An Improved Sliding Mode Control (SMC) Approach for Enhancement of Communication Delay in Vehicle Platoon System

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Abstract: Vehicle platoon systems are widely recognized as a key enabler to address mass-transport. Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) are two technologies that drive platooning. The inter-vehicle spacing and the collaboration velocity in the platoon are main important parameters that must be controlled. Recently, a new mass-transport system has been proposed, called the Tracked Electric Vehicles (TEV). In TEV, the inter-vehicular spacing is reduced to only a quarter of the regular car length and cars drive at 200km/h which enable mass transport at uniform speed. However, conventional radar based Adaptive Cruise Control (ACC) system fail to control each vehicle in these scenarios. Lately, Sliding Mode Control (SMC) has been applied to control platoons with communication technology but with low speed and without delay. This paper proposes a novel SMC design for TEV using global dynamic information with the communication delay. Also, graph theory has been employed to investigate different V2V communication topology structures. To address the issues of node vehicle stability and string stability, Lyapunov candidate function is chosen and developed for in-depth analysis. In addition, this paper, uses first-order vehicle models with different acceleration and deceleration parameters for simulation validations under communication delay. The results show that this novel SMC has a significant tolerance ability therefore meet the design requirements of TEV.

1 Introduction

Presently, there are several transport challenges such as congestion, pollution, provision of stress-free travel which are all user-centric problems. One must also be considered that the increasing data use and connectivity will have a greater role to play in the future as well. Moreover, new technologies have to scrutinize the cost, safety and efficiency which are also the three main demands of mass-transport systems as well as Highway Transportation Systems (HTS)[1]. In order to operate platoons in an effective manner, an Intelligent Transportation System (ITS) is required [2]. Vehicle-to-vehicle (V2V) communication and vehicle-to-infrastructure (V2I) communication are the two key technologies in ITS [3]. At the heart of this are its On-Board Units (OBU) and sensors (embedded in vehicles and the infrastructure) which are used to collect, relay and share information such as the driver’s control decision [4], vehicle’s position, velocity and acceleration of each car.

The impact that a platoon can have on fuel saving was first studied extensively by the Program for the Advanced Technology Highway (PATH) [5]. Nowadays, platoons are being considered by many organizations world-wide such as the Grand Cooperative Driving Challenge (GCDC) in the Netherlands [6], Safe Road Trains for the Environment (SARTRE) in Europe [7] and Energy-ITS in Japan [8]. Despite large programs, individual companies such as the car manufacturer Volvo are also experimenting in this area of research. Volvo drew wide-spread media attention when the company successfully built a highway truck platoon where each truck drives at a speed of 100km/h with a 1.5s time gap [9]. With advancements in communication technology such as 4G, 5G, and even the future 6G, vehicle communication can be greatly aided by these technologies. It also removes distance as a constraint on the topology of inter-vehicle communication. As a result of the topological structure variety, new challenges emerge, which is especially important when considering time delay and packet loss in communications. For example, Alipour-Fanid et. al. conduct a comprehensive analysis on the stability and safety of the platoon under the wireless Rician fading channel model and jamming attacks [10]. The problem of centralized control for a platoon of non-identical vehicles under constant time headway strategy (CTHS) is investigated using multi predecessors following (MPF) topology [11]. Muehlebach et. al. propose a synthesis procedure for designing the agents’ state estimators and the event triggering thresholds [12]. The optimization algorithm performs the computation of the control input in a control horizon window and ensures that the spacing error take only positive values [13]. Real-world vehicular data from video traffic detection (VTD) are used for minimizing the travel delay at intersections and a real-time traffic optimization model, based on the SUMO traffic simulation software, is established accordingly [14]. Sau et. al. present the state-space representation of the linearised dynamical system [15]. Especially Mohammed et. al. aim to improve the Greedy Traffic Aware Routing (GyTAR) protocol to support QoS in IoV networks [16].

At the hub of this technology is a platoon controller which must dynamically control all vehicles within the platoon. Mostly a one-dimensional longitudinal control method is the common approach to be applied in platoon controllers. This concept is used to keep a desired distance between neighbour vehicles and maintain a desired velocity for each car within the platoon system [17]. For example, Swaroop et al. [18][19] applied the classic spring effect to control a platoon. To control the inter-vehicular spacing, the Constant Distance (CD) and the Constant Time Headway (CTH) methods have been proposed. CTH is preferred when safety is the main concern and CD is typically employed when the goal is efficiency. In parallel to the development of platoon control, there have to be advancements in SMC platoon theory. In 2014, Ji-Wook Kwon and Dongkyoung Chwa [22] proposed a bidirectional control with a sliding mode method. With the development of communication, the node vehicle in a platoon system can obtain more information depending on communication structures. Following this research, in 2019 Li et al [23] designed an experimental 4-vehicle-platoon...
system with a global SMC method. A key feature of this work involved V2I communication. In platooning stability, Tae Soo No et al. [24] improved the original Lyapunov stability theorem by using the concept of ‘Expected Spacing Error’ and implemented it in various platooning scenarios. In this work, they have the Lyapunov Function (LF) approach by adding V2V communication with the topology structure matrix to demonstrate the stability of the whole platoon. The proposed LF proves the reaching law stability and sliding surface stability.

