The hydraulic characteristics of Roman lead water pipes: an experimental investigation

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Abstract

An experimental investigation was conducted on two lengths of Roman lead water pipe excavated near Corbridge, Northumberland, England. The pipes date from approximately AD 80. One length of pipe contained a sleeve joint. The pipes were linked to a pump system using 3D-printed connectors, and the hydraulic head loss along the pipe was measured. The pipe without a joint was used to determine a value for the wall roughness height, fitting the results to the Darcy-Weisbach equation. The wall roughness height obtained was assumed to apply to the pipe with the joint, and the additional head loss observed was then assumed to be due to the joint, allowing determination of a local loss coefficient. The pipes have a sinter encrustation, indicating that they had typically flowed partially full. Using this as an indication of water depth and deriving topographical information from the excavation report, the likely flow the pipes carried during their operational life was estimated. It was concluded that the pipe wall roughness coefficient k_s was 0.9 mm, the joint local loss coefficient was 1.159, and that during operation the pipe probably carried around 17 litres/minute.

Keywords

Roman, pipe, friction, coefficient, joint, head losses

Introduction and Previous Work

The manufacture and use of lead water pipes by the Romans is widely reported and was addressed in the contemporary works of Vitruvius (Rowland, Howe et al. 2007) and Frontinus (Rodgers 2003). As these authors allude, and many archaeological finds confirm, pipes were formed by casting a sheet of lead, rolling it up, probably around some kind of former, and soldering a joint along the top (Figure 1). Lengths of pipe were then joined *in situ* with further lead castings (see for example (Cochet and Hansen 1986) for a detailed summary of their manufacture). Pipes were used in many contexts, including urban water distribution networks such as that of Pompeii, where they connected water towers together and thence to street fountains, monumental water displays and private houses.

Figure 1: Example of a Roman lead water pipe formed from a sheet of metal with a soldered joint at the top. The pipe internal dimensions are 76 x 43 mm

Recently there has been considerable interest in gaining a greater understanding not just of the archaeology but of the function of Roman water systems, for example studies of Pompeii (Monteleone, Crapper et al. 2021), Roman Constantinople (Crapper, Ruggeri et al. 2016, Ward, Crow et al. 2017), Rome itself (Motta, Keenan-Jones et al. 2017), and Apamea (Haut and Viviers 2007). Water flows in such water systems can be modelled using what are now standard engineering approaches, but to do this requires an engineering understanding of the properties of the various pipes and other conduits and structures, in particular the frictional characteristics of the materials used.

The requirement for such knowledge gives rise to difficulty, particularly in the case of lead pipes where modern frictional characteristics are unrelatable to the Roman artefacts, as outlined by (Smith 2007), primarily due to the soldered joint in the top, which is not replicated in any modern component for which standard wall roughness height data is available.

Smith was able to find only one investigation, that of (Garbrecht 1982), in which genuine Roman pipes had been tested; these were large earthenware pipes from the Madradag Aqueduct at Pergamon. Details of these tests are sketchy, but a Darcy Friction Factor λ is recorded. This, in itself, is not of much use: λ is not constant but depends on the wall roughness height of the pipe material and the Reynolds Number of the flow (ratio of inertial forces to viscous forces within the fluid), so the figure is by no means transferrable to other contexts. (Smith 2007) bemoaned the unlikeliness of ever being able to conduct more rigorous tests on a genuine Roman specimen and so gain more useful engineering information.

