is larger than the predictive time. Then, the control command would be used immediately, where we have that the actuation update time is equal to the arrival time, i.e., $t_{k+1} = t_k' = t_k+d$, and the AoI calculation for the k-th actuation update would be finished at time t_k'. If the predictive time is greater than or equal to the arrival time at the actuator, i.e., $\tau \geq d$, the control command would be stored in the buffer and wait for its usage at time $t_k^A = t_k+\tau$, where we have $t_{k+1} = t_k^A$ and the AoI calculation for the k-th actuation update would be finished at time t_k^A. Then, we conclude that the AoI calculation for actuation update in predictive control system falls into the following two cases.

- *Case I*: $t_k^A < t_k'$. In this case, the predictive time length $\tau = t_k^A - t_k$ is less than the communication time delay $d = t_k' - t_k$ from the controller to the actuator, i.e., $\tau < d$, which means that the control command arrives at the actuator with time delay $d-\tau = t_k' - t_k^A$ compared with its required time. In this case, when $\tau = t_k^A - t_k = 0$, i.e., $t_k^A = t_k$, the control command is for the current sampling time, which is the same as the traditional status update. Thus, the age calculation for traditional status update is a specific case in Case I.
- *Case II*: $t_k^A \geq t_k'$. In this case, the predictive time length is greater than or equal to the communication time delay from the controller to the actuator, i.e., $\tau \geq d$, which means that the control command arrives at the actuator with time $t_k^A - t_k'$ in advance compared with its required time. In addition, when $t_k^A = t_k'$, the predictive time length is equal to the communication time delay from the controller to the actuator, which means that the control command arrives at the actuator exactly when it is required.

We need to note that we only focus on the process from the controller to the actuator, where the process from the sensor $X(t_k-\alpha)$ to the controller $X(t_k)$ is the status update of the traditional case and out of scope of this paper.

To calculate the AoI, we consider a typical queueing model, i.e., first-come-first-service (FCFS) $M/M/1/1$ queueing model, where the new arrival packet would be dropped while a packet is being served, i.e., only one packet is allowed in the system. In such a queueing model, the samples arriving at the controller are exponentially distributed with parameter λ. The communication time[1] for control command from the controller to the actuator is exponentially distributed with parameter μ.

Based on the above system model and statement, we discuss the average AoI calculation for actuation update in the next section.

III. Average Age of Information Calculation

In this section, we first introduce the definition of the average AoI for actuation update in real-time feedback control systems. Then, we characterize the average AoI for the above two cases with FCFS $M/M/1/1$ queueing model.

A. Definition of AoI in Actuation Update and Preliminary Calculations

1) Average Age: According to [5], we can obtain the definition of the time average age as follows.

Definition 1 (*Time average age* [5]):

Assuming ergodicity of process $\Delta(t)$, we can use time average during an interval $(0,\tau)$ to represent the average age, which can be expressed as

$$\Delta_\tau = \frac{1}{\tau}\int_0^\tau \Delta(t)dt. \tag{1}$$

Then, the time average age is calculated by

$$\Delta = \lim_{\tau\to\infty} \Delta_\tau. \tag{2}$$

■

To calculate the time average age in (1), we need to integrate the area under the curve of $\Delta(t)$, which can be done by calculating the sum of the parts labeled by Q_k in Fig. 2. As shown in this figure, Q_k can be characterized by the parameters in actuation update process. Then, we provide a new definition for AoI in actuation update.

Definition 2 (*Age of information for actuation update*):

In the actuation update process, the age of information for each update is defined by

$$\Delta(t) = t - t_{k-1}^A, \tag{3}$$

[1]Note that the communication time consists of the process time at the remote controller and the transmission time from the controller to the actuator.

where t_{k-1}^A is the predictive time of the most recently received actuation update and t is the represents the observation time during the k-th actuation update.

■

The whole path of the age in (3) is a sawtooth form as shown in Fig. 2. In this figure, from the above discussion, the AoI definition in actuation update is significantly different from that in status update. The initial time of the age in the new definition is the predictive time t_{k-1}^A of the most recently received actuation update since the latest predictive actuation update time is the original time for the arrangement of the current predictive actuation update. The age calculation for each actuation update ends when the control command is performed, i.e., t_k' or t_k^A. Taking Case I in Fig. 2(a) as an example, the age for each actuation update starts from the latest predictive time t_{k-1}^A and increases with time elapsing. When the control command is received by the actuator at t_k', the age is reset to $\Delta(t_k') = t_k' - t_{k-1}^A$. Furthermore, we can obtain that the initial time in Case I is overlapped with the generated time t_k when $t_k^A = t_k$, where t_k is the initial time of the traditional AoI. This indicates that the traditional AoI is a special case in this paper.

