ORIGINAL RESEARCH

An improved sliding mode control (SMC) approach for enhancement of communication delay in vehicle platoon system

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Abstract
Vehicle platoon systems are widely recognized as key enablers to address mass-transport. Vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) are two technologies that drive platooning. The inter-vehicle spacing and collaboration velocity in the platoon are important parameters that must be controlled. A new mass-transport system called the Tracked Electric Vehicles (TEV) has been proposed which has reduced the inter-vehicular spacing to only a quarter of the regular car length. This enables mass transport at uniform speed for cars with speed of 200km/h. However, conventional radar based adaptive cruise control (ACC) system fail to control each vehicle in these scenarios. Lately, sliding mode control (SMC) has been applied to control platoons with communication technology but with low speed and without delay. This paper proposes a novel SMC design for TEV using global dynamic information with the communication delay. Also, graph theory has been employed to investigate different V2V communication topology structures. To address issues of node vehicle stability and string stability, Lyapunov candidate function is chosen and developed. Additionally, this paper uses first-order vehicle models with different acceleration/deceleration parameters for simulation validations under communication delay. The results show that this SMC has a significant tolerance ability and meets the design requirements of TEV.

1 | INTRODUCTION

Presently, there are several transport challenges such as congestion, pollution, provision of stress-free travel which are all user-centric problems. One must also be considered that the increasing data use and connectivity will have a greater role to play in the future as well. Moreover, new technologies have to scrutinize the cost, safety and efficiency which are also the three main demands of mass-transport systems as well as highway transportation systems (HTS) [1]. In order to operate platoons in an effective manner, an intelligent transportation system (ITS) is required [2]. Vehicle-to-vehicle (V2V) communication and vehicle-to-infrastructure (V2I) communication are the two key technologies in ITS [3]. At the heart of this is its on-board units (OBU) and sensors (embedded in vehicles and the infrastructure) which are used to collect, relay and share information such as the driver’s control decision [4], vehicle’s position, velocity and acceleration of each car.

The impact that a platoon can have on fuel saving was first studied extensively by the program for the advanced technology highway (PATH) [5]. Nowadays, platoons are being considered by many organizations world-wide such as the Grand Cooperative Driving Challenge (GCDC) in the Netherlands [6], Safe Road Trains for the Environment (SARTRE) in Europe [7] and Energy-ITS in Japan [8]. Despite large programs, individual companies such as the car manufacturer Volvo are also experimenting in this area of research. Volvo drew wide-spread media attention when the company successfully built a highway truck platoon where each truck drives at a speed of 100 km/h with a 1.5 s time gap [9]. With advancements in communication technology such as 4G, 5G, and even the future 6G, vehicle communication can be greatly aided by these technologies. It also
removes distance as a constraint on the topology of inter-vehicle communication. As a result of the topological structure variety, new challenges emerge, which is especially important when considering time delay and packet loss in communications. For example, Alipour-Fanid et al. conducted a comprehensive analysis on the stability and safety of the platoon under the wireless Rician fading channel model and jamming attacks [10]. The problem of centralized control for a platoon of non-identical vehicles under constant time headway strategy (CTHS) is investigated using multi predecessors following (MPF) topology [11]. Muehlebach et al. propose a synthesis procedure for designing the agents’ state estimators and the event triggering thresholds [12]. The optimization algorithm performs the computation of the control input in a control horizon window and ensures that the spacing error takes only positive values [13]. Real-time vehicular data from video traffic detection (VTD) are used for minimizing the travel delay at intersections and a real-time traffic optimization model, based on the SUMO traffic simulation software, is established accordingly [14]. Sau et al. present the state-space representation of the linearised dynamical system [15]. Especially Mohammed et al. aim to improve the greedy traffic aware routing (GyTAR) protocol to support QoS in IoV networks [16].

At the hub of this technology is a platoon controller which must dynamically control all vehicles within the platoon. Mostly a one-dimensional longitudinal control method is the common approach to be applied in platoon controllers. This concept is used to keep a desired distance between neighbour vehicles and maintain a desired velocity for each car within the platoon system [17]. For example, Swaroop et al. [18, 19] applied the classic spring effect to control a platoon. To control the inter-vehicular spacing, the constant distance (CD) and the constant time headway (CTH) [20, 21] methods have been proposed. CTH is preferred when safety is the main concern and CD is typically employed when the goal is efficiency. In parallel to the development of platoon control, there have to be advancements in SMC platoon theory. In 2014, Ji-Wook Kwon and Dongkyoung Chwa [22] proposed a bidirectional control with a sliding mode method. With the development of communication, the node vehicle in a platoon system can obtain more information depending on communication structures. Following this research, in 2019 Li et al. [23] designed an experimental 4-vehicle-platoon system with a global SMC method. A key feature of this work involved V2I communication. In platooning stability, Tae Soo No et al. [24] improved the original Lyapunov stability theorem by using the concept of ‘Expected Spacing Error’ and implemented it in various platooning scenarios. In this work, they have the Lyapunov function (LF) approach by adding V2V communication with the topology structure matrix to demonstrate the stability of the whole platoon. The proposed LF proves the reaching law stability and sliding surface stability.