The Pipes from Red House Baths

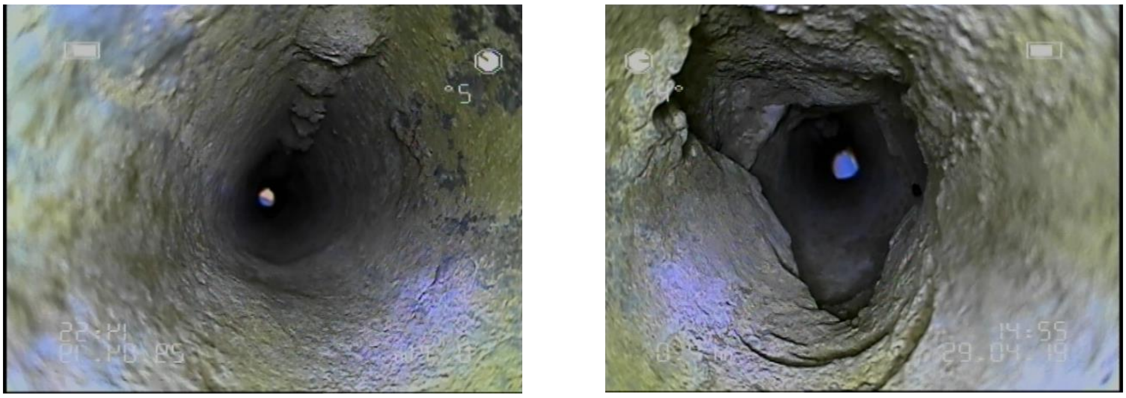
Sometime around AD 80, a Roman cavalry unit, *Ala Petrania*, arrived in the area of what is now Corbridge, Northumberland, England, and set about organizing the construction of an encampment and a bath house. This, the Red House Baths, was used until around AD 98 when it was closed and demolished. Thereafter it lay relatively undisturbed until excavated in the mid-20th century, as reported in (Daniels 1959). On excavation, one of the finds was an 18.52 m length of lead water pipe made up of lengths of approximately 3 m joined with cast sleeve joints. Short sections of this were recovered, became the property of the Society of Antiquaries of Newcastle upon Tyne, and found their way into the collection of the Great North Museum: Hancock in Newcastle upon Tyne, where one length is on display. The others were stored, and have recently been made available to the authors with permission to test them under the flow of water to determine their frictional characteristics.

Figure 2: The two pipe sections tested: A - jointed pipe and B - non-jointed pipe. Both are sections from the same run of pipe at Red House Baths, Corbridge. The pipe sections shown are all approximately 1 m in length

Two broadly similar sections of pipe were considered, each approximately 1 m in length (Figure 2). These were both sections of the same 18.52 m length of pipe excavated and reported by (Daniels 1959). One section has a sleeve joint in its centre; the other does not. This gave the potential for examining the hydraulic significance of the joint as well as the pipe itself, by comparing the performance of the two specimens under similar conditions.

Endoscope Analysis

The interior of each specimen was examined by means of an endoscope (Figure 3). It is apparent that the interior of the pipe is very rough; Figure 3 (a) clearly shows the intrusion of lumps of the soldering material at the seam in the top of the pipe, presumably an artefact of its original manufacture. Figure 3 (b) shows that the sleeve joint is crudely made, with the ends of the pipe sections joined not meeting flush in its centre. All these features are likely to have a significant effect on the hydraulic performance of the pipe.



(a) (b)

Figure 3: Endoscope images of (a) Non-jointed pipe showing the soldered seam at the top; and (b) the inside of the sleeve joint in the jointed Pipe

A further feature of note is that the pipes had a noticeable deposit of sinter (limestone) over the lower part of their section, as shown in Figure 4. The distinctive ‘tide line’ at the top of this deposit is parallel to the pipe, indicating that it arose from the flowing partially full, leading to the sinter deposit on the lower, wetted section of the pipe, but not on the drier, top part. Had the sinter formed from standing water following the baths falling into disuse, the tide line would have been horizontal rather than parallel to the pipe; in any case, sinter precipitation is the result of degassing of CO_2 from the water, a process which is enhanced by turbulence and much more likely to occur in flowing than in standing water (Buhmann and Dreybrodt 1985, Sürmelihindi, Passchier et al. 2013). Partially full flow suggests that that the soldered joint may not have been wetted all that often, and so would have had little effect on the actual operation of the pipeline. The gradual build-up of sinter would, of course, have affected the hydraulic performance somewhat over time, changing the wall roughness.

