ACADEMIA | Letters

The Role of Mathematics in Economics

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1. Introduction

Since Economics was recognised as a discipline there have been debates about the role of mathematics. The Historical School of Economics, which flourished in the 19th and early 20th centuries, saw economics as resulting from careful empirical and historical analysis instead of from logic and mathematics, promoting instead the role of history, politics and social theories above that of mathematical modelling. However, Schumpeter in his unfinished History of Economic Analysis (1954) acknowledged that 'mathematical models of reasoning played a significant and indeed decisive role in the pure theory of our science'. Keynes in the General Theory was critical of the then burgeoning profile of mathematics in economics; arguing '*Too large a proportion of recent mathematical economics are merely concoctions, as imprecise as the initial assumptions they rest on, which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols'* (Keynes 1936). Keynes, in a review for the League of Nations, attacked the use of mathematical modelling by Tinbergen. (Keynes 1939). In his obituary of Keynes, Schumpeter contends '*theorists will recognize the mathematical quality of mind that underlies the purely scientific part of Keynes's work, …some of them may wonder why he kept aloof from the current of mathematical economics which gathered decisive momentum at just about the time. Nor is this all. Though never definitely hostile to mathematical …. he never threw the weight of his authority into its scale. The advice that emanated from him was almost invariably negative. Occasionally his conversation revealed something akin to dislike*". (Schumpeter 1946) After the second world war, Von Neumann (1971), Samuelson (1952), Nash (1950), Arrow

and Debreu (1954) re-established mathematics as a main element of economic practice but recently behaviourists such as Kahneman and Thaler have pointed out several inconsistencies. The authors therefore contend that the role of mathematics in economics is by no means established. This is the rationale for this short note which revisits the debate in the light of current practice and recent advances in mathematical thinking.

2. What is Mathematics

The ontology and epistemology of mathematics is difficult to pin down. In 1903, on the eve of the publication of the second volume of his masterpiece 'Grundgesetze der Arithmetik' concerning the foundations of mathematics, Gottlieb Frege, the father of analytic philosophy, received a letter from Russell[1] concerning the self-referential paradox which seriously undermined his work.(Frege 1903) Russell later sidestepped the paradox by introducing the theory of types (i.e. propositions could not reference other propositions of the same type) and in 1910 published Principia Mathematica in collaboration with Whitehead. The aim of both these works was to ground mathematics firmly on a basis of logic. On page 362 of the four-volume work, Russell proved a proposition which he purported showed that 1+1=2. In 1936, Russell's work was challenged by Kurt Gödel who showed that Russell had proved 1+1=2 by assigning a specific meaning (numbers) to his logical propositions but he (Gödel) could equally assign different meanings which showed that *Principia Mathematica,* that formidable intellectual fortress so painstakingly erected as a bastion against the horrid scourge of self-referentiality, was in fact riddled through and through with formulas allegedly stating all sorts of absurd and incomprehensible things about themselves.(Hofstadter 2007) Later, Russell famously defined mathematics as ' *the subject where no one knew what they were talking about or whether what they said was true*.'[2] Russell was working with a narrow definition of mathematics which sought to characterise elemental features of complex mathematical issues involving abstract mathematical entities. This is no longer a mainstream approach. A contrasting one is the transcendental-idealist approach formulated by Da Silva (2017). Another approach could be to stimulate understanding and to spread knowledge This is seen as a meaningful activity and is an approach adopted by many economists. The availability of computer power has been a significant contributor to recent advances in mathematics. This can be seen on the development of non-linear dynamics whose equations could not be solved by hand. It has opened new horizons especially in complexity, chaos and fractal theory. (Peters 1994)

There has always been attempts to connect mathematics to the behaviour of the universe

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[1] https://sites.tufts.edu/histmath/files/2015/11/Frege-Letter-to-Russell.pdf

[2] https://www.shmoop.com/quotes/mathematics-subject-we-never-know-what-we-are-talking-about.html

which can be loosely defined as applied mathematics. (Tegmark 2008) The implicit inference is that there is a latent mathematical quality in Nature, a quality which the casual observer would not suspect, but which nevertheless plays an important role in Nature's grand design. In pursuing this dream, applied mathematics has had many spectacular successes but within limited conditions. There is a view that is becoming prevalent that mathematics can only give approximate explanations of a restricted range of phenomena.

Hilbert associated mathematical truth with "mathematical consistency". Gödel then proved that such a system could never be complete in the sense that there will always be "undecidable" propositions. (Davis 2005) This raises the question as to the meaning of the phrase "verifiable mathematically." Mathematics, through its abstract corpus of knowledge, has a body of rules. If these rules are respected, then the resulting principles are deemed "mathematically proved". Forrester restated Godel's theorem in the context of establishing truth when he proposed: *'…any objective model-validation procedure rests eventually at some lower level on a judgement or faith that either the procedure or its goals are acceptable with objective proof.'* (Forrester 1969)

3. Mathematics and the Natural Sciences

In 1940, the Nobel physicist, Paul Dirac, made the following remark '…*the study of natural phenomena, has two methods of making progress: (1) the method of experiment and observation, and (2) the method of mathematical reasoning.'* Let us consider observations, experiments and measurements.