However, with the development of the communication technology, the vehicular platoon controlled only by radar cannot meet the current traffic demand. One of the latest platooning scenarios is described in Tracked Electric Vehicle (TEV) [25]. The TEV system is a fully automated highway system for Electric Vehicles (EVs) to achieve HTS zero emissions. The TEV lane is designed as a single lane with no restricted access. EVs drive fully automatically where 10 vehicles form one platoon and each vehicle drives at a constant velocity of 200km/h. The inter-vehicle spacing is only a quarter of a car’s length. This short length reduces the overall aerodynamic drag coefficient of all cars including the front car. Such arrangement can save power and in [26] it is claimed that a power saving of 40% for the whole platoon is possible as compared to a scenario in which all 10 cars are driving individually (not in a platoon). The challenge with TEV is the control of short distance at high speed. Currently, SMC has been applied for platoons but without communication topology structures and high speed. To tackle the challenge of reducing the inter-vehicle distance of less than 1 car length, an advanced SMC capable of including V2V is proposed and developed in this paper.

The proposed SMC controller can ensure stable vehicle platoon system in terms of enhancing traffic efficiency and road utilization by reducing inter-vehicle distance. The final experimental results verify the effectiveness of the controller which demonstrate that the inter-vehicle distance can be effectively maintained as 0.4-0.6m under different communication structures. In the case of communication delay, the inter-vehicle distance depends on the value of the communication delay. Therefore, this paper makes the following four main contributions:

- The design of a novel SMC with V2V and V2I communication to control the vehicular distance in a non-homogeneous platoon system.
- In-depth investigation of vehicular communication structures in influencing system stability by employing LF.
- Demonstration of the vehicle system lumped delay in a vehicle dynamic model utilizing system identification method.
- Exploring the features of SMC and its tolerance for communication delays in the simulation.

The rest of the paper is organized as follows: Section 2 presents the background theory and related work. The mathematical model and the stability analysis are described in section 3 for the platoon system. Section 4 presents the variety of controllers that use SMC and in particular those that can be used for the platoon system. The simulation results for the proposed SMC are shown in section 5. In section 6, it studies the features of the SMC and adds the dynamic communication delay in this system. Finally, Section 7 presents the concluding remarks showing the inter-vehicular spacing error (5.5m) is rapidly decreased with V2V communication SMC controller, compared with the error of (0.5m) in platoon systems without the lead vehicle information.

2 Background Theory

In order to characterize the information topology structure in the vehicle platoon system, Zheng et al. [27] [28] use matrices and graph theory. This technology is widely used in communication flow structure. Assuming there is an N-size vehicle platoon system, we use a directed graph $G_N = (V_N, W_N)$ to describe the information transmission. In this set, $V_N$ denotes the vehicle set and $W_N = V_N \times V_N$ denotes the edge set. In a platoon system, there are two random vehicles $i$ and $j$ ($i,j \in N$). Here, the edge set $w(i,j)$ represents vehicle $j$ that can receive the vehicle’s dynamic information from vehicle $i$. The definition of a directed path is a sequence of edge set $w(1,2), w(2,3), w(3,4), ..., w(k-1, k), (k \leq N)$. This set is the directed path from node $i$ to node $k$ within the platoon system. Another definition is of the directed spanning tree, assuming that there is at least one vehicle that can acquire information from any other vehicle(s) directly or indirectly. In other words, this vehicle (node) has a directed path towards/or for the others. This directed path is known as a directed spanning tree and the vehicle (node) is known as the root of this directed spanning tree. In [8], to control the platoon system, an assumption is made that $G_{N-1}$ has a directed spanning tree with the lead vehicle ($N = 0$) as the root. Usually, the lead vehicle is globally reachable.

Before using a directed graph $G_N$, we provide a brief introduction to how we define our matrices using the following steps:

1. Adjacency Matrix $A = [a_{ij}]$:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in W_N \\ 0 & \text{if } (i,j) \notin W_N \end{cases} \quad (1)$$

In this case, $a_{ij}$ denotes vehicle $i$ that can acquire the dynamic information from vehicle $j$, $a_{ii} = 0$ indicates there is no self-loop in this graph.