Figure 4: Distinctive sinter line observable in pipe. This indicates a substantial pattern of partially-full flow

Method

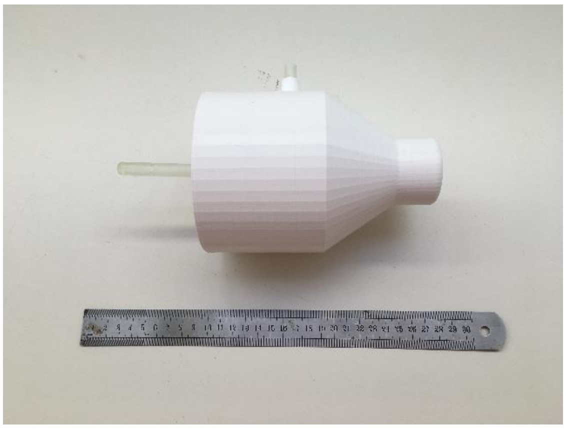
The internal surface of each section of pipe was scanned at each end using a David Light Scanner; wetted perimeters and cross section areas for pipe full conditions were determined from the scans. Those for partially full flow, as indicated by the distinctive sinter (limestone) deposit on the inside of the pipe (Figure 4), were obtained by scaling from photos, as the sinter line did not show up on the scan.

The pipes were then carefully cleaned with tap water, whilst dust was monitored to ensure that there was no meaningful risk of lead contamination for the persons involved. Appropriate PPE was used by the experimentalists, and the flushing water was disposed of in the contaminated waste stream at the laboratory at Northumbria University.

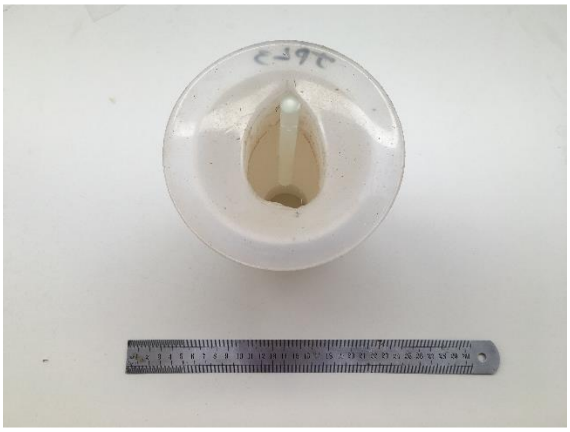
When the first section of pipe was flushed on 26th March 2019, it is reasonable to suppose that this was the first time water had flowed in any section of the pipeline since the demolition of Red House Baths in AD 98.

The pipes were linked to the experimental rig by means of 3D-printed connectors (Figure 5). These were based on the scans, which were used to create a 3D digital replica of each pipe end, and from these, individual slices were made to create 2D CAD drawings. The drawn cross-sections were decreased to 96% of their original size to ensure a tight seal and used to create plywood moulds for the casting of sealing pieces, using ECOFLEX silicon two-part mix. This created a soft seal on the outside of the pipes, where it would not interfere hydraulically or cause damage to the ancient artefacts. The soft seals were held in outer connectors that were drawn in SolidWorks and 3D-printed in PLA. These connected to the experimental pipework by means of jubilee clips. The connectors were fitted with L-shaped static tubes (Figure 5 and Figure 6), again drawn in SolidWorks and 3D-printed in Formlabs clear resin. These allowed static pressure measurements within the Roman pipe, having their end facing the flow capped off, but a series of pressure tapping holes around their circumference, normal to the direction of flow. The longitudinal length of the tube between the point where it emerged from the side of the connector and the static pressure tappings was 100 mm, and when connected to the pipe the pressure measurement point was around 60 mm away from the end of the lead; this was considered a reasonable distance away from the junction with the connector to avoid hydraulic effects of the joint, bearing in mind the already quite short length of the whole section of pipe. The connector and pressure tapping arrangement is shown in Figure 5. The effective length of each pipe section under test was therefore the measured distance between the centre lines of the static tube where they emerged from the side of the connectors, less 2 x 100 mm to allow for the L-piece at the pipe centre line.

Flow was controlled by means of a G-clamp compressing the flexible hose attached to the pump (Figure 6); this simple arrangement allowed quite fine control. Flow was measured by recording the time taken to fill a 30 litre container, averaged over at least two similar readings. The static pressure difference between each end of the pipe section was measured using a simple manometer arrangement with a metre rule and flexible tubes taped securely in place (Figure 6). At the start of each test, air was carefully expelled from the manometer tubes, and the manometer reading was photographed three times to avoid difficulties in reading it caused by small water level fluctuations; the average reading was taken from three photographs.