However, with the development of the communication technology, the vehicular platoon controlled only by radar cannot meet the current traffic demand. One of the latest platooning scenarios is described in tracked electric vehicle (TEV) [25]. The TEV system is a fully automated highway system for Electric Vehicles (EVs) to achieve HTS zero emissions. The TEV lane is designed as a single lane with no restricted access. EVs drive fully automatically where 10 vehicles form one platoon and each vehicle drives at a constant velocity of 200 km/h. The inter-vehicle spacing is only a quarter of a car's length. This short length reduces the overall aerodynamic drag coefficient of all cars including the front car. Such arrangement can save power and in [26] it is claimed that a power saving of 40% for the whole platoon is possible as compared to a scenario in which all 10 cars are driving individually (not in a platoon). The challenge with TEV is the control of short distance at high speed. Currently, SMC has been applied for platoons but without communication topology structures and high speed. To tackle the challenge of reducing the inter-vehicle distance of less than 1 car length, an advanced SMC capable of including V2V is proposed and developed in this paper. The proposed SMC controller can ensure stable vehicle platoon system in terms of enhancing traffic efficiency and road utilization by reducing inter-vehicle distance. The final experimental results verify the effectiveness of the controller which demonstrate that the inter-vehicle distance can be effectively maintained as 0.4–0.6 m under different communication structures. In the case of communication delay, the inter-vehicle distance depends on the value of the communication delay. Therefore, this paper makes the following four main contributions:

- The design of a novel SMC with V2V and V2I communication to control the vehicular distance in a non-homogeneous platoon system.
- In-depth investigation of vehicular communication structures in influencing system stability by employing LF.
- Demonstration of the vehicle system lumped delay in a vehicle dynamic model utilizing system identification method.
- Exploring the features of SMC and its tolerance for communication delays in the simulation.

The rest of the paper is organized as follows: Section 2 presents the background theory and related work. The mathematical model and the stability analysis are described in Section 3 for the platoon system. Section 4 presents the variety of controllers that use SMC and in particular those that can be used for the platoon system. The simulation results for the proposed SMC are shown in Section 5. In Section 6, it studies the features of the SMC and adds the dynamic communication delay in this system. Finally, Section 7 presents the concluding remarks showing the inter-vehicular spacing error (5.5 m) is rapidly decreased with V2V communication SMC controller, compared with the error of (0.5 m) in platoon systems without the lead vehicle information.

2 | BACKGROUND THEORY

In order to characterize the information topology structure in the vehicle platoon system, Zheng et al. [27, 28] use matrices and graph theory. This technology is widely used in communication flow structure. Assuming there is an N-size vehicle
platoon system, we use a directed graph $G_N = (v_N, w_N)$ to describe the information transmission. In this set, $v_N$ denotes the vehicle set and $w_N = v_N \times v_N$ denotes the edge set. In a platoon system, there are two random vehicles $i$ and $(i, j \in N)$. Here, the edge set $w(i, j)$ represents vehicle $j$ that can receive the vehicle’s dynamic information from vehicle $i$.

The definition of a directed path is a sequence of edge set $w(1, 2), w(2, 3), w(3, 4), \ldots, w(k - 1, k), (k \leq N)$. This set is the directed path from node $i$ to node $k$ within the platoon system. Another definition is of the directed spanning tree, assuming that there is at least one vehicle that can acquire information from any other vehicle(s) directly or indirectly. In other words, this vehicle (node) has a directed path towards/for the others. This directed path is known as a directed spanning tree and the vehicle (node) is known as the root of this directed spanning tree. In [8], to control the platoon system, an assumption is made that $G_{N+1}$ has a directed spanning tree with the lead vehicle ($N = 0$) as the root. Usually, the lead vehicle is globally reachable.

Before using a directed graph $G_N$, we provide a brief introduction to how we define our matrices using the following steps:

1. Adjacency matrix ($A = [a_{ij}]$):

   \[
   a_{ij} = \begin{cases} 
   1 & \text{if } (i, j) \in w_N \\
   0 & \text{if } (i, j) \notin w_N 
   \end{cases}
   \]  
   \(a_{ij}\) denotes vehicle $i$ that can acquire the dynamic information from vehicle $j$. $a_{ij} = 0$ indicates there is no self-loop in this graph.

2. Laplacian matrix ($L = [l_{ij}]$):

   \[
   L_{ij} = \begin{cases}  
   l_{ij} = \sum_{j \neq i=1}^{N} a_{ij} \\
   l_{ij} = -a_{ij} 
   \end{cases}
   \]  
   \(l_{ij}\) expresses the vehicle information that vehicle $i$ can obtain through V2V communication or radar detection in the following vehicle.