2. Laplacian Matrix $L = [l_{ij}]$:

$$l_{ij} = \begin{cases} l_{ii} = \sum_{k \neq i,j=1}^{N} a_{ij} \\ l_{ij} = -a_{ij} \end{cases} \quad (2)$$

In the Laplacian matrix, each element expresses the vehicle information that vehicle $i$ can obtain through V2V communication or radar detection in the following vehicle.

3. Leader adjacency Matrix or Running Matrix:

The Leader Adjacency Matrix (LAM) represents that the followers can obtain the information from the leading vehicle. And so LAM can be defined as:

$$P_N = \begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix} \quad (3)$$

Here, $p_i$ can be 0 or 1, if $p_i = 1$ then vehicle $i$ can receive the information from the lead vehicle, otherwise $p_i = 0$.

Fig.1 shows typical structures of vehicle communication: Predecessor Following (PF) means the followers can only receive information from those in front. Bi-Directional (BD) allows the vehicles to share their information with its neighbours. Predecessor-Following Leader (PFL) permits the vehicles to receive dynamic information from the lead vehicle and its predecessor. Using Graph theory, we can write the LAMs and Laplacian Matrices for these structures as:
\[ L_{PF} = \begin{bmatrix} 1 & \cdots & \cdots & \cdots & 1 \\ -1 & \cdots & \cdots & \cdots & -1 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & 1 \end{bmatrix}, \quad P_{PF} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \]

\[ L_{BD} = \begin{bmatrix} 1 & -1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -1 \\ -1 & 2 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -1 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ -1 & \cdots & \cdots & \cdots & 1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -1 \\ 1 & \cdots & \cdots & \cdots & -1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 1 \end{bmatrix}, \quad P_{BD} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \]

\[ L_{PLF} = \begin{bmatrix} 1 & \cdots & \cdots & \cdots & 1 \\ -1 & \cdots & \cdots & \cdots & -1 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & 1 \end{bmatrix}, \quad P_{PLF} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix} \]

$L + P$ matrix is important in the platoon system closed-loop stability study. In [29], it has been shown that if the directed graph $G_{N-1}$ has a directed spanning tree, then $L + P$ is defined as positive. This matrix is used to calculate the stability margin for linear node dynamic. In our new proposed SMC design, we use $L + P$ matrix methodology to prove the string stability of the platooning system.

### 3 System Modeling

In order to model a vehicle platoon for TEV, modelling of an individual vehicle, spacing policies describing the distance between vehicles and string stability must be introduced which is explained in this Section.

#### 3.1 Individual Vehicle Dynamic Model

Below are the assumptions made for the vehicle dynamic model:

1. Vehicles only experience rolling friction and aerodynamic force.
2. Vehicles used in this paper do not use gear shift for torque conversion.
3. Only a 1-D longitudinal dynamics model is considered.
4. Vehicles are treated as ideal rigid thus ignoring the unbalanced left and right movement of cars.

Based on the above forces acting on a vehicle can be written as:

\[ m\ddot{x} = F_t - F_r - F_aero \]  \hspace{1cm} (4)

Where $m$ is the mass of the car, $\ddot{x}$ is the acceleration, $F_t$ is the force due to rolling resistance and $F_aero$ is the aerodynamic force. Thus, if the term $(F_t + F_aero)$ is equal to $F_t$ the car is driving at a constant speed. Equation (4) assumes that the wheel rolling resistance for each wheel is the same and that the car is driving in a straight lane without any elevation. If the term $(F_t + F_aero)$ is less than $F_t$ the vehicle accelerates and if it is greater than $F_t$ the vehicle decelerates. In order to determine the velocity of the car we use the following equation:

\[ v = \dot{x} = R_r \dot{\omega}_e \]  \hspace{1cm} (5)

where, $\dot{\omega}_e$ is the motor speed, $R$ is the gear ratio and $r_e$ is the effective tire radius. With the details of the vehicle drive train technology the derivative of $\dot{\omega}_e$ can be expressed as:

\[ \ddot{\omega}_e = \frac{T_{net} - c_0 R^2 \dot{\omega}_e^2 - R(r_e F_v)}{J_e} \]  \hspace{1cm} (6)

where, $\dot{\omega}_e$ represents the acceleration/deceleration of the motor-shaft speed. $T_{net}$ is the net motor torque, $c_0$ is the aerodynamic drag coefficient and $J_e$ is the motor inertia respectively. From equation (5) and combining it with equation (6), we have:

\[ \ddot{x} = R_r \dot{\omega}_e = R_r \left( \frac{T_{net} - c_0 R^2 \dot{\omega}_e^2 - R(r_e F_v)}{J_e} \right) \]  \hspace{1cm} (7)

In equation (7) none of the parameters can be influenced by the driver except the net motor torque $T_{net}$ which is the required torque to produce $F_v$ in (4). In EVs, electric drive trains are torque controlled and therefore $T_{net}$ is a demand value. Consequently, as shown in (7) the demand of $T_{net}$ results in an acceleration/deceleration represented by $\ddot{x}$.