(a)



(b)

(c)

Figure 5: The 3D-printed connector and pressure measuring arrangement: (a) side view; (b) end view; (c) in situ on pipe

The Roman pipe section was arranged to be in its correct orientation with its soldered seam uppermost, as it (like all such pipes) was found *in situ* when excavated, and to be level within the tolerance defined by its naturally uneven and not-straight nature. The pipe was at all times below the surface of water in the sump tank (Figure 6), ensuring it remained in pipe-full conditions throughout the test.

A full view of the experimental arrangement is shown in Figure 6.

Figure 6: Overview of the experimental rig showing the jointed pipe under test, with manometer and flow control. To the left is the sump tank in which the pump was located

Results

Pipe dimensions and values of hydraulic radius and diameter derived from them are recorded in Table 2.

Table 1: Pipe dimensions and calculated hydraulic radii and diameters

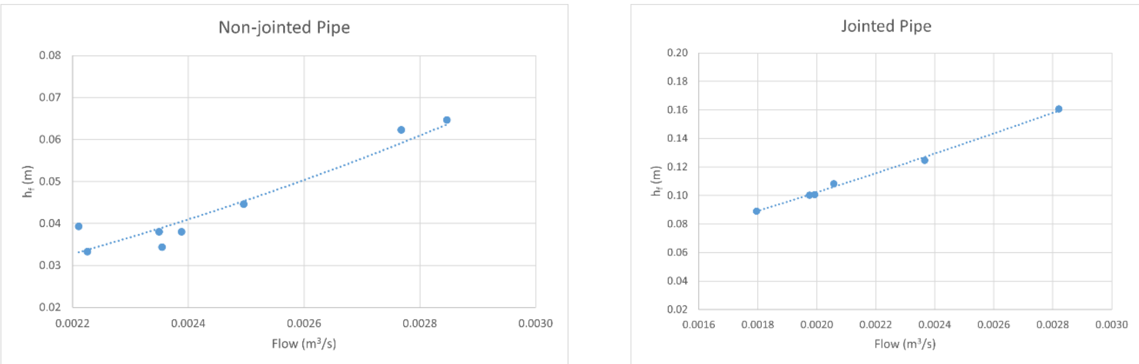
Pipe Full Conditions						
		Area (m^2)	**Perimeter (m)**	**Hydraulic Radius (m)**	**Hydraulic Diameter (m)**	**Length between Pressure Tappings (m)**
Non-jointed Pipe	End 1	0.00240	0.187	0.013	0.051	0.828
	End 2	0.00230	0.181	0.013	0.051	
	Average	**0.0023**		**0.013**	**0.051**	
Jointed Pipe	End 1	0.00156	0.149	0.010	0.042	0.809
	End 2	0.00267	0.196	0.014	0.054	
	Average	**0.0021**		**0.012**	**0.048**	
Partially Full Conditions						
		Height of Sinter Line (m)	**Area (m^2)**	**Perimeter (m)**	**Hydraulic Radius (m)**	
Non-jointed Pipe		0.022	0.0007	0.072	0.009	
Joint Pipe		0.024	0.0008	0.078	0.010	

Results took the form of time for 30 litres of flow and the head difference between the pressure tappings at each end of the pipe. These were converted into flow rate and head loss (h_f) along the pipe. Results are shown in Table 2, together with calculated flow velocities and Reynolds numbers (Re) based on the hydraulic diameter of the relevant section of pipe, and are plotted in Figure 7.