3. Leader adjacency matrix or pinning matrix:

The leader adjacency matrix (LAM) represents that the followers can obtain the information from the leading vehicle. And so LAM can be defined as:

\[
P_N = \begin{bmatrix} 
   p_1 \\
   \vdots \\
   p_N 
\end{bmatrix}
\]  

Here, $p_i$ can be 0 or 1, if $p_i = 1$ then vehicle $i$ can receive the information from the lead vehicle, otherwise $p_i = 0$.

\[L + P\] matrix is important in the platoon system closed-loop stability study. In [29], it has been shown that if the directed graph $G_{N+1}$ has a directed spanning tree, then $L + P$ is defined as positive. This matrix is used to calculate the stability margin for linear node dynamic. In our new proposed SMC design, we use $L + P$ matrix methodology to prove the string stability of the platooning system.

\section{SYSTEM MODELLING}

In order to model a vehicle platoon for TEV, modelling of an individual vehicle, spacing policies describing the distance between vehicles and string stability must be introduced which is explained in this section.
3.1 Individual vehicle dynamic model

Below are the assumptions made for the vehicle dynamic model:

1. Vehicles only experience rolling friction and aerodynamic force.
2. Vehicles used in this paper do not use gear shift for torque conversion.
3. Only a 1D longitudinal dynamics model is considered.
4. Vehicles are treated as ideal rigid thus ignoring the unbalanced left and right movement of cars.

Based on the above forces acting on a vehicle can be written as:

\[
m \ddot{x} = F_c - F_r - F_a \tag{4}
\]

where \( m \) is the mass of the car, \( \ddot{x} \) is the acceleration, \( F_c \) is the traction force, \( F_r \) is the force of rolling friction and \( F_a \) is the aerodynamic force. Thus, if the term \( (F_c + F_a) \) is equal to \( F_r \), the car is driving at a constant speed. Equation (4) assumes that the wheel rolling resistance for each wheel is the same and that the car is driving in a straight lane without any elevation. If the term \( (F_c + F_a) \) is less than \( F_r \), the vehicle accelerates and if it is greater than \( F_r \) the vehicle decelerates. In order to determine the velocity of the car we use the following equation:

\[v = \dot{x} = Rr\omega_e\tag{5}\]

where \( \omega_e \) is the motor speed, \( R \) is the gear ratio and \( r_e \) is the effective tire radius. With the details of the vehicle drive train technology the derivative of \( \omega_e \), can be expressed as:

\[\dot{\omega}_e = \frac{T_{net} - c_a R^2 r_e \omega_e^2 - R(r_e F_r)}{J_e}\tag{6}\]

where \( \dot{\omega}_e \) represents the acceleration/deceleration of the motor-shaft speed. \( T_{net} \) is the net motor torque, \( c_a \) is the aerodynamic drag coefficient and \( J_e \) is the motor inertia respectively. From Equation (5) and combining it with Equation (6), we have:

\[\ddot{x} = Rr\dot{\omega}_e = Rr_e \left( \frac{T_{net} - c_a R^2 r_e \omega_e^2 - R(r_e F_r)}{J_e} \right) \tag{7}\]

In Equation (7) none of the parameters can be influenced by the driver except the net motor torque \( T_{net} \) which is the required torque to produce \( F_r \) in (4). In EVs, electric drive trains are torque controlled and therefore \( T_{net} \) is a demand value. Consequently, as shown in (7) the demand of \( T_{net} \) results in an acceleration/deceleration represented by \( \ddot{x} \).

Although Equations (4)–(7) are sufficient to study the vehicle dynamics [30] vehicle platooning requires a different set of equations. The most common mathematical model for studying vehicle dynamics of a platoon is called double-integrator model [31] and its equation is:

\[
\begin{aligned}
\dot{x}_i(t) &= v_i(t) \\
v_i(t) &= u_i(t)
\end{aligned} \tag{8}
\]

In this equation, \( u_i(t) \) is the output acceleration in the platoon of each vehicle, where \( i \) ranges from 0 to \( N \) and \( N + 1 \) is the platoon size including the lead vehicle. Despite the fact, that this model considers the details of vehicle dynamics it is not suitable as a true representation of vehicles’ behaviour in a platoon as delays within the platoon system are not reflected. Therefore, many studies have used a ‘lumped’ delay \( \tau_i \) to represent a delay in vehicle dynamics [8, 30, 32] and the fundamental equation used is:

\[\dot{x}_i = \frac{1}{\tau_i} + u_i \tag{9}\]

Adding \( \tau_i \), changes (9) to:

\[\dot{x}_i = A_i x_i(t) + B_i u_i(t), x_i(0) = \begin{bmatrix} \rho_i \\ v_i \\ a_i \end{bmatrix} \]

\[A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1/\tau_i \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ 1/\tau_i \end{bmatrix} \tag{10}\]

where \( \rho_i \) and \( a_i \) are the position and acceleration of vehicle \( i \) respectively.