Although equations (4)-(7) are sufficient to study the vehicle dynamics [30] vehicle platooning requires a different set of equations. The most common mathematical model for studying vehicle dynamics of a platoon is called double-integrator model [31] and its equation is:

\[
\begin{cases}
\dot{x}_i(t) = v_i(t) \\
\dot{v}_i(t) = u_i(t)
\end{cases}
\]

\[ \text{In this equation, } u_i(t) \text{ is the output acceleration in the platoon of each vehicle, where } i \text{ ranges from } 0 \text{ to } N \text{ and } N + 1 \text{ is the platoon size including the lead vehicle. Despite the fact, that this model considers the details of vehicle dynamics it is not suitable as a true representation of vehicles' behavior in a platoon as delays within the platoon system are not reflected. Therefore, many studies have used a 'lumped' delay } \tau_i \text{ to represent a delay in vehicle dynamics [8] [30] [32] and the fundamental equation used is:}
\]

\[ \dot{x}_i = \frac{1}{\tau_i s + 1} u_i \]  \hspace{1cm} (9)

Adding $\tau_i$, changes (9) to:

\[ \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t), x_i(t) = \begin{bmatrix} p_i \\ v_i \\ \dot{v}_i \end{bmatrix}, \quad A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix} \]  \hspace{1cm} (10)

Where $p_i$ and $a_i$ are the position and acceleration of vehicle $i$ receptively.

#### 3.2 Spacing Policies

In a platoon system study, the main control target is maintaining the desired space and velocity consistency. When using a CD policy, it is important to maintain a small but safe distance, hence it can achieve higher traffic efficiency. In the CTH policy the distance follows a linear relationship with self-velocity; somehow it is similar to a driver’s behavior but the distance between the two vehicles is larger. As a result, it cannot deliver efficiency as high as the CD policy, however, is safer than using the CD policy.

##### 3.2.1 CD Policy: As Fig.2 shows, the inter-vehicular spacing is defined as:

\[ \varepsilon_i = x_{i-1} - x_i - l_i - 1 \]  \hspace{1cm} (11)

where $x$ is the inter-vehicular spacing, $x_i$ and $x_{i-1}$ are the vehicle head position for vehicle $i$ and vehicle $i-1$; $l_i$ is the vehicle length of vehicle $i$. Then the spacing error for vehicle $i$ can be defined as:
The string stability polynomial is
\[ z^\alpha \text{ or 'weak string stability', otherwise it is not stable. Now polynomial so that if the roots are inside the unit cycle it means it is } Z \text{ equal to or less than 1. To consider critical scenarios and using the In this equation, to ensure the string does not diverge, } α \text{ is appropriate for typical feedback control. Hedrick [36] defines a typical slide surface } S_i \text{ that combines the dynamic information with the lead and preceding vehicles. In this paper, the slide surface has changed to the following equation:}

\[ S_i = q_1 \delta_i + q_2 \delta_i + q_3 (x_i - x_0) + q_4 (x_i - x_0 + \sum_{j=1}^i d_{j,dx}) \]  

where } S_i \text{ is the } i\text{th vehicle slide surface in the platoon system, } \delta_i \text{ is the velocity error of the } i\text{th vehicle with respect to its preceding vehicle, } d_{j,dx} \text{ is the fixed distance from vehicle } i \text{ to the leading vehicle, } q_1, q_2, q_3 \text{ and } q_4 \text{ are the coefficients for the slide-controller. It is assumed that the length of each vehicle has been ignored. The reaching law for the vehicle in the platoon system can be defined as:}

\[ \dot{\delta}_i = -\lambda S_i \]  

where } \lambda > 0 \text{ is the turning parameter. It is used in equation (18) and (19) to calculate the input of each vehicle in the platoon system:}

\[ u_i = -\frac{1}{q_1 + q_3} \left[ q_1 \dot{x}_i - 1 + q_3 x_0 - (q_2 + \lambda q_1) \dot{\delta}_i - (q_4 + \lambda q_3) \cdot (x_i - x_0) - \lambda q_2 \delta_i - \lambda q_4 (x_i - x_0 + \sum_{j=1}^i d_{j,dx}) \right] \]  