Table 2: Experimental results and calculated flow velocities and Reynolds numbers

	Time to fill 30 litres (s)	Flow rate (m^3/s)	Observed head loss h_f (m)	Flow velocity (m/s)	Re
Non-Jointed Pipe	13.57	0.00221	0.039	0.961	49021
	13.48	0.00223	0.033	0.968	49348
	12.77	0.00235	0.038	1.021	52092
	12.74	0.00235	0.034	1.024	52215
	12.56	0.00239	0.038	1.038	52963
	12.02	0.00250	0.045	1.085	55343
	10.84	0.00277	0.062	1.203	61367
	10.54	0.00285	0.065	1.238	63114
Jointed Pipe	16.71	0.00180	0.089	0.855	41036
	15.19	0.00197	0.100	0.940	45142
	15.06	0.00199	0.101	0.949	45532
	14.58	0.00206	0.108	0.980	47031
	12.68	0.00237	0.125	1.127	54078
	10.64	0.00282	0.161	1.343	64447

Whilst not specifically recorded for each test, the absolute value of static head reached a maximum of about 1 m above the pipe invert, corresponding to around 10 kPa.



(a) (b)

Figure 7: Plots of experimental observations for (a) Non-jointed and (b) Jointed Pipes

Discussion

In commencing a discussion of the results, it must be remembered that the pipe sections tested were only around 1 m in length; ideally, a much longer section would have been tested to avoid the possibility of end effects at the connectors and pressure tappings and to give a more representative result. Unfortunately, no longer length of Roman pipe was available, so the following discussion must be viewed in the context of this limitation.

The Reynolds numbers of the pipe flow were all greater than 4 x 10^5, indicating turbulent flow.

It is apparent from Figure 7 (a) that in the case of the Non-Jointed Pipe, there is considerable scatter in the experimental results, particularly at the lower end of the flow readings. There are many possible reasons for this, including misreading the flow time, the flow volume or the head difference, as well as possible fluctuations in flow, e.g. due to a partial blockage in the system, during the course of a set of readings. None of these is known to have occurred, but neither can they be ruled out with absolute certainty. What was observed, particularly in the case of the Non-jointed pipe, were a number of small leaks through the soldered seam at the top of the pipe, which would have of course contributed to some small error in the flow measurement, increasing with pressure as the flow was increased. It was not possible to quantify these leaks, but visual observation suggested they would be of only minor importance in the overall test. In view of the pipes' age of almost 2000 years and their lack of maintenance for all but the first twenty of these, they proved remarkably robust.

It is clear that, as expected, the absolute values of head loss are much greater for the Jointed Pipe for given flows. This is no-doubt due to the losses attributable to the sleeve joint.

Pipe Roughness Value k_s

The Darcy-Weisbach equation for head loss in a pipe in terms of the Moody friction factor λ is well established for pipe flow (Chadwick, Morfett et al. 2013) (page 111):

$$h_f = \frac{\lambda L u^2}{2gD} \tag{1}$$

Here *L* is the pipe length, *u* the flow velocity, *g* the acceleration due to gravity and *D* the hydraulic diameter (or actual diameter for circular pipes).

The unknown here is λ, which can be computed using the well-known Colebrook-White equation (Chadwick, Morfett et al. 2013) (Page 114):

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{R_e\sqrt{\lambda}}\right) \tag{2}$$

Here k_s is the pipe material roughness height, which is the principal unknown for Roman pipes, given their ancient and specific construction with a rough soldered joint.

Taking the non-jointed pipe section, it was assumed that all head loss over the test resulted from pipe friction loss as given by equations (1) and (2), and a curve fit was carried out to the experimental results, varying k_s until the best fit in terms of the minimum sum of residuals squared was obtained, residuals being the difference between the calculated head loss for the observed flow, and the observed head loss. This gave a fit with a correlation coefficient of 0.961 corresponding to a k_s value of 0.9 mm, which is indicative of a rough pipe. To put this in context, a value commonly used for a modern concrete pipe is 1.5 mm, and that for a PDPE pipe is typically 0.03 mm.

It is not possible to claim that our value of 0.9 mm is in any way generally applicable to Roman pipes; quite apart from the limitations of measuring using short lengths of pipe already mentioned, it is unlikely that manufacturing processes and tolerances were well controlled over the whole Empire. There would also need to be allowance made for the effect of the sinter encrustation over the lower part of the pipe surface. We are also unable to determine how much of the roughness actually arises from the wall roughness *per se*, and how much from the intrusions along the soldered joint, which is a manufacturing issue and perhaps not well represented by k_s at all, since this parameter is intended to be more of a material property.