Again, in Fig. 2, we need to calculate the area Q_k to obtain the age for the two cases in Section II. In the following, we take Case I as an example to obtain a general expressions to calculate Q_k for the two cases. In Case I as shown in Fig. 2(a), the communication time from the remote controller to the actuator is $Z_k = t_k' - t_k$, the interval departure time is expressed as $T_k = t_k' - t_{k-1}'$, and the time difference between the predictive time interval is $Y_k = t_k^A - t_k$. Then, we can obtain the expression for the area of Q_k as

$$Q_k = \frac{1}{2}(Z_{k-1} - Y_{k-1} + T_k)^2 - \frac{1}{2}(Z_k - Y_k)^2. \tag{4}$$

Taking the mean value on Q_k, we can obtain the average age for each actuation update. To obtain the average age for all actuation updates, the *effective arrival rate* should be calculated, which is defined as the ratio of the completed control loops to the time interval τ, and can be expressed as

$$\lambda_e := \lim_{\tau \to \infty} \frac{N(\tau)}{\tau}, \tag{5}$$

where $N(\tau) := \max\{k | t_k' \leq \tau\}$ is the index of the most recently actuation update. Then, the average age can be obtained as

$$\Delta = \lambda_e \mathbb{E}[Q_k]. \tag{6}$$

2) Preliminary Calculations: Considering $M/M/1/1$ queueing model, we assume that Ψ_k represents that the system is empty when the k-th control command packet leaves the remote controller, where the arriving packet will be served immediately at the empty controller. Furthermore, we assume that $\bar{\Psi}_k$ represents that the system is not empty upon departure of the k-th control command, i.e. there is one packet serving at the controller. In this case, the new arrived packet would be discarded. Next, we calculate the probability for the empty or busy system.

An $M/M/1/1$ queue can be described using a two-state Markov chain with "0" and "1" [22], where "0" represents the system is empty and "1" represents the system is busy. Then, we can obtain

$$\begin{cases} \lambda p_0 = \mu p_1, \\ p_0 + p_1 = 1, \end{cases} \tag{7}$$

where p_0 is the probability that the system is empty and p_1 is the probability that the system is busy. Solving (7), we have

$$\begin{cases} p_0 = \frac{\mu}{\lambda+\mu}, \\ p_1 = \frac{\lambda}{\lambda+\mu}. \end{cases} \tag{8}$$

Then, in $M/M/1/1$ queueing model, a sampling packet is accepted in the system only if it is empty. The effective arrival rate can be expressed as

$$\lambda_e = \lambda(1 - p_1) = \frac{\lambda\mu}{\lambda + \mu}. \tag{9}$$

The new arrival sampling packet only can be accessed when the controller is empty. Then, the variables T_k and Z_{k-1} are conditionally independent when Ψ_k holds. The statistic characteristics of Z_{k-1} can be easily obtained by the exponentially distribution with parameter μ. Furthermore, the statistic characteristics of T_k can be obtained by the convolution of two exponentially distributions with parameter λ and μ. The *probability distribution function* (PDF) of T_k can be expressed as

$$\begin{aligned} f(t|\Psi_k) &= \int_0^t \lambda e^{-\lambda\tau} \cdot \mu e^{-\mu(t-\tau)} d\tau \\ &= \frac{\lambda\mu}{\mu - \lambda}\left(e^{-\lambda t} - e^{-\mu t}\right). \end{aligned} \tag{10}$$