where } x_0, x_0 \text{ and } x_0 \text{ are the dynamic information of the lead vehicle, } \delta_i \text{ is the spacing error between the } i\text{th vehicle and the } i-1\text{th vehicle. Using equations (18) and (20), numerical simulations for the platooning system can be performed and this is shown in the next section. If we use the vehicle model as described in equation (9), it means it must consider the actuator and communication lags, which then changes to:}

\[ \frac{d}{dt} \bar{u}_i + u_i = u_{id} \]  

where } u_{id} \text{ is the input with 'humped' lags for vehicles. We now need to analyze the stability of this controller. As shown in equation (15), the string stability polynomial can be used to calculate the stability margin. Moreover, the transfer function } \tilde{H}(s) \text{ for the error propagating in the platoon has to be bound to a constant } \alpha. \text{ In this case, in order to construct the transfer function for spacing error it can use } S_i(s) - S_{i-1}(s) \text{ in s-domain, such that,}

\[ \tilde{H}(s) = \frac{\Delta_i(s)}{\Delta_{i-1}(s)}, \]  

In a string stability study, these can be summarized if the following condition is met, which have also been validated in [34]:

\[ \| \tilde{H}(s) \|_\infty \leq 1 \]
We can now define a relationship between velocity error and spacing quick response time. Then we can change the classical method which ing law which has less parameters need to be set and produces a performance. Now there are several reaching laws for a designer to sliding surface in any state for a limited time and to reach the desired the reaching law. The task here is to enable the system to enter the next step is to design a topology SMC for a platoon system is

\[ \Delta_t(s) = \frac{q_1x^2 + q_2}{(q_1 + q_3)x^2 + q_2 + q_4} \Delta_{t-1}(s) + \left( S(s) - \Delta_{t-1}(s) \right) + (q_1 + q_3)\delta(0) - \delta_{t-1}(0) \]

\[ \frac{(q_1 + q_3)x^2 + q_2 + q_4} \]

if \( t \to \infty \), we obtain the string stability polynomial which is \( z = \frac{q_1}{q_2 + q_4} \). Therefore, considering the coefficients they must satisfy \( \frac{q_1}{q_2 + q_4} < 1 \). From these equations it can be known that \( q_1 \) is independent of string stability.

### 4.2 Proposed SMC with Global Information

The purpose of the proposed controller is to make the system converge to the sliding surface \( s_i = 0 \) as soon as possible. Then we can consider the system’s stability of string stability such that \( s_i = s_{i-1} = 0 \). The SMC with the lead vehicle information can be designed in two steps: Step 1, Sliding surface design, which depends on the types of error involved. Step 2, the reaching law design, which must be able to reach the slide surface quickly and ensure there is no chattering effect near the surface. Hence with the elements \( a_{ij} \) and \( p_i \) from the adjacency matrix and adjacency matrix, equation (18) becomes:

\[ s_i(t) = \sum_{j=1,j\neq1}^{N} a_{ij}(\dot{\delta}_{ij} + (x_i - x_j + \sum_{k=1}^{\lceil j \rceil - 1} d_{kj,des})) + p_i(\dot{\delta}_{ij} + \Delta x_i) \]  

\[ + (\dot{x}_i - \dot{x}_0) + (x_i - x_0 + \sum_{k=1}^{\lceil j \rceil - 1} d_{kj,des}) \]

To simplify equation (22), we set:

\[ \Delta x_i = x_i - x_0 + iD_{des} \]

Now equation (22) can be rewritten as:

\[ s_i(t) = \sum_{j=1,j\neq1}^{N} a_{ij}(\dot{\delta}_{ij} + (\Delta x_i - \Delta x_j)) + p_i(\dot{\delta}_{ij} + \Delta x_i) \]  

We can now define a relationship between velocity error and spacing error which is:

\[ \dot{\delta}_{ij} = -\lambda_1(\Delta x_i - \Delta x_j) = -\lambda_1 \delta_{ij} \]

Then equation (24) becomes

\[ s_i(t) = \sum_{j=1,j\neq1}^{N} a_{ij}(1 - \lambda_1) \delta_{ij} + p_i(1 - \lambda_1) \delta_{ij} \]

Using the communication topology matrix \( L + P \) we obtain the platoon system sliding surface:

\[ S(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_{N}(t) \end{bmatrix} = (1 - \lambda_1)(L + P) \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_N \end{bmatrix} \]