A plot of the curve-fit is shown in

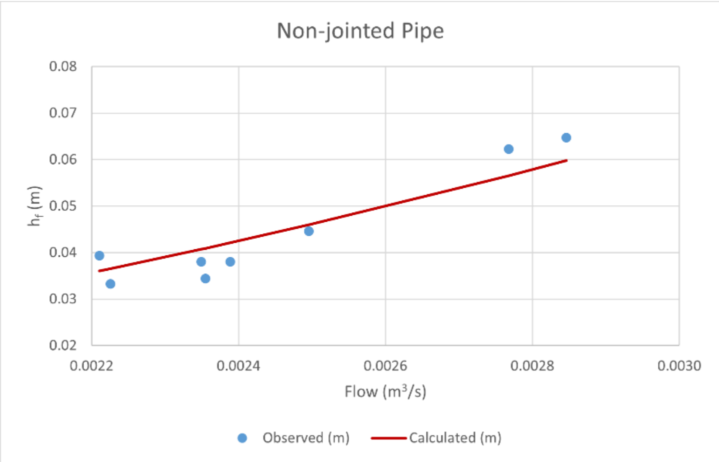


Figure 8: Observed and calculated head loss against flow for Non-jointed Pipe. Calculated values use the Darcy-Weisbach and Colebrook-White Equations, with k_s = 0.9 mm

. Whilst the raising of the velocity to the power of two in equation (1) arises from dimensional analysis, it is clear that in order to gain an exact fit to the observed data, a higher power is required. In fact, using a power of 2.6 gives a much better fit at higher flow values, whilst not making dimensional sense. Varying this power would also offer no help in any subsequent use of the information in network modelling using typical application software, where use of the established pipe-flow equations is mandated and only parameters such as k_s or D can easily be changed.

A longer length of pipe than those available to us would provide a more robust test, but none is currently available. However, an obvious future step here is to obtain suitable equipment and internally scan the Roman pipes so that a detailed computational mesh can be obtained and a Computational Fluid Dynamics (CFD) simulation carried out to determine the flow patterns in detail and relate those to fluid shear forces and corresponding friction losses.

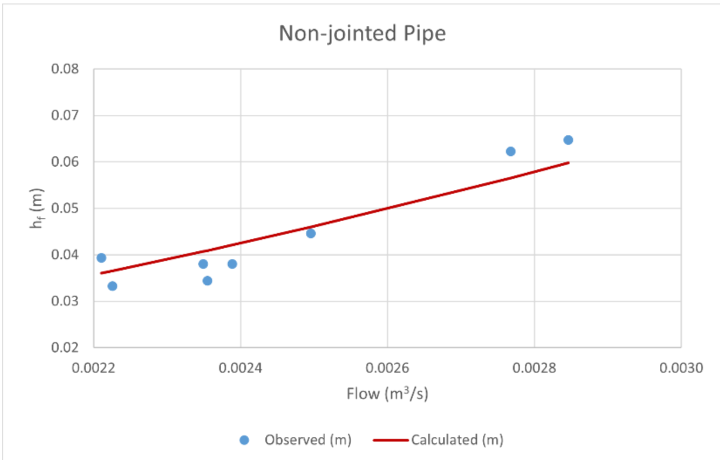


Figure 8: Observed and calculated head loss against flow for Non-jointed Pipe. Calculated values use the Darcy-Weisbach and Colebrook-White Equations, with k_s = 0.9 mm

Joint Local Loss Coefficient k_{joint}

The other significant unknown for Roman pipes is the head loss contribution of the sleeve joint, which as discussed is rather crude, particularly internally.

In order to determine this, it was assumed that the k_s value determined for the non-jointed pipe applied also to the jointed pipe, and any losses above this were a result of the Roman sleeve joint, according to the usual local head loss formulation:

$$h_{f\,joint} = k_{joint}\frac{u^2}{2g} \tag{3}$$

A curve-fitting exercise was done as previously, this time varying the local loss coefficient for the joint until a good fit was obtained between observed and calculated head loss, as before. A joint local loss coefficient of 1.159 gave a R^2 correlation of 0.998. Results are plotted in Figure 9.