3.2 Spacing policies

In a platoon system study, the main control target is maintaining the desired space and velocity consistency. When using a CD policy, it is important to maintain a small but safe distance, hence it can achieve higher traffic efficiency. In the CTH policy the distance follows a linear relationship with self-velocity; somehow it is similar to a driver’s behaviour but the distance between the two vehicles is larger. As a result, it cannot deliver efficiency as high as the CD policy, however, is safer than using the CD policy.

3.2.1 CD policy

As Figure 2 shows, the inter-vehicular spacing is defined as:

\[\xi = x_{i-1} - x_i - l_{i-1} \tag{11}\]

where \( \xi \) is the inter-vehicular spacing, \( x_i \) and \( x_{i-1} \) are the vehicle head position for vehicle \( i \) and vehicle \( i - 1 \). \( l_{i-1} \) is the vehicle length of vehicle \( i \). Then the spacing error for vehicle \( i \) can be
SMC CONTROLLERS AND STRING STABILITY

CTH policy

In CTH Policy, the desired spacing in the platoon system is not constant. It has a linear relationship with the velocity, and it has an introduced time-gap parameter $b$ as shown in the following equation:

$$D_{des} = l_{i-1} + b \dot{x}_i$$  \hspace{1cm} (13)

For example, assuming that $b = 2$, the velocity of a vehicle $i$ is 144 km/h and the length of vehicle is 5 m, then with the Equation (13) $D_{des}$ is 83 m without any initial safety distance. In comparison in CD policy, $D_{des}$ would be 3 m which equal to the length of the vehicle $i - 1$. For safety consideration, it increases the desired spacing to a constant value for example, 10 m. As CD always employs a smaller distance compared to CTH, CTH is often termed slack platoon structure and CD is a termed compression structure. The spacing error for the CTH controller can be written as:

$$\delta_i = \epsilon_i - b \dot{x}_i$$ \hspace{1cm} (14)

String stability

In order to determine the stability of a platoon, Swaroop and Hedrick [18] introduced string stability. According to this, a platoon is string stable if and only if the disturbances in the platoon system cannot be amplified towards the platoon. This implies that string stability is defined as the ratio of spacing errors between front and rear adjacent vehicles and this stability is achieved when the spacing error ratio is less than or equal to 1. They analysed the I/O properties and built a string stability polynomial to judge the stability of the platoon system. To study the signal propagation in the string, the norms of signals must be identified first which are defined by Doyle et al. [33].

Assume that we have a vehicle string system as shown in Figure 2. The spacing error $\delta_i$ is the input of vehicle $i$. Every input signal should be bound and the maximum upper bound is $\alpha_i$. If the vehicle platoon system is consistent, the gain has to be the same. So, the error signal will be:

$$\|\delta_i\|_\infty \leq \alpha_i \|\delta_{i-1}\|_\infty$$ \hspace{1cm} (15)

In this equation, to ensure the string does not diverge, $\alpha_i$ should be equal to or less than 1. To consider critical scenarios and using the $Z$-transform, we can change Equation (15) to $\|\delta_i\|_\infty = \alpha_i \|\delta_{i}\|_\infty e^{-1}$. Then string stability polynomial is $\alpha - \alpha_i$. Using the unit cycle of the polynomial so that if the roots are inside the unit cycle it means it is stable, if the roots are on the unit cycle it means it is critical stable or ‘weak string stability’, otherwise it is not stable. Now $\alpha_i$ is the maximum upper bound of the error transfer function, which is,

$$\hat{H}(s) = \frac{\Delta_i(s)}{\Delta_{i-1}(s)}$$ \hspace{1cm} (16)

In a string stability study, these can be summarized if the following condition is met, which have also been validated in [34]:

$$\|\hat{H}(s)\|_\infty \leq 1$$ \hspace{1cm} (17)

Considering the conditions in Equation (17), if the platoon system cannot obtain the information from the lead vehicle, it cannot make the system achieve its string stability. This has been explained more in detail in [35]. In the following section we use SMC controller with lead vehicle information (classical method) and the conditions in Equation (17) to prove string stability. In addition to this, we also show the stability improvement by using global information for SMC control in the following section.