The next step is to design a topology SMC for a platoon system is the reaching law. The task here is to enable the system to enter the sliding surface in any state for a limited time and to reach the desired performance. Now there are several reaching laws for a designer to choose, such as constant reaching law, exponential reaching law or power reaching law. In this system we chose the exponential reaching law which has less parameters need to be set and produces a quick response time. Then we can change the classical method which is equation (19) to the proposed topology SMC slide reaching law, which is:

\[ \dot{s}_i(t) = -k \delta_{ij}(t) \]

where the value of \( k > 0 \). Then the collective topological approach law becomes:

\[ \dot{S}(t) = \begin{bmatrix} \dot{s}_1(t) \\ \dot{s}_2(t) \\ \vdots \\ \dot{s}_N(t) \end{bmatrix} = -(1 - \lambda)(L + P) \cdot kS(t) \]

If we take the derivative of (27) and compare it with (29) it gives the full rank matrix. It can cancel \( (1 - \lambda)(L + P) \), so:

\[ \begin{bmatrix} \dot{\Delta x}_1(t) \\ \dot{\Delta x}_2(t) \\ \vdots \\ \dot{\Delta x}_N(t) \end{bmatrix} = -kS(t) \]

If we consider an individual vehicle control mode, the equation becomes:

\[ \Delta x_i = -k \delta_{ij}(t) \]

By comparing the derivative of (31) with (21) we can obtain the input of the controller, which is:

\[ u_{il} = \tau_i \dot{x}_l + \dot{x}_0 + k^2(t - 1) \delta_{il}(t) \]

It is difficult to use the string stability polynomial to analyze stability in the topology platoon system. Thus, let us consider a platoon system with 2 preceding vehicles as the lead vehicles so the string stability polynomial is \( |\delta|_1 = \alpha_1 |\delta_1 - 1| + \alpha_2 |\delta_2 - 1| \). If the preceding lead vehicles become \( N \) as a result, the polynomial will have \( N \) items. We need to analyze the stability by considering different situations and topology structures. To circumvent this problem researchers have used the Lyapunov method. In a typical sliding mode control the stability analysis has been separated into two parts, which are: reaching law stability and sliding surface stability.

#### 4.2.1 Reaching Law Stability:

For a scalar function \( V(x) \) with continuous first-order partial derivatives, if \( V(x) \) is positive definite and the derivative of \( V(x) \) is negative definite, then the equilibrium state of the system is asymptotically stable, and such \( V(x) \) is the system A Lyapunov function. The Lyapunov candidate for the topology platoon system is:

\[ V(t) = \frac{1}{2} \delta(t)^T S(t) \]

The derivative of the Lyapunov candidate equation is:

\[ \dot{V}(t) = -S(t)^T (L + P) \cdot kS(t) \]

From the graph theory we know \( L + P \) is positive. So, it has the property that \( x^T (L + P) x > 0 \). Due to this, \( \dot{V}(t) \) is negative \( (\dot{S}(t) \neq 0, \dot{V}(t) < 0) \). So, when \( t \to \infty \), \( S(t) \) moves towards zero \( (\dot{S}(t) \to 0) \). This shows that this surface can be reached asymptotically.

#### 4.2.2 Sliding Surface Stability

We choose the Lyapunov candidate individual vehicle function as:

\[ V_i = \frac{1}{2} \delta_{ij}^2 \]

which is clearly positive. By taking the derivative of equation (35) we obtain:

\[ \dot{V}_i = \delta_{ij} \dot{\delta}_{ij} = -\lambda \delta_{ij}^2 \]

Now by choosing \( \lambda > 0 \), \( \dot{V}_i \) become negative. Thus, the proposed SMC controller is able to change the matrix into different topology structures by changing its elements to make it more flexible for various vehicle platooning applications. Additionally, the design parameters are fewer than those in the classical SMC.
method. Using global communication by adding V2V communication, the platoon size is expandable. Due to the obvious constraints in communication range within a platooning scenario, it becomes difficult for the vehicle at the end of a platoon to obtain the lead vehicle’s information. To overcome this limitation, the proposed method allows the application of a potentially viable topology structure where vehicles other than the lead and rear vehicles can act as repeaters to their respective consecutive vehicles and pass on the desired information.

5 Simulation for TEV Platform

![Fig. 3: Diagram of platoon system with virtual lead vehicle](image)

In the simulation we have 10 vehicles forming one platoon, all driving at a constant speed of 165.6km/h and all cars must reach 200km/h which is the TEV requirement. This platoon system is towed by a reference vehicle which is considered non-existent, thus it is a virtual vehicle as shown in Fig.3. The topological structure of the communication and the identification of the model parameters of the node vehicle will be generated in the master controller. This simulation used the Matlab System identification toolbox to identify the node model parameter \( \tau \). The master controller broadcasts the control target to every vehicle in the platoon system. The distributed controller of this structure in it uses the classical SMC. In contrast, the following simulation uses the proposed SMC controller with BDL vehicular communication structure as shown in Fig.1. The designed parameters table of this paper can be shown as Table 1.