There is an immediate problem with observation which involves the paradox that one can only identify what one already knows. A neuroscientific perspective on this is: *'Knowledge is constructed in the mind of the learner.'* (Bodner 1986) A second problem is that of analysis – how to divide a whole into identifiable meaningful parts The scientific paradigm has been successful in this regard simply because it deals with inert entities. ('Inert' signifies that although the entities maybe connected in complex ways, they are incapable of autonomous decision making, learning or growth.) These problems were exacerbated in the early 1900's with the discovery of quantum effects. (Rovelli 2021) It was difficult to understand how the micro could be so different from the macro. How could they be reconciled? The onus today is to explain why we are treating the macro so differently from the micro i.e., behaviour at the quantum level is being gradually accepted as the norm. (Merali 2015)

To address Dirac's second point, it can be argued that the use of mathematical reasoning is only truly appropriate within the discipline of mathematics. As discussed, mathematics can be defined as a consistent corpus of logically derived theorems finding its perfection within, not

outside, the confines of its own disciplinary boundaries. New mathematical theories can be formed but should not refute old ones - only extend ranges and add refinements. New meanings can be assigned. The advent of quantum phenomena revealed that participant/observer dichotomy was not appropriate. It proved impossible to precisely define an electron and its behaviour. Probability was introduced much against the opinion of Einstein who declared that "*god does not play dice.*"

4. Mathematics and Economics

The difficulty in applying mathematics to economics is that one is no longer dealing with inert systems but with human beings whose behaviour can be characterised as non-linear and chaotic. Many of the entities they deal with are fractal in nature (the stock market) Behavioural economists recognise that the measurer is part of the measurement process, and that inherent biases and preconceptions are endemic. (Kahneman 2011) Many of these difficulties can be circumvented by concentrating on the relationships or the connections rather than the entities themselves. Data gathered from "empirical facts" is distinguished from proofs of "economic facts" since these are, in essence, esoteric concepts. (Hart 2005) Leijonhufuvd (1968) was one of the first to address these issues when he suggested that we treat an 'economy' as an 'economic system'.

One consequence of thinking systemically is that the strict causality demanded by mathematics is not applicable If mathematics is used, it should not be the strict logical mathematics of Russell and Hilbert. A common use of mathematics by economists is to first derive predictions from reasonable postulates; then, make use of statistics to check their validity. An example is Arrow and Debreu's (ibid) investigation of general equilibrium where they showed that the conditions which must be fulfilled for reaching such a state have not the least chance to be met in reality. The core economic theories currently use highly sophisticated statistical and stochastic models. Although, these techniques are mathematically consistent, there is little experimental evidence that corroborates economic theory and reality. (McCloskey 2005) The predictive power of statistical models in economics has been challenged notably by Keynes. In a letter to Koopmans, he argued '*... that is the dilemma of many of these enquiries, which they do not seem to me to face. Either they are dependent on too few observations, or they cannot rely on the stability of the environment. It is only rarely that this dilemma can be avoided.*' (Keynes 1971) Friedman followed this outburst in (1966,1968) with his papers on prediction where he classified models solely on their predictive power and not the strength of their postulates which could never be true. This has a resonance with Gödel's remarks.

5. Summary

One issue thus facing Economics is whether it is a discipline making sense of physical inter-actions, or does it take the abstract mathematical trajectory? Initially, Economics was eager to embrace the abstract world and has been reluctant to leave it. In this sense, like mathematics, it has become a tautological practice. Many mathematical techniques used in Economics have not been motivated towards rationalising markets nor to the emulation of markets, but to give a logical basis concerning the creation of efficient markets. (Muth 1961, Aumann & Brandenberger 1995)

The argument against the current use of mathematics in Economics is not its validity, it is its relationship to corporeal reality. This view coincides with that of Ziliak and McCloskey. (2008) Economic models attempt to replicate self-generated contemplative / introspective theory. Many philosophers believe that even under a seemingly chaotic reality, there exists a concealed but profound mathematical structure that is yet to be discovered. Is it such a deep understanding which Economic Theory is searching for? The authors do not believe that such a structure exists. There is no Economic theorem that could be classed as eternal or ideal. If this is the "Holy grail" that economists are hunting, then they may well be disappointed. Mathematics and Economics are fundamentally different. Economics must recognise the non-logical, emotional human input and use mathematics to understand rather than predict. The role of mathematics in Economics can thus be meaningful but its perception by economists needs to change.

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