4 | SMC CONTROLLERS AND STABILITY

4.1 | Conventional SMC with lead vehicle information

Using V2V communication gives us the possibility of the position, velocity and acceleration to be broadcast from any neighbouring vehicles. The first step in sliding mode control is to locate the slide surface $S(x)$. The purpose of the slide surface $S(x)$ is to allow the system to finally reach the surface $S(x)$ and move along it, so the slide surface must be stable, that is, when the $x$ error moves along $S(x)$, the error eventually becomes 0, indicating that the equilibrium point has been reached. The error in this paper is the spacing error, and the velocity error is the derivative of the spacing error, which is appropriate for typical feedback control. Hedrick [36] defines a typical slide surface $S_i$ that combines the dynamic information with the lead and preceding vehicles. In this paper, the slide surface has changed to the following
where $\dot{S}_i$ is the $i$th vehicle slide surface in the platoon system, $\delta_i$ is the velocity error of the $i$th vehicle with respect to its preceding vehicle, $d_{j,\text{det}}$ is the fixed distance from vehicle $i$ to the leading vehicle, $q_1$, $q_2$, $q_3$, and $q_4$ are the coefficients for the slide-controller. It is assumed that the length of each vehicle has been ignored. The reaching law for the vehicle in the platoon system can be defined as:

$$\dot{S}_i = -\lambda S_i$$

(19)

where $\lambda > 0$, is the turning parameter. It is used in Equations (18) and (19) to calculate the input of each vehicle in the platoon system:

$$u_i = \frac{1}{q_1 + q_3} \left( q_1 \dot{x}_{i-1} + q_3 \dot{x}_0 - (q_2 + \lambda q_1) \delta_i + (q_4 + \lambda q_3) \cdot (\dot{x}_i - \dot{x}_0) + \sum_{j=1}^{i} d_{j,\text{det}} \right)$$

(20)

where $x_0$, $\dot{x}_0$ and $\dot{x}_0$ are the dynamic information of the lead vehicle, $\delta_i$ is the spacing error between the $i$th vehicle and the $i-1$th vehicle. Using Equations (18) and (20), numerical simulations for the platooning system can be performed and this is shown in the next section. If we use the vehicle model as described in Equation (9), it means it must consider the actuator and communication lags, which then changes to:

$$\tau \frac{d}{dt} u_i + u_i = u_{ij}$$

(21)

where $u_{ij}$ is the input with 'lumped' lags for vehicles. We now need to analyze the stability of this controller. As shown in Equation (15), the string stability polynomial can be used to calculate the stability margin. Moreover, the transfer function $\frac{\Delta}{\delta}$ for the error propagating in the platoon has to be bound to a constant $\alpha$. In this case, in order to construct the transfer function for spacing error it can use $\Delta_i = \Delta_{i-1}$ in $s$-domain, such that,

$$\Delta_i(s) = \frac{\frac{q_1}{q_1 + q_3} s + q_2 + q_4}{(q_1 + q_3) s + q_2 + q_4} \Delta_{i-1}(s)$$

if $\tau \rightarrow \infty$, we obtain the string stability polynomial which is $z = \frac{q_3}{q_2 + q_4}$. Therefore, when considering the coefficients they must satisfy $\frac{q_3}{q_2 + q_4} < 1$. From these equations it can be known that $q_1$ is independent of string stability.

### 4.2 Proposed SMC control with global information

The purpose of the proposed controller is to make the system converge to the sliding surface $\epsilon = 0$ as soon as possible. Then we can consider the system's stability of string stability such that $\epsilon = \epsilon_{i-1} = 0$. The SMC with the lead vehicle information can be designed in two steps: Step 1, Sliding surface design, which depends on the types of error involved. Step 2, the reaching law design, which must be able to reach the sliding surface quickly and ensure there is no chattering effect near the surface. Hence with the elements $a_{ij}$ and $p_i$ from the leader adjacency matrix and adjacency matrix, Equation (18) becomes:

$$\dot{\epsilon}_i(t) = \sum_{j=1, j \neq i}^{N} a_{ij} \left( \delta_{i,j} + \left( \frac{\epsilon_i - \epsilon_j + \sum_{k=1}^{j-1} d_{k,\text{det}}}{\sum_{k=1}^{j-1} d_{k,\text{det}}} \right) \right)$$

(22)

To simplify Equation (22), we set:

$$\Delta \epsilon_i = \epsilon_i - \epsilon_0 + iD_{\text{det}}$$

(23)

Now Equation (22) can be rewritten as:

$$\dot{\epsilon}_i(t) = \sum_{j=1, j \neq i}^{N} a_{ij} \left( \delta_{i,j} + (\Delta \epsilon_i - \Delta \epsilon_j) \right) + p_i \left( \delta_{i,0} + \Delta \epsilon_i \right)$$

(24)

We can now define a relationship between velocity error and spacing error which is:

$$\dot{\delta}_{i,j} = -\lambda \Delta \epsilon_i - \Delta \epsilon_j = -\lambda \delta_{i,j}$$

(25)

Then Equation (24) becomes

$$\dot{\epsilon}_i(t) = \sum_{j=1, j \neq i}^{N} a_{ij} \left( 1 - \lambda \right) \delta_{i,j} + p_i \left( 1 - \lambda \right) \delta_{i,0}$$

(26)

Using the communication topology matrix $L + P$ we obtain the platoon system sliding surface:

$$S(t) = \begin{bmatrix} \epsilon_1(t) \\ \epsilon_2(t) \\ \vdots \\ \epsilon_N(t) \end{bmatrix} = -\frac{1}{1+\lambda} + (L + P) \begin{bmatrix} \Delta \epsilon_1 \\ \Delta \epsilon_2 \\ \vdots \\ \Delta \epsilon_N \end{bmatrix}$$