<table>
<thead>
<tr>
<th>Table 1 Simulation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation parameter</td>
</tr>
<tr>
<td>Classic SMC parameters</td>
</tr>
<tr>
<td>Lumpded delay of vehicle ( i )</td>
</tr>
<tr>
<td>Acceleration of the reference Vehicle (LV)</td>
</tr>
<tr>
<td>Control parameter</td>
</tr>
<tr>
<td>Proposed SMC parameter</td>
</tr>
<tr>
<td>Vehicle velocity</td>
</tr>
<tr>
<td>Spacing error</td>
</tr>
<tr>
<td>Sample time</td>
</tr>
<tr>
<td>Communication delay</td>
</tr>
</tbody>
</table>

5.1 Generate vehicle model parameter

EVs that enter the TEV lane will undergo an acceleration of \( 2m/s^2 \). This accelerating procedure is required to obtain output data from the vehicle. Then this simulation adds a zero mean and 0.1 variance Gaussian white noise to the output data as mechanical noise. The vehicle dynamic model is equation (9) with \( \tau=0.5 \). The simulations are performed using Matlab System Identification toolbox, the result can identify the value of \( \tau=0.4834s \) and the fitting rate is 87.74%.

Fig.4 shows the measured value of the identification which is the red line in the upper figure. The black trace in the upper figure is the output from the tested vehicle which is the input of the identification system. The lower figure shows the error between input and output with a \( \pm0.2 \) magnitude. With this error the position error can be calculated within 0.05m which is too small to be ignored. So, in real time if the fitting rate can reach over 85% the error can be ignored in a platoon system.

5.2 Results at Different Communication Structures

The platoon receives a disturbing signal of \( +2m/s^2 \) and \( -2m/s^2 \) to test its robustness and so it reach the upper speed of the TEV requirement. Therefore, the acceleration of the reference vehicle \( a_r \) can be defined as:

\[
a_r(t) = \begin{cases} 
0 & 0 < t < 5s \\
2 & 5s < t < 10s \\
0 & 10s < t < 20s \\
-2 & 20s < t < 25s \\
0 & t > 25s 
\end{cases}
\] (37)

5.2.1 Conventional SMC Control with Lead Vehicle Information

In this simulation, each vehicle in the platoon system can receive the dynamic information for the reference lead vehicle and the vehicle in front this PLF structure is shown in Fig.1. This has been achieved by assuming all the vehicles have no initial spacing error, velocity error and acceleration error. Overall, to begin with all vehicles operate as normal in the TEV lane and the reference lead vehicle transmits signals as described in the piece-wise function shown in the piece-wise function (37). In this simulation, the sample time is 0.01 s therefore the abscissa unit is 0.01s.

5.2.2 BDL Structure for Platoon System

It can improve the BD structure by adding the lead reference vehicle’s information to each node vehicle as Fig.1. Then this structure can be called the Bidirectional-Lead (BDL) structure. Then the BDL matrix can be derived as:

\[
L_{BD} = L_{BDL} = \begin{bmatrix} 
1 & -1 \\
-1 & 2 \\
& & \ddots \\
& & & -1 \\
& & & & 1 
\end{bmatrix}
\]
6 The features and effects of the proposed SMC

The above sections show the corresponding matrices for BDL platoon typology. Their general structures can also be described using graph theory [27]. This means that the properties of the graph can be transformed into the properties of the corresponding matrix (eigenvalues, eigenvectors, etc.). Note that this description is only based on the topology structures between nodes and it does not consider communication characteristics, such as communication error, packet loss and delay. In the BDL topology structure, the following vehicle can also attain the information of the preceding vehicle. Therefore, in this simulation it applies the BDL structure for research on the designed SMC with the parameters of controller $k^2$, the communication delay and vehicle parameter $\tau_r$.

6.1 Controller coefficient $k^2$

In this paper, we analyze the influence of the controller parameter on the spacing between vehicles, the vehicles’ velocity and the acceleration of vehicles in a platoon. It changes the parameter of the controller to $k^2$ from 1 to 10. Fig.8 shows the max spacing error and acceleration oscillations results in the platoon system with the change of $k^2$. It can be seen that as $k^2$ increases, the space error in the platoon system decreases. However, the oscillation of the acceleration of this system will increase significantly. So Compromising the acceleration oscillations and the space error, $k^2 = 6$ is selected as an optimal value.

