



## Fractal-Based Orthonormal Basis for Compressing and Reducing the Dimensionality of Full-Waveform Inversion for Hydrogeological Applications Using Ground-Penetrating Radar

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Full-waveform inversion (FWI) using ground-penetrating radar (GPR) is gaining momentum as a powerful hydrogeological tool for inferring the hydraulic properties of soils between boreholes [1]. Nonetheless, the large computational requirements of FWI make it often unattainable with limited practical uptake [2]. In addition, the inability to accurately reconstruct the loss mechanisms and the need for a good initial model, further reduce the applicability of FWI [1], [2].

In order to overcome the aforementioned limitations, we suggest a novel framework that substantially reduces the optimization space of FWI which consequently reduces the overall computational requirements [2]. This methodology assumes that the water fraction of the investigated medium follows a fractal distribution [3]. Based on that, using a principal components analysis on 3000 randomly generated fractals, we build an orthonormal basis that is fine-tuned for fractal correlated noise. Furthermore, it is proven [2], that fractal correlated noise is highly compressible and can be sufficiently represented with just 30-40 principal components. This reduces the optimization space since now FWI needs to fine-tune just these 30-40 parameters instead of every cell of the investigated medium [2].

The involved fractals describe the distribution of the water fraction that is subsequently transformed to dielectric properties via a semi-empirical formula that relates readily available soil properties to the frequency depended complex electric permittivity [4], [5]. Via this approach, we overcome the need for a simultaneous FWI for both permittivity and conductivity [6]. This further reduces the optimization space and overcomes pitfalls associated with reconstructing loss mechanisms [2].

### References

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