(27)

The next step is to design a topology SMC for a platoon system is the reaching law. The task here is to enable the system to enter the sliding surface in any state for a limited time and to reach the desired performance. Now there are several reaching laws for a designer to choose, such as constant reaching law, exponential reaching law or power reaching law. In this system
we chose the exponential reaching law which has less parameters need to be set and produces a quick response time. Then we can change the classical method which is Equation (19) to the proposed topology SMC slide reaching law, which is:

\[ \dot{\gamma}_i(t) = -kS_i(t) \quad (28) \]

where the value of \( k > 0 \). Then the collective topological approach law becomes:

\[
\dot{S}(t) = \begin{bmatrix}
\dot{s}_1(t) \\
\dot{s}_2(t) \\
\vdots \\
\dot{s}_N(t)
\end{bmatrix}
= -(1 + \lambda_i) + (L + P) \cdot kS(t) \quad (29)
\]

If we take the derivative of (27) and compare it with (29) it gives the full rank matrix. It can cancel \((1 - \lambda_i)(L + P)\), so:

\[
\begin{bmatrix}
\Delta\dot{s}_1(t) \\
\Delta\dot{s}_2(t) \\
\vdots \\
\Delta\dot{s}_N(t)
\end{bmatrix}
= -kS(t) \quad (30)
\]

If we consider an individual vehicle control mode, the equation becomes:

\[ \Delta\dot{s}_i = -kS_i(t) \quad (31) \]

By comparing the derivative of (31) with (21) we can obtain the input of the controller, which is:

\[ u_i = \tau_i \ddot{x}_i + \dot{x}_i + \dot{k}_i (\tau_i + 1) s_i(t) \quad (32) \]

It is difficult to use the string stability polynomial to analyze stability in the topology platoon system. Thus, let us consider a platoon system with 2 preceding vehicles as the lead vehicles so the string stability polynomial is \( \parallel \delta_i \parallel_{\infty} = \alpha_1 \parallel \delta_{i-1} \parallel_{\infty} + \alpha_2 \parallel \delta_{i-2} \parallel_{\infty} \). If the preceding lead vehicles become \( N \) as a result, the polynomial will have \( N \) items. We need to analyze the stability by considering different situations and topology structures. To circumvent this problem researchers have used the Lyapunov method. In a typical sliding mode control the stability analysis has been separated into two parts, which are: reaching law stability and sliding surface stability.

### 4.2.1 Reaching law stability

For a scalar function \( V(x) \) with continuous first-order partial derivatives, if \( V(x) \) is positive definite and the derivative of \( V(x) \) is negative definite, then the equilibrium state of the system is asymptotically stable, and such \( V(x) \) is the system A Lyapunov function. The Lyapunov candidate for the topology platoon system is:

\[ V'(t) = \frac{1}{2} S(t)^T S(t) \quad (33) \]

The derivative of the Lyapunov candidate equation is:

\[ \dot{V}'(t) = -S(t)^T (L + P) kS(t) \quad (34) \]

From the graph theory we know \( L + P \) is positive. So, it has the property that \( x^T (L + P) \chi > 0 \). Due to this, \( V'(t) \) is negative \((S(t) \neq 0, \dot{V}'(t) < 0)\). So, when \( t \to \infty \), \( S(t) \) moves towards zero \((S(t) \to 0)\). This shows that this surface can be reached asymptotically.

### 4.2.2 Sliding surface stability

We choose the Lyapunov candidate individual vehicle function as:

\[ V_i = \frac{1}{2} \delta_{i,j}^2 \quad (35) \]

which is clearly positive. By taking the derivative of Equation (35) we obtain:

\[ \dot{V}_i = \delta_{i,j} \dot{\delta}_{i,j} = -\lambda \delta_{i,j}^2 \quad (36) \]

Now by choosing \( \lambda > 0 \), \( \dot{V}_i \) becomes negative.

Thus, the proposed SMC controller is able to change the matrix into different topology structures by changing its elements to make it more flexible for various vehicle it becomes difficult for the vehicle at the end of a platoon to obtain the lead vehicle’s information. To overcome this limitation, the proposed method allows the application of a potentially viable topology structure where vehicles other than the lead and rear vehicles can act as repeaters to their respective consecutive vehicles and pass on the desired information.

### 5 SIMULATION FOR TEV PLATFORM

In the simulation we have 10 vehicles forming one platoon, all driving at a constant speed of 165.6 km/h and all cars must reach 200 km/h which is the TEV requirement. This platoon system is towed by a reference vehicle which is considered non-existent, thus it is a virtual vehicle as shown in Figure 3. The topological structure of the communication and the identification of the model parameters of the node vehicle will be generated in the master controller. This simulation used the MATLAB System identification toolbox to identify the node model parameter \( \tau \). The master controller broadcasts the control target to every vehicle in the platoon system. The distributed controller of this structure in it uses the classical SMC. In contrast, the following simulation uses the proposed SMC controller with BDL vehicular communication structure as shown in Figure 1.
The designed parameters table of this paper can be shown as Table 1.