Fig. 6: Proposed SMC with BDL structure in platoon system

Fig. 5: Conventional SMC with Lead Vehicle Information

Fig. 7: Parameter relationship
6.2 Vehicle parameter $\tau_i$

$\tau_i$ is the time delay constant of the vehicle longitudinal dynamic system. In fact, $\tau_i$ is the reaction time of the vehicle to the input signal. In the discussion above it chooses $k^2 = 6$ and changes the value of $\tau_i = 0.5$ as [27] set. The $\tau_i$ corresponding to the acceleration and deceleration of the vehicle should be different. So in order to simulate the real-time situation, it will choose the $\tau_i$ with 0.01 variance Gaussian white noise to represent the vehicle acceleration time delay constant and vehicle deceleration time delay constant. In the simulation, with the proposed controller, when $\tau_i = 1$ the system is critically stable. So it knows that with an increase of $\tau_i$, the system overshoot and oscillation of the system are greatly increased. Therefore, the faster the vehicles’ response to acceleration and deceleration, the smaller $\tau_i$ will be. In practice, improving mechanical efficiency, reducing wind resistance, and improving road conditions can reduce the value of the time lag coefficient $\tau_{ave}$.

6.3 Communication delay

6.3.1 Constant communication time delay: In this simulation, it changes the $\tau_i$ for a single vehicle, for which the acceleration is $\tau_i = 0.5$ and the deceleration is $\tau_i = 0.6$ with 0.01 variance Gaussian white noise. Then, we add the communication delay $t = 0.02s$ and $t = 0.04s$ to test the performance of the controller. Since the sampling time in this article is 0.01s, the setting of this time delay is equivalent to a delay of 2 and 4 sampling times. Fig.9 shows the space error results, velocity results and acceleration results with respect to the 0.02s and 0.04s communication time delay. In the comparison figures, there is only a 0.02s communication delay difference. However, the overshoot and oscillation of the system are greatly increased. The system can become unstable and difficult to control. Therefore, communication delay will be a prerequisite for vehicle platoon and vehicular driver-less technology.

6.3.2 Random communication time delay: In order to make the simulation realistic, it changes the communication delay to a random 0.01s to 0.03s i.e. 1 to 3 sampling time delay. According to the results in Fig.10, the controller has been trying to adjust the controlled object to the set parameters. However, it can be seen that the inter-vehicle spacing of the entire vehicle platoon is convergent. The acceleration changes are relatively large, so the hardware requirements for the acceleration of the vehicle will be very high. Therefore, the vehicle equipment in the vehicle platoon control, such as sensors, radar, and power output equipment, is very demanding due to a floating communication delay.

7 CONCLUSION

This paper proposes an SMC controller with a virtual lead vehicle information and V2V and V2I communication (global communication) to control EVs driving along a TEV lane. Moreover, this paper successfully studied the influence of the controller parameters and the communication delay. TEV with this controller is a possible solution for HTS. The main idea of TEV is that EVs drive within a dedicated lane at 200km/h with an inter-vehicle distance of 0.25 car lengths. The short distance is the biggest challenge for every platoon

![Fig. 8: Parameter relationship](image)

![Fig. 9: Communication delay for the platoon system](image)
controller to achieve accuracy and stability. The proposed controller is able to achieve these targets by introducing a proposed SMC with global information for determining the first order vehicle linear system identification. This paper assumes that all vehicles will be able to obtain dynamic information through communication technology. The roadside unit (RSU) has been considered as a way to support the TEV system’s V2I communication while also increasing V2V connectivity. [37] specifies a maximum latency of 100ms for V2V/V2I and a minimum latency of 1ms for autonomous driving. The latency design requirement for 5G-V2C communication technology is 1ms, which meets the need of the scene in this paper. However, due to 5G technology is still in its early stages, such a short delay is not currently possible. As a result, the design of this paper must be based on 5G development and RSU construction. In this simulation, it is shown that the performance with the designed SMC under the BDL structures. Compared to the classical SMC control method with limited V2I communication, the new controller has the flexible use of different communication topology structures. In the study of the features of the SMC controller itself, this controller has strong robustness. With less communication delay and mechanical delay, it can still guarantee the vehicular platoon quality. The designed controller shows heterogeneous string stability in a platoon. Future research may include the effectiveness of the real vehicular platoon scenarios with the proposed SMC controller and use 5G (5th generation mobile networks) for the V2C communication, making the entire system to be more user-centric. The algorithm’s complexity, efficiency, and running time should all be considered from the standpoint of the vehicle platoon algorithm. This part’s running delay can also be added to the total time delay or discussed separately in future work.

ACKNOWLEDGMENT

The research presented in this paper is supported by the Tracked Electric Vehicle (TEV) Project, Philadelphia Scientific (UK) Limited.

8 References


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