In some ways this value is surprisingly low given the apparent rough nature of the joint: it is comparable with the losses at a T-connection in a modern piping system. However, a test calculation indicates that for the friction contribution of the joints in a run of pipe to become negligible (taken, rather liberally, as less than 10% of the total head losses), the joint spacing would have to be at least 10 m. Sections of pipe this long would be very difficult to manufacture and handle, and there is no evidence the Romans attempted to do this. The pipe sections recorded by (Daniels 1959) were of 3.048 m (i.e. 10') length, which we assume to be an approximate average rather than a detailed measurement.

It may be concluded that an allowance for the sleeve joints in pipe runs should be made in any network modelling of Roman water systems involving such lead pipes.

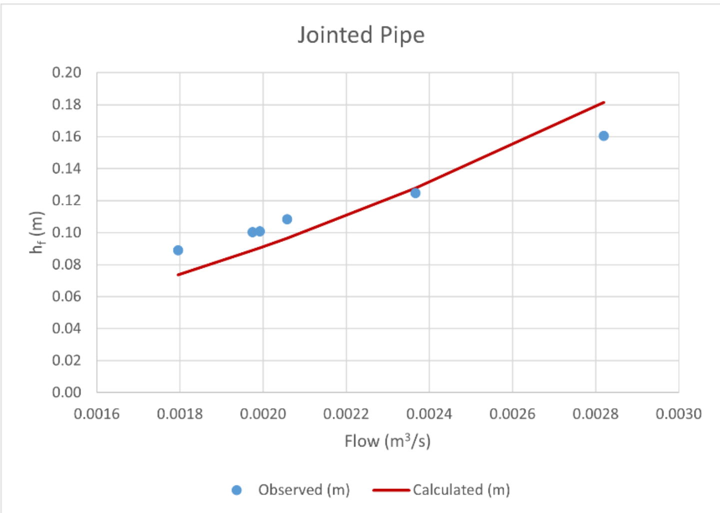


Figure 9: Observed and calculated head loss against flow for Jointed Pipe. Calculated values use the Darcy-Weisbach and Colebrook-White Equations, with k_s = 0.9 mm and k_{joint} = 1.159

Pipe *in Situ*

Our experiments were intended to determine general engineering parameters describing the hydraulic performance of the Roman pipe and pipe joint; in doing so we generated a maximum flow of 0.00285 m^3/s (Table 3), which converts to 171 litres/minute. It was apparent in the experiments that this level of flow required careful management as it led to rapid filling of the measuring tank and a powerful jet. It is now interesting to consider the flow that might have been carried by the pipe in its original situation at Red House Baths. Topographical information in (Daniels 1959) is sparse, but there is enough to make some estimates.

Daniels' Fig. 4 shows the length of pipe in the south east corner of the bath house courtyard. Where it crosses the southern wall, it is noted that this is beneath a doorway where a sleeper wall had had the lower three of four courses of stone removed to make space for the pipe. Considering Daniels' Fig. 6, centre section across Rooms II, III and IV, the pipe at the eastern side of the courtyard (its upstream end) can be observed, and the courses of stone on the southern wall are shown and can be roughly scaled at 0.2' per course. Assuming the pipe invert was about the level of the second course up from the foundation, that would be three courses or 0.6', i.e. 0.183 m below the courtyard surface. Similarly, the same section shows the upstream end of the pipe to be 0.24' or 0.073 m below the courtyard surface. This gives an approximate drop along the length of pipe within the courtyard of 0.110 m.

Turning back to Daniels' Fig. 4, the length of pipe within the courtyard can be roughly scaled at 14.8 m; six sleeve joints are shown, though this contradicts the record that the pipe was in 3.048 m sections (which would give 4 or 5 joints only).