### Table 1 Simulation parameters

<table>
<thead>
<tr>
<th>Simulation parameter</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classic SMC parameters</td>
<td>$q_1, q_2, q_3, q_4$</td>
</tr>
<tr>
<td>Lumped delay of vehicle $i$</td>
<td>$\tau_i$</td>
</tr>
<tr>
<td>Acceleration of the reference vehicle (LV)</td>
<td>$a_r$</td>
</tr>
<tr>
<td>Control parameter</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Proposed SMC parameter</td>
<td>$k^2$</td>
</tr>
<tr>
<td>Vehicle velocity</td>
<td>$v_i$</td>
</tr>
<tr>
<td>Spacing error</td>
<td>$\delta_i$</td>
</tr>
<tr>
<td>Sample time</td>
<td>0.01 s</td>
</tr>
<tr>
<td>Communication delay</td>
<td>0.02–0.04 s, Random</td>
</tr>
</tbody>
</table>

5.1 Generate vehicle model parameter

EVs that enter the TEV lane will undergo an acceleration of $2 \text{ m/s}^2$. This accelerating procedure is required to obtain output data from the vehicle. Then this simulation adds a zero mean and 0.1 variance Gaussian white noise to the output data as mechanical noise. The vehicle dynamic model is Equation (9) with $\tau = 0.5$. The simulations are performed using Matlab System Identification toolbox, the result can identify the value of $\tau = 0.4834$ s and the fitting rate is 87.74%.

Figure 4 shows the measured value of the identification which is the red line in the upper figure. The black trace in the upper figure is the output from the tested vehicle which is the input of the identification system. The lower figure shows the error between input and output with a $\pm 0.2$ magnitude. With this error the position error can be calculated within 0.05 m which is too small to be ignored. So, in real time if the fitting rate can reach over 85% the error can be ignored in a platoon system.

5.2 Results at different communication structures

The platoon receives a disturbing signal of $+2 \text{ m/s}^2$ and $-2 \text{ m/s}^2$ to test its robustness and so it reaches the upper speed of the TEV requirement. Therefore, the acceleration of the reference vehicle $a_r$ can be defined as:

$$a_r(t) = \begin{cases} 
0 & 0 < t < 5 \\
2 & 5 < t < 10 \\
0 & 10 < t < 20 \\
-2 & 20 < t < 25 \\
0 & t > 25
\end{cases}$$

(37)

5.2.1 Conventional SMC control with lead vehicle information

In this simulation, each vehicle in the platoon system can receive the dynamic information for the reference lead vehicle and the vehicle in front this PLF structure is shown in Figure 1. This has been achieved by assuming all the vehicles have no initial spacing error, velocity error and acceleration error. Overall, to begin with all vehicles operate as normal in the TEV lane and the reference lead value is 0.5. For other vehicles in the platoon system, we assume they have 10 similar random values distributed by the system. Figure 5 gives the spacing error, velocity and acceleration respectively of the platoon system. The result of this controller
in the platoon system has an approximate 0.5 error band, 5 s settling time and an acceleration overshoot of nearly 50%. The spacing error and velocity for the platoon system are within the acceptable ranges. Note that the setting of acceleration of the vehicle has an upper limit.

5.2.2 BDL structure for platoon system

It can improve the BD structure by adding the lead reference vehicle’s information to each node vehicle as Figure 1. Then this structure can be called the bidirectional-lead (BDL) structure. Then the BDL matrix can be derived as:

\[
L_{BD} = L_{BDL} = \begin{bmatrix}
1 & -1 & & & \\
-1 & 2 & \ddots & & \\
& \ddots & \ddots & -1 & \\
& & -1 & 1 & \\
\end{bmatrix}
\]

\[
P_{BDL} = \begin{bmatrix}
1 & & & & \\
& 1 & & & \\
& & \ddots & & \\
& & & 1 & \\
\end{bmatrix}
\]

Figure 6 shows the results of the BDL communication system for the platoon system. The node vehicle can obtain the lead vehicle’s dynamic information directly as shown in this structure. It uses the model as shown in Equation (32) and the parameters are set to: \( \lambda_1 = 0.5 \) and \( k^2 = 6 \). The features and effects of this parameters will be shown in next section. The maximum velocity of the last vehicle is less than 205.2 km/h. From these results, it has successfully shown that the proposed SMC with BDL structure has improved the conventional SMC structure keeping the initial conditions same. A more significant and essential point is that in the case of directly or indirectly obtain dynamic information from the lead vehicle, the proposed SMC can be more conveniently used in a variety of communication topology structures. Also it is gaining greater information from vehicles will lead to less spacing errors between vehicles. Consequently, this design method is more suitable for the requirement of the vehicle platoon control.