Then, based on the PDF in (10), we can further obtain the mean value and covariance as [5]

$$\mathbb{E}[T_k|\Psi_k] = \frac{1}{\lambda} + \frac{1}{\mu}, \tag{11}$$

and

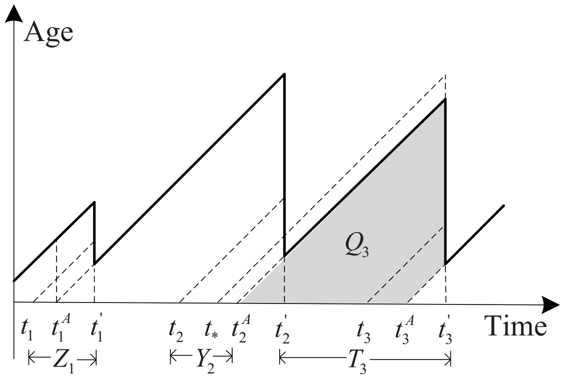
$$\mathbb{E}[T_k^2|\Psi_k] = \frac{2(\lambda^2 + \lambda\mu + \mu^2)}{\lambda^2\mu^2}. \tag{12}$$

In the rest of this section, we discuss the average AoI for the two cases in detail.

B. Average AoI for Case I

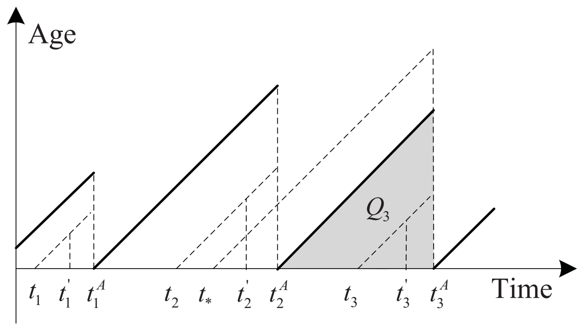
As shown in Fig. 2(a), in Case I, the predictive control command arrives at the actuator later than predictive actuation update time, which means $t_k^A < t_k'$. Then, the control command is used immediately once it arrives at the actuator. Thus, the calculation of the age for each actuation update k begins from t_{k-1}^A and ends at the k-th control command received by the actuator t_k'.

Recall that Ψ_k represents the event that the k-th control process finished with an empty system left behind. In $M/M/1/1$ model, this is a certain event since there is only one packet served at a time in the system. In this model, the expected



(a) AoI for Case I

Fig. 2: Age of information for different cases with $M/M/1/1$ queueing model (con't)



(b) AoI for Case II

Fig. 2: Age of information for different cases with $M/M/1/1$ queueing model.

time for the actuation update is $\mathbb{E}[T_{k-1}] = 1/\mu$, and it is independent of the interdeparture time. From (6), we can obtain the average age for Case I as

$$\begin{aligned}\Delta_I &= \lambda_e\big(\frac{1}{2}\mathbb{E}[(Z_{k-1} - Y_{k-1} + T_k)^2] - \frac{1}{2}\mathbb{E}[(Z_k - Y_k)^2]\big) \\ &= \lambda_e\big(\frac{1}{2}\mathbb{E}[T_k^2] - y\mathbb{E}[T_k] + \mathbb{E}[Z_{k-1}]\mathbb{E}[T_k]\big) \\ &= \frac{\lambda\mu}{\lambda+\mu}\big[\frac{1}{2}\frac{2(\lambda^2+\lambda\mu+\mu^2)}{\lambda^2\mu^2} \\ &\quad - y\frac{\lambda+\mu}{\lambda\mu} + \frac{1}{\mu}\frac{\lambda+\mu}{\lambda\mu}\big] \\ &= \frac{1}{\lambda} + \frac{2}{\mu} - \frac{1}{\lambda+\mu} - y,\end{aligned} \tag{13}$$

where we assume that the predictive length $Y_k = y$ is constant for each control loop k.

C. Average AoI for Case II

As shown in Fig. 2(b), in Case II, the predictive control command is for the time after the control command received by the actuator, which means $t_k^A > t_k'$. Then, the calculation of age of information for each actuation update k begins from t_{k-1}^A and ends at the k-th predictive time t_k^A. Then, the average age for Case II is calculated as