Meanwhile, the sinter line in the pipes gives an indication of the likely pattern of partially full flow in the pipe, allowing partially full calculations to be made. The following equation was used (Chadwick, Morfett et al. 2013)(Page 129) to determine the proportion of the pipe-full flow, Q_p, that flows in a partially-full pipe:

$$Q_p = \left(1 + \frac{\log_{10} R_p}{\log_{10}\left({}^{3.7D}/_{k_s}\right)}\right) A_p R_p^{1/2} \qquad (4)$$

Here R_p is the proportional hydraulic radius and A_p the proportional area of the partially full flow, which as discussed was determined from scaling from photos showing the sinter line in the pipe sections.

Head losses for various pipe full flows were calculated using the Darcy-Weisbach/Colebrook-White approach (equations (1) and (2)) together with the appropriate number of local losses for the joints (equation (3)), in the normal way, and using the values of k_s and k_{joint} obtained from our earlier analysis. These were balanced with the assumed drop in height. Proportional flows corresponding to the observed depth from the sinter line were then determined from equation (4) above. We have assumed that equation (4) continues to hold true in the presence of local losses such as joints.

Results of calculations are shown in Table 3. Various calculations were made for each pipe section (Non-jointed and Jointed) but those shown are for their average dimensions (bearing in mind they were part of the same run of pipe) and with a sensitivity analysis showing the effect of under- or overestimating the length and drop of the pipe (by 10% and 20% respectively) and by incorporating 4, 5 or 6 sleeve joints. The total variation range in flows calculated for all analyses was only between 15.0 and 18.5 litres/minute, which is comparable with the flow from a modern bath tap.

One thing to note here is that in the partially full situation, the soldered joint in the top of the pipe would have no impact on the flow as it would not be wetted. This may mean that the k_s value of 0.9 mm is an overestimate for this circumstance, and the actual flows would be somewhat greater than our calculations indicate.

In fact, so many assumptions are inherent in this analysis that more detailed quantification is pointless, but it does give an idea of the flow likely carried by the pipe when in its design situation almost 2000 years prior to this study.

Table 3: Partially full flow calculation results for various assumptions

Assumed Pipe Length (m)	Assumed Number of Joints	Assumed Drop in Height (m)	Pipe Full Velocity (m/s)	Pipe Full Re	Partially Full Q (l/min)
14.8	5	0.088	0.395	19554	15.0
14.8	5	0.110	0.444	21979	16.8
14.8	5	0.132	0.488	24179	18.5
14.8	4	0.110	0.472	23341	17.9
14.8	5	0.110	0.444	21979	16.8
14.8	6	0.110	0.421	20835	16.0
13.3	5	0.110	0.455	22524	17.2
14.8	5	0.110	0.444	21979	16.8

We believe that our experiment is the first ever attempt to quantify the hydraulic characteristics of Roman lead water pipes, including in ancient times, when lack of the ability to accurately measure small intervals of time would have made this kind of analysis impossible in any form.

Conclusions

An experimental study has been carried out in which Roman lead pipes manufactured in approximately AD 80 for Red House Baths, near Corbridge, Northumberland, England, were tested to determine their hydraulic frictional characteristics. This is the first occasion, including in ancient times, that such a test has been carried out on lead pipes of Roman manufacture.

Two sections of the same pipe run were loaned by the Great North Museum: Hancock on behalf of the Society of Antiquaries of Newcastle upon Tyne, these being sections of the excavated pipe approximately 1 m in length. One is a plain pipe and the other had a cast lead sleeve joint in its centre. The pipes were connected to a water supply and measurement system using 3D-printed connectors and measurements of flow and head loss taken.

It was concluded that the pipes have a wall roughness height k_s of 0.9 mm and that the Roman cast lead sleeve joint in one of them has a local loss coefficient k_{joint} of 1.159.

A sinter line was observed in the pipes suggesting that they had run partially full during their operational life. Reference to the excavation report from the 1950s enabled assumptions to be made about the length and gradient of the pipe, which in turn allowed a calculation of the likely flow carried by the pipe running partially full up to the sinter line. It was concluded that the flow may have been in the region of 17 litres/minute, which is comparable with that obtained from a modern-day bath tap.

It is recommended that suitable equipment be obtained to conduct a detailed scan of the inside of the pipe sections to provide the basis for a computational mesh for a CFD investigation for further understanding of the pipe characteristics.

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