6 THE FEATURES AND EFFECTS OF THE PROPOSED SMC

The above sections show the corresponding matrices for BDL platonic typology. Their general structures can also be described using graph theory [27]. This means that the properties of the graph can be transformed into the properties of the corresponding matrix (eigenvalues, eigenvectors, etc.). Note that this description is only based on the topology structures between nodes and it does not consider communication characteristics, such as communication error, packet loss and delay. In the BDL topology structure, the following vehicle can also attain the information of the preceding vehicle. Therefore, in this simulation it applies the BDL structure for research on the designed SMC with the parameters of controller \( k^2 \), the communication delay and vehicle parameter \( \tau \).
changes the value of $\tau_j = 0.5$ as [27] set. The $\tau_j$ corresponding to the acceleration and deceleration of the vehicle should be different. So in order to simulate the real-time situation, it will choose the $\tau_j$ with 0.01 variance Gaussian white noise to represent the vehicle acceleration time delay constant and vehicle deceleration time delay constant. In the simulation, with the proposed controller, when $\tau_j = 1$ the system is critically stable. It also knows that, from the Figure 8, with an increase of $\tau_j$, the system overshoot and oscillation of the max space error, velocity and acceleration will increase. Therefore, the faster the vehicles’ response to acceleration and deceleration, the smaller $\tau_j$ will be. In practice, improving mechanical efficiency, reducing wind resistance, and improving road conditions can reduce the value of the time lag coefficient $\tau$.

6.2 Communication delay

6.2.1 Constant communication time delay

In this simulation, it changes the $\tau_j$ for a single vehicle, for which the acceleration is $\tau_j = 0.5$ and the deceleration is $\tau_j = 0.6$ with 0.01 variance Gaussian white noise. Then, we add the communication delay $t = 0.02$ s and $t = 0.04$ s to test the performance of the controller. Since the sampling time in this article is 0.01 s, the setting of this time delay is equivalent to a delay of 2 and 4 sampling times. Figure 9 shows the space error results, velocity results and acceleration results with respect to the 0.02 s and 0.04 s communication time delay. In the comparison figures, there is only a 0.02 s communication delay difference. However, the overshoot and oscillation of the system are greatly increased. The system can become unstable and difficult to control. Therefore, communication delay will be a prerequisite for vehicle platoon and vehicular driver-less technology.

6.2.2 Random communication time delay

In order to make the simulation realistic, it changes the communication delay to a random 0.01–0.03 s, that is, 1 to 3 sampling time delay. According to the results in Figure 10, the controller has been trying to adjust the controlled object to the set parameters. However, it can be seen that the inter-vehicle spacing of the entire vehicle platoon is convergent. The acceleration changes
are relatively large, so the hardware requirements for the acceleration of the vehicle will be very high. Therefore, the vehicle equipment in the vehicle platoon control, such as sensors, radar, and power output equipment, is very demanding due to a floating communication delay.

7 | CONCLUSION

This paper proposes an SMC controller with a virtual lead vehicle information and V2V and V2I communication (global communication) to control EVs driving along a TEV lane. Moreover, this paper successfully studied the influence of the controller parameters and the communication delay. TEV with this controller is a possible solution for HTS. The main idea of TEV is that EVs drive within a dedicated lane at 200 km/h with an inter-vehicle distance of 0.25 car lengths. The short distance is the biggest challenge for every platoon controller to achieve accuracy and stability. The proposed controller is able to achieve these targets by introducing a proposed SMC with global information for determining the first order vehicle linear system identification. This paper assumes that all vehicles will
be able to obtain dynamic information through communication technology. The roadside unit (RSU) has been considered as a way to support the TEV system’s V2I communication while also increasing V2V connectivity. [37] specifies a maximum latency of 100 ms for V2V/V2I and a minimum latency of 1 ms for autonomous driving. The latency design requirement for 5G-V2V communication technology is 1 ms, which meets the needs of the scene in this paper. However, due to 5G technology is still in its early stages, such a short delay is not currently possible. As a result, the design of this paper must be based on 5G development and RSU construction. In this simulation, it is shown that the performance with the designed SMC under the BDL structures. Compared to the classical SMC control method with limited V2I communication, the new controller has the flexible use of different communication topology structures. In the study of the features of the SMC controller itself, this controller has strong robustness. With less communication delay and mechanical delay, it can still guarantee the vehicular platoon quality. The designed controller shows heterogeneous string stability in a platoon. Future research may include the effectiveness of the real vehicular platoon scenarios with the proposed SMC controller and use 5G (5th generation mobile networks) for the V2V communication, making the entire system to be more user-centric.

CONFLICT OF INTEREST
All authors declare that they have no conflicts of interest.

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DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the corresponding author (H.W.) upon reasonable request.

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