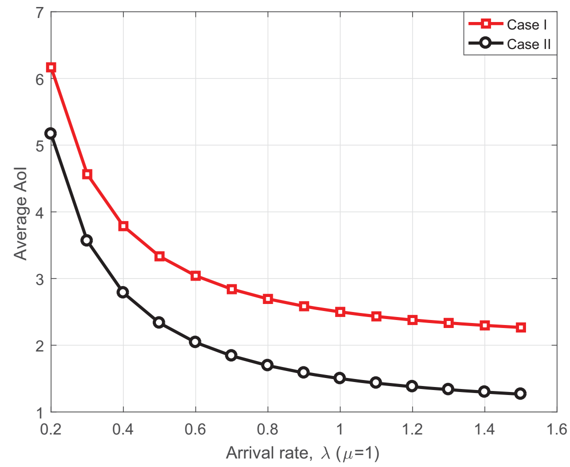


Fig. 3: Average AoI for the two different cases with different arrival rate λ when $\mu = 1$.

$$\begin{aligned}\Delta_{II} &= \lambda_e\left(\frac{1}{2}\mathbb{E}[\big(T_k - (Y_{k-1} - Z_{k-1}) + (Y_k - Z_k)\big)^2]\right) \\ &= \lambda_e\left(\frac{1}{2}\mathbb{E}[\big(T_k + Z_{k-1} - Z_k)\big)^2]\right) \\ &= \lambda_e\big(\frac{1}{2}\mathbb{E}[T_k^2]\big) \\ &= \frac{\lambda\mu}{\lambda+\mu}\big[\frac{1}{2}\frac{2(\lambda^2+\lambda\mu+\mu^2)}{\lambda^2\mu^2}\big] \\ &= \frac{1}{\lambda} + \frac{1}{\mu} - \frac{1}{\lambda+\mu},\end{aligned} \tag{14}$$

where we assume that Y_k is constant for each k.

IV. Simulation Results

In this section, we provide numerical results to demonstrate the average age and its relationship with the overall system performance. For $M/M/1/1$ queueing model, we assume that the parameter μ for transmission service is equal to 1, i.e., $\mu = 1$. Then, the effect of the $M/M/1/1$ queueing model can be obtained by different arrival rate λ.

Fig. 3 shows the average age of the two different cases with different arrival rate λ when $\mu = 1$, where we assume that the predictive length is $y = 0.5$ for Case I. From the figure, all the curves strictly decrease with the arrival rate λ, which means that larger arrival rate leads to smaller average AoI. Furthermore, given arrival rate λ and transmission process rate μ, the average age decreases from Case I to Case II, which means that longer predictive length leads to smaller average AoI.

Fig. 4 illustrates the relationship between the average AoI and predictive length, where different process rates are adopted, i.e., $\mu = 0.4$, $\mu = 0.6$, and $\mu = 1.0$. In this figure, all the curves first decrease with predictive length, and then becomes

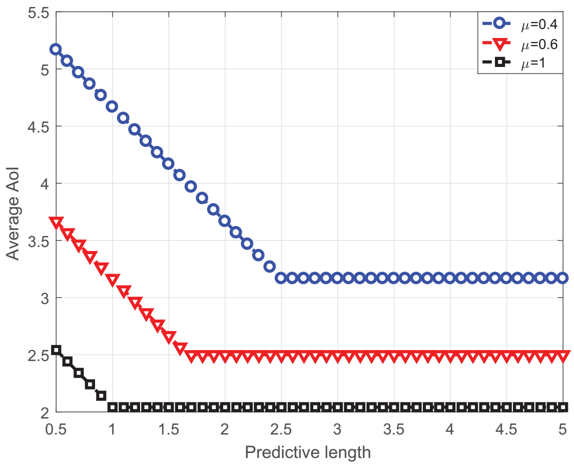


Fig. 4: AoI with different predictive length.

horizontal when the predictive length is larger than the average process time from the controller to the actuator. In addition, the average AoI with larger process rate is higher than that with lower process rate. The reason can be obtain by (13) and (14). Thus, to decrease the average AoI, the average predictive length needs to be no less than the average process time $1/\mu$ and the process rate μ needs to be larger enough.

V. Conclusion

In this paper, a generalized age of information was proposed for actuation updates. With a general first-in-first-service $M/M/1/1$ queueing model in a typical predictive wireless control system, we found that the initial time of the age is the predictive time of the latest actuation update, which is significantly different from the traditional calculation only considering the queueing model in status update. Considering different predictive lengths, the calculation for the average age in the proposed system was divided into two cases, where the traditional age calculation method is a specific case in this paper. Furthermore, we provided the closed form expressions of the average age for the two cases. Based on the obtained AoI for actuation update, further works can be done on the whole process in real-time wireless control